# Big problems, Big computers and a Big man:

## Towards ab initio hadron physics with basis light-front quantization

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Nuclear Theory in the Supercomputing Era
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"For distinguished contributions to our understanding of nuclear and hadronic structure, creating new computational frameworks for the many-body problem, and for fostering international collaboration in science."

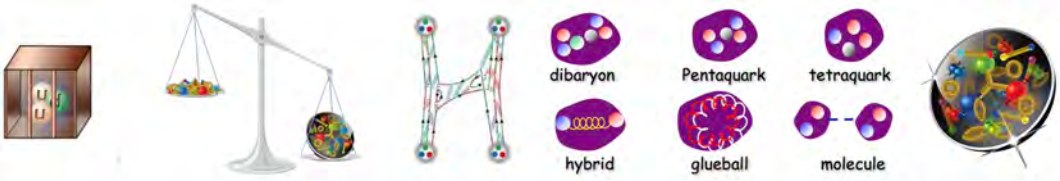
Citation for his fellowship of American Association for the Advancement of Science (2021)

My first problem from Prof. Vary: solve for the structure of the proton


Disclosure: I was not able to solve the problem -- but I still got my Ph.D. at the mercy of Prof. Vary

# Big questions with hadron structures

- Why is there no free quark or gluon observed in nature?
- Why is the proton much heavier than its constituent quarks?
- What is the origin of the nuclear force?
- What are the structures of the exotic hadrons?
- What are the distributions of quarks and gluons inside the hadrons?



# A brief (and biased) history of hadron structures

- Proton magnetic moment (1930s)
- Elastic scattering of proton (1950s)
- Quark model (early 1960s)
- Chiral symmetry breaking (1960s)
- Deep inelastic scattering (late 1960s)
- Quantum chromodynamics (1970s)

# 50 Years of Quantum Chromodynamics

**Franz Gross[a,1,2], Eberhard Klempt[b,3],**

**Stanley J. Brodsky[c,4], Andrzej J. Buras[c,5], Volker D. Burkert[c,1], Gudrun Heinrich[c,6], Karl Jakobs[c,7], Curtis A. Meyer[c,8], Kostas Orginos[c,1,2], Michael Strickland[c,9], Johanna Stachel[c,10], Giulia Zanderighi[c,11,12],**

## 5.4 Light-front quantization

**James Vary, Yang Li, Chandan Mondal and Xingbo Zhao**

In this section, we discuss non-perturbative light-front Hamiltonian quantization methods. We primarily focus on introducing the Hamiltonians for QED and QCD derived in the light-cone gauge (for extensive reviews, see Refs [792, 908]). We introduce methods of solution and results for mesons and baryons. We focus on the Discretized Light Cone Quantization (DLCQ) and Basis Light Front Quantization (BLFQ) methods due to their ability to include gluons and sea quarks dynamically.

Light-front quantization is the natural language for

$$H = \sum_i \frac{p_i^{\perp 2} + m_i^2}{x_i} + H_{int} + \lambda_{CM} H_{CM}\,. \qquad (5.4.2)$$

Here, the sum is over all partons and $m_i$ is the mass of the $i^{th}$ parton. The role of the Lagrange multiplier term ensures factorization of the state vector's transverse component into an internal, boost invariant, component times a center of mass (CM) component [911].

We note that this eigenvalue problem applies to systems with arbitrary baryon number so that, for example, it applies to atomic nuclei as well. An eigenstate of a system can be written in terms of a Fock-space expansion over sectors with $N$-partons as

$$|P, \Lambda\rangle = \sum_N \sum_{\lambda_1,\ldots,\lambda_N} \int \frac{\prod_{i=1}^N \mathrm{d}x_i \mathrm{d}p_i^\perp}{[2(2\pi)^N]^2 \sqrt{x_1 x_N}} \delta(1 - \sum_{i=1}^N x_i)$$

# QCD as the next frontier

[Reviews: 50 years of QCD, 2212.11107]

- Electron-ion colliders: eRHIC, EicC, LHeC [AbdulKhalek:2021gbh, Anderle:2021wcy, LHeC:2020van]
- Electron-positron colliders: superKEKB, Super $\tau$-charm facility (STCF) [Peng:2020orp]
- Heavy ion colliders: NICA, HIAF, FAIR

# Wilson's idea: put QCD into big computers

**AB INITIO QUANTUM CHEMISTRY:**
**A SOURCE OF IDEAS FOR LATTICE GAUGE THEORISTS**

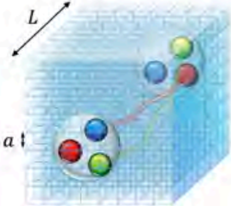
Ken Wilson, 1989
cf. Ken Wilson, 2005

Kenneth G. WILSON

The Ohio State University, Department of Physics, 174 W. 18th Avenue, Columbus, OH 43210 USA

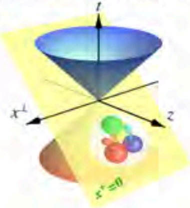
Ab initio quantum chemistry is an emerging computational area that is fifty years ahead of lattice gauge theory, a principal competitor for supercomputer time, and a rich source of new ideas and new approaches to the computation of many fermion systems. An overview of the history, current prospects and future frontiers of quantum chemistry is given, with special emphasis on lessons for lattice gauge theory. Particular reference is given to the role of Gaussian basis functions (in place of grids) and analytic (as opposed to Monte Carlo) methods. The main recommendation to lattice gauge theorists is for greater emphasis on infinite momentum frame studies, using Gaussian basis functions.

Lattice gauge theory





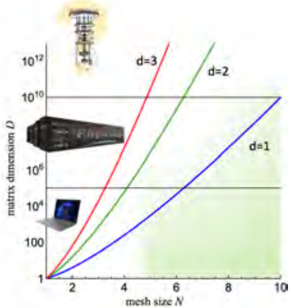
Light-front Hamiltonian theory



# LFQCD as a strong-coupling relativistic quantum many-body problem

$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{x_i} + U_i + \sum_{ij} V_{ij}^{(\text{QCD})} - \delta_{ij} U_i$$

- Exponential wall: $\dim \mathcal{H} = N^{dN}$ [Hornbostel:1988fb, Anand:2020gnn]
- Moore's law, Quantum advantage? [Kreshchuk:2020dla]
- Semi-classical first approximation

# Jacob's ladder


# Semiclassical LFQCD

[Review: Brodsky:2014yha, Chabysheva:2012fe, Li:2021jqb, deTeramond:2021yyi]

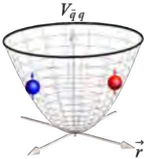
$$\Big(\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + U\Big)\psi(x, \vec{k}_\perp) = M^2\psi(x, \vec{k}_\perp)$$

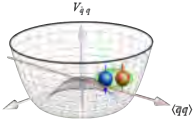
*factorization ansatz* $\Downarrow$ $\psi = \varphi(\vec{\zeta}_\perp)\chi(x)$

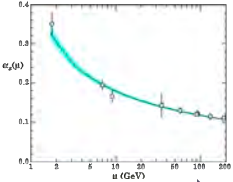
$$\Big[-\nabla^2_{\zeta\perp} + U_\perp(\vec{\zeta}_\perp)\Big]\varphi(\vec{\zeta}_\perp) = M_\perp^2\varphi(\vec{\zeta}_\perp), \quad \Big[\frac{\cancel{m_q^2}}{x} + \frac{\cancel{m_{\bar{q}}^2}}{1-x} + U_\parallel(\tilde{z})\Big]\chi(x) = M_\parallel^2\chi(x)$$

where $\vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$.

$U_{\text{eff}}(\zeta_\perp)$



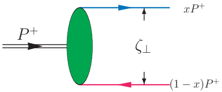

$\Lambda_{\text{QCD}}\sim 0.3$ GeV — $\Lambda_\chi \sim 1$ GeV — $\mu \gg 1$ GeV

# Holographic LFQCD

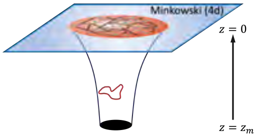
[Review: Brodsky:2014yha]

Light-front holography provides a unique correspondence between semiclassical LFQCD and semiclassical fields in 5D anti-de Sitter space

| Strong coupling | | Weak coupling |
|---|---|---|
| QCD | AdS/QCD | ? |
| ↑ | | ↑ |
| Semiclassical LFQCD | LF holography | Semiclassical fields in $AdS_5$ |
| ↑ | | ↑ |
| N=4 SUSY Yang-Mills theory (CFT) | AdS/CFT | Type IIB string theory in $AdS_5 \times S^5$ |




| | | |
|---|---|---|
| semiclassical LFQCD | $\leftrightarrow$ | semiclassical field theory in AdS |
| $\zeta_\perp = \sqrt{x(1-x)}r_\perp$ | $\leftrightarrow$ | fifth coordinate $z$, |
| LF amplitude | $\leftrightarrow$ | string amplitude |
| confining potential | $\leftrightarrow$ | dilation field $\Phi$ |
| $L^2 - (J-2)^2$ | $\leftrightarrow$ | $(\mu R)^2$ |
| form factors | $\leftrightarrow$ | form factors |



# Confinement & chiral symmetry breaking in holographic QCD

$$S = -\int \mathrm{d}^5x\sqrt{-g}e^{-\Phi(z)}\mathrm{Tr}\Big\{\big|DX\big|^2 - V[X] + \frac{1}{4g_5^2}F^2\Big\},$$

- Confinement: breaking of the conformal symmetry at the IR ($z \to \infty$) [Erlich:2005qh]
  - *Dilaton field* $\Phi(z)$
  - *Soft-wall model reproduces Regge trajectory:* $\Phi(z\to\infty) \sim z^2 \Rightarrow U(z) \sim z^2$ [Karch:2006pv]
- Chiral symmetry breaking: Higgs condensate $\langle X\rangle = \frac{1}{2}\chi(z)$ [Gherghetta:2009ac]
  - *Scalar field* $X$ *is dual to* $\bar{q}q$
  - *Higgs potential:* $V[X] = -m_X^2|X|^2 + \kappa|X|^4$, $m_X^2 = -3$ [Kapusta:2010mf]
  - *Condensate at CFT boundary (*$z\to 0$*):* $\chi(z) \sim m_q z + \Sigma z^3 \Rightarrow U(z) \sim -z^4$ [Klebanov:1999tb]
- $\chi$SB & confinement dictate the UV & IR behaviors of $U(z)$, respectively [Li:2022ytx]

# Chiral symmetry breaking on the light front

## $\chi$SB is encoded in the hadronic LFWFs

[Casher:1974xd, Burkardt:1996pa, Wu:2003vn, Beane:2013oia, Beane:2015ufo]

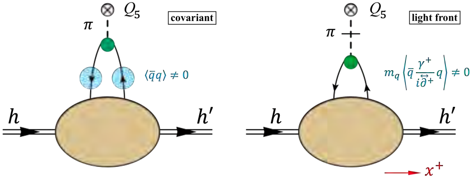
- Light-front vacuum & zero modes: $Q_5|0\rangle_{\rm LF} = 0$ [Brodsky:2010xf]
- The most general covariant structure of pion LFWFs including zero modes [Carbonell:1998rj]

$$\psi_{s\bar{s}/P}(x, \vec{k}_\perp) = \bar{u}_s(p_1)\Big[\gamma_5\,\phi_1(x, k_\perp) + \hat{f}_\chi \frac{\gamma_5 \not{\omega}}{\omega\cdot p}\phi_2(x, k_\perp)\Big] v_{\bar{s}}(p_2),$$

- Partially conserved axial-vector current (PCAC) leads to a light-front chiral sum rule: [Li:2022ytx]

$$\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \frac{k_\perp^2}{x(1-x)} \psi^{(0)}_{\uparrow\downarrow-\downarrow\uparrow/P}(x, \vec{k}_\perp) = 0 \quad \overset{\text{LFH}}{\Rightarrow} \quad f_P \nabla_\perp^2 \varphi_P(\zeta_\perp = 0) = 0$$

- Gell-Mann-Oakes-Renner relation: $f_P^{(0)2} M_P^2 = 2 m_q g_P^{(0)} + O(m_q^2)$, where $g_P = \langle 0 | j_5 | P(p)\rangle$
- Similar idea in DSE/BSE: AvWTI leads to $f_\pi E_\pi(k, 0) = B_q(k^2), \cdots$ [Maris:1997hd]



# Semiclassical LFQCD

[Review: Brodsky:2014yha]

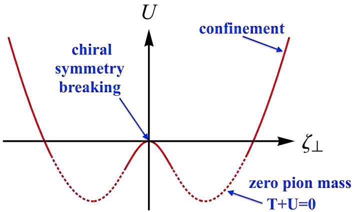
$$\Big[-\nabla_\perp^2 + U(\zeta_\perp)\Big]\varphi_h(\zeta_\perp) = M_h^2\varphi_h(\zeta_\perp), \qquad \nabla_\perp^2\varphi_P(\zeta_\perp=0)=0, \quad U(\zeta_\perp) \to \begin{cases} \zeta_\perp^2 & \zeta_\perp\to\infty \\ -\zeta_\perp^4 & \zeta_\perp\to 0 \end{cases}$$

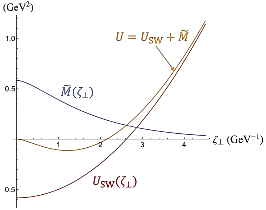
$$\Big[\frac{m_q^2}{x} + \frac{m_{\bar q}^2}{1-x} + U_\parallel\Big]\chi(x) = M_\parallel^2\chi(x),$$

The pion: $f_\pi \neq 0, M_\pi = 0, \;\Rightarrow\; U(\zeta_\perp=0)=0$ (cf. $U_{\text{Higgs}} \sim -\zeta_\perp^4 \to 0$),

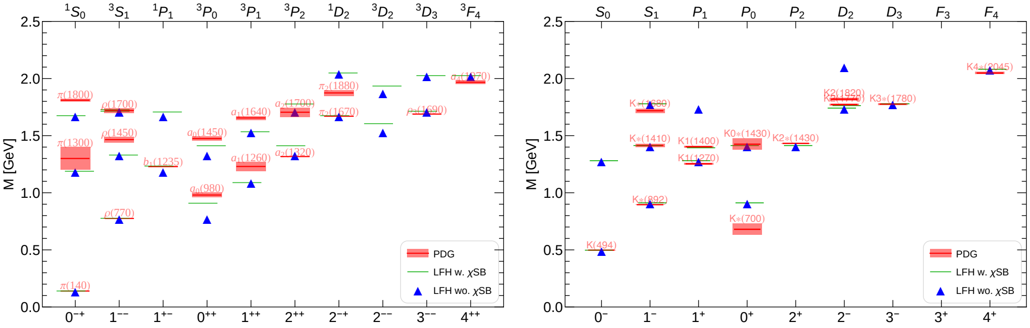
- Transverse: Mexican hat potential [Li:2022ytx]
- Longitudinal: Li-Maris-Zhao-Vary potential $U_\parallel = -\sigma^2\partial_x(x(1-x)\partial_x)$ similar to 't Hooft model, Gell-Mann Oakes Renner relation

[Li:2021jqb, cf. deTeramond:2021yyi, Ahmady:2021lsh, Ahmady:2021yzh, Lyubovitskij:2022rod, Ahmady:2022dfv]



# Spectra of unflavored light mesons




- Semiclassical LFQCD with confinement and with/without chiral symmetry breaking

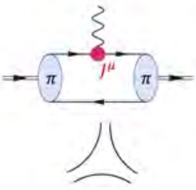
[Brodsky:2014yha, Li:2021jqb]

- State identification with relativistic quantum numbers, $n, m, l, J, S \to J^{PC}(N\ ^{2S+1}L_J)$
  - $\mathrm{C} = (-1)^{m+l+S}, (-1)^J \mathrm{P} = (-1)^{m+S+1}$

# Pion electromagnetic form factor & parton distribution function

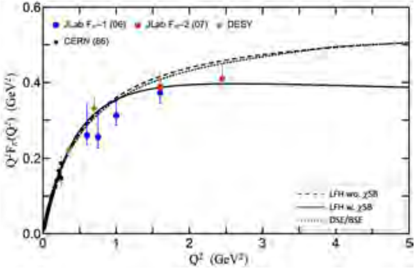
- Holographic current: Veneziano amplitude [Grigoryan:2007wn, deTeramond:2018ecg, Liu:2019vsn]
- $\chi$SB incorporates high-twist contributions: Brodsky-Farrar counting rule ($\tau = N$) [Li:2022ytx]

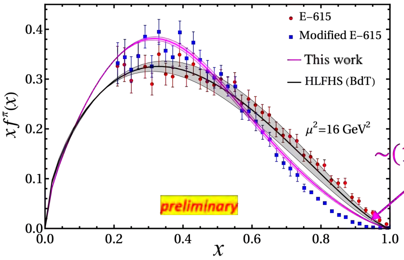
$$F_\pi(q^2) = N\Big\{\underbrace{F_{\tau=2}(q^2)}_{[q\bar{q}]} + \underbrace{F_{\tau=3}(q^2)}_{[q\bar{q}g]} + c_1\,\underbrace{F_{\tau=4}(q^2)}_{[q\bar{q}q\bar{q}]} + \cdots\Big\}$$




- Generalized parton distribution function (GPD)

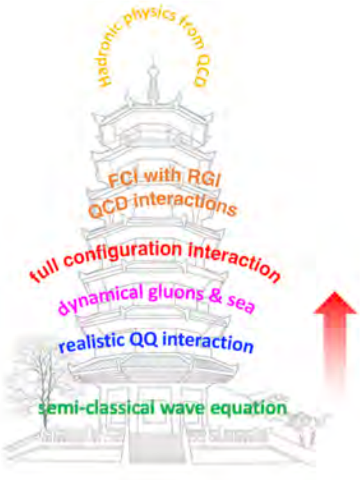
$$F_\pi(q^2) = \int \mathrm{d}x\, H_\pi(x, t = q^2), \quad f_v(x) = H_\pi(x, t = 0)$$



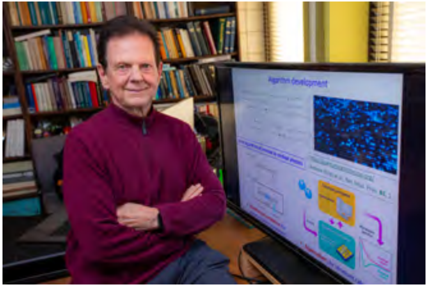




Global analysis:
$\sim (1-x)^{1.0\sim1.3}$
[Barry:2021osv]

# Jacob's tower



# Big computers to solve big problems



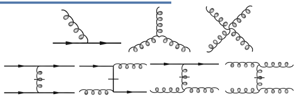
Yang, we are not much smarter than those big names in the 70s, but we certainly have much bigger computers, and they get bigger.

# Lessons from quantum many-body theory

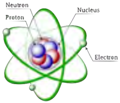
[Review: Brodsky:1997de, cf. Wilson:1994fk]

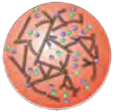
Light-front QCD in
light cone gauge $A^+ = 0$

$$H_{\rm LFQCD} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{x_i} + \sum_{ij} V_{ij}^{\rm (QCD)}$$



| Atoms | Nuclei | Hadrons |
|---|---|---|

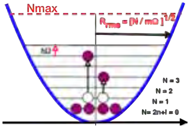


| Atoms | Nuclei | Hadrons |
|---|---|---|
| Coulomb interaction | NN, NNN interactions | QCD interactions |
| ↓ | ↓ | ↓ |
| Bohr Model | Shell Model | Light Front Holography |
| $\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$ | $\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2}r^2\right)\psi = E\psi$ | $\left(\frac{k_\perp^2 + m_q^2}{x(1-x)} + \kappa^4\zeta_\perp^2 + U_\parallel\right)\psi = M^2\psi$ |

It is important to preserve all kinematical symmetries under the truncation

- Discretized momentum basis (DLCQ) → Lagrange mesh (continuum method)
  [Hornbostel:1988fb, Trittmann:1997xz, Lamm:2013oga, Li:2015iaw]
- Basis light-front quantization (BLFQ) [Vary:2009gt]




[Review: Barrett:2013nh (NCSM)]

# Basis light-front quantization

- Configuration interaction: [Vary:2009gt]

$$H|\psi\rangle = M^2|\psi\rangle \quad \Rightarrow \quad \sum_j H_{ij}c_j = M^2 c_i$$

where, $H_{ij} = \langle\phi_i|H|\phi_j\rangle$, $c_i = \langle\phi_i|\psi\rangle$.

- Single-particle basis $\{|\phi_i\rangle\}$ is chosen to preserve all kinematical symmetries of QCD
  - 3 Boosts + 1 rotation:

$$\phi_{nml}(\vec{x}_\perp, x^-) = \mathcal{N} e^{\frac{i\pi l x^-}{L} + im\theta_\perp - \frac{\rho^2}{2}} \rho^{|m|} L_n^{|m|}(\rho), \quad (\rho = \Omega\sqrt{l/K}x_\perp)$$

  - Longitudinal direction: discretized momentum basis, Jacobi polynomials on a $k$-simplex [Chabysheva:2013oka, Li:2017mlw]
  - Other symmetries of QCD
- Consistent with holographic LFQCD
- Large sparse matrix eigenvalue problem: suitable for modern HPC (& quantum computing?)

# Finite basis truncations in BLFQ

- Energy cutoffs/regularization:

$$\sum_i [2n_i + |m_i| + 1] \leq N_{\max}, \quad \sum_i m_i + s_i = m_J, \quad \sum_i l_i = K.$$

- UV & IR regulators:

$$\Lambda_{\text{UV}} = \Omega\sqrt{N_{\text{max}}}, \qquad \Lambda_{\text{IR}} = \Omega/\sqrt{N_{\text{max}}}$$

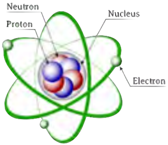
- Truncation on the many-body basis:
  - Fock sector truncation
  - $N_{\max}$-truncation, $K$-truncation
  - Coupled cluster/coherent basis [Hiller:2016itl, More:2014rna]
  - DMRG, matrix product states, tensor network, ...
- Variational theorem
- Continuum limit (need a proof):

$$N_{\max} \to \infty, \quad K \to \infty, \quad A \to \infty$$

# Semi-classical first approximation: first order

$$H = \sum_i \frac{p_{i\perp}^2 + m_i^2}{x_i} + U_i^{(0)} + \sum_{ij} \alpha_s U_{ij}^{(1)} + \sum_{ij} V_{ij} - U_{ij}$$

**Atoms**




Bohr Model

$$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$$

↓

L-S coupling, …

**Nuclei**

Shell Model

$$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$$

↓

L-S coupling, …

**Hadrons**

Light Front Holography

$$\left(\frac{\vec{k}_\perp^2}{x(1-x)} + \kappa^4 \zeta_\perp^2 + U_{||}\right)\psi = M^2\psi$$

↓

One-gluon exchange

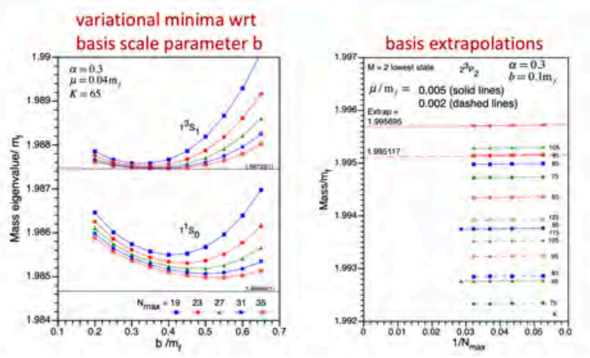
# Positronium

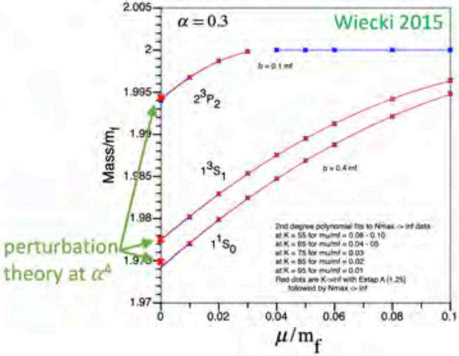
[Wiecki:2014ola]

- Bloch-Wilson interaction: perturbative solution to the OSL effective Hamiltonian [Krautgartner:1991xz]

$$\alpha_s U^{(1)} = \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')$$

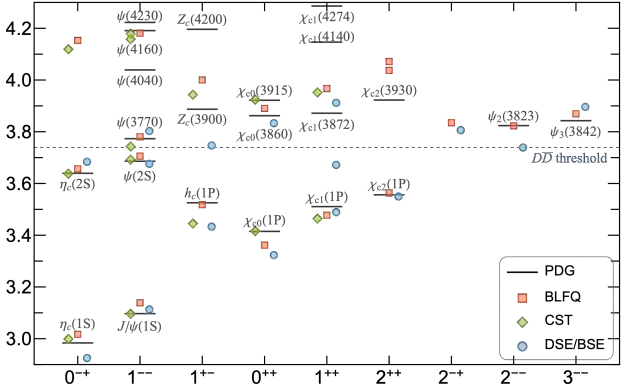
- Benchmark (LFQED): striking agreement with perturbative QED results up to $O(\alpha^4)$:
  $H = T + H_{\rm Col} + H_{\rm Dar} + H_{\rm rel} + H_{\rm LS} + H_{\rm SS}$ [cf. Lamm:2016djr]

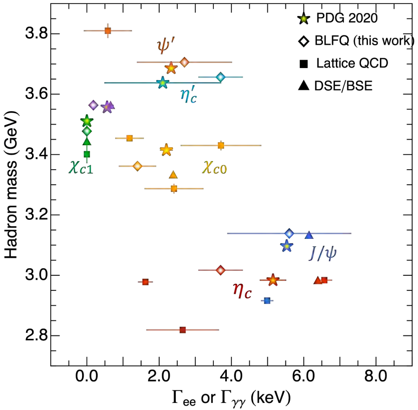


# Application to charmonium

[Li:2017mlw, Li:2021ejv]



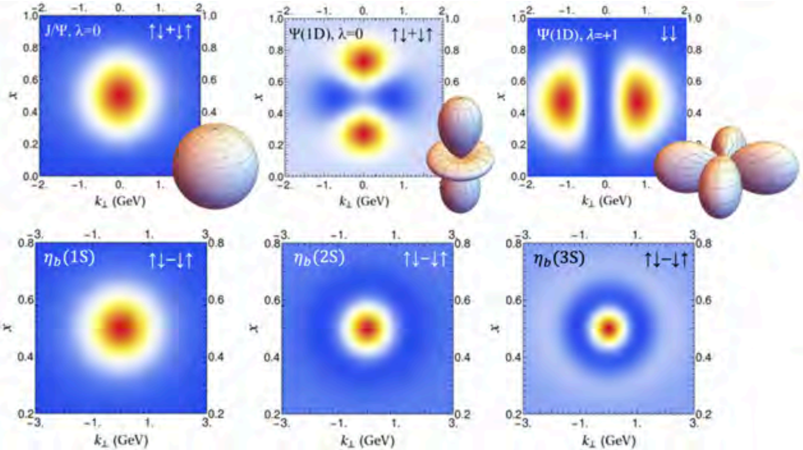

[Gross:2022hyw]

- Two free parameters $(m_c, \kappa)$, rms deviation: $30$ MeV
- Good agreement with the PDG data for both the masses and the widths

# Quarkonium light-front wave functions

angular excitations

radial excitations

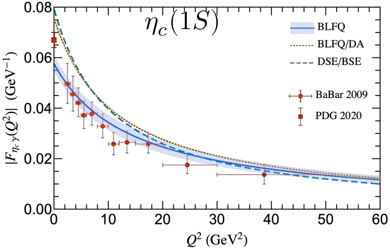


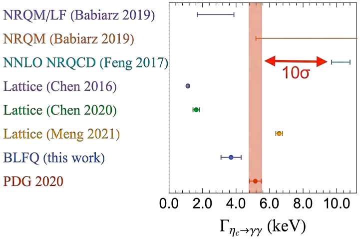

[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

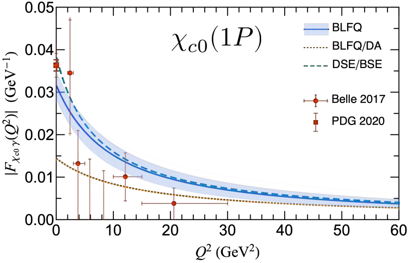
# Parameter-free predictions for the diphoton transition form factors

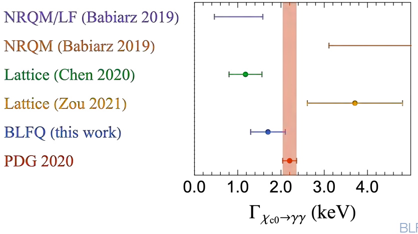
Very challenging for NRQCD, Lattice QCD and quark models

[Feng:2015uha, Feng:2017hlu, Meng:2021ecs, Colquhoun:2023zbc, Christ:2022rho]



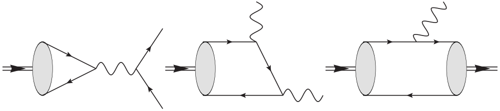



# All radiative transitions

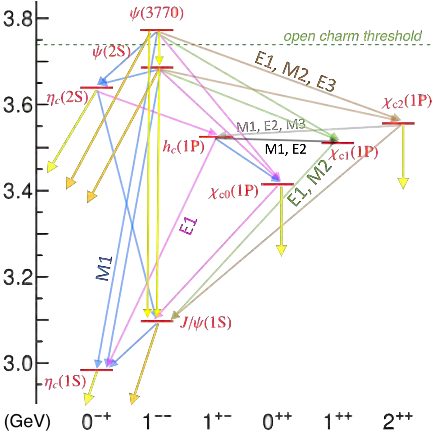
Leptonic and radiative transitions probe the fundamental structure of the hadrons:

[Review: Barnes & Yuan, Int. J. Mod. Phys. A 2009]



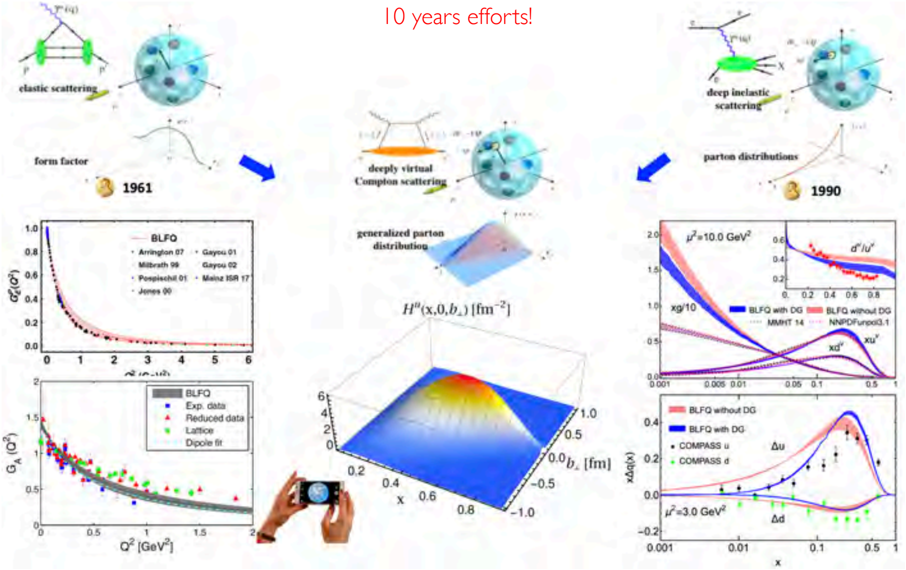
| decay width (keV) | | $\Gamma_{ee}$ | $\Gamma_{\gamma\gamma}$ | $\Gamma_{\eta_c\gamma}$ | $\Gamma_{J/\psi\gamma}$ |
|---|---|---|---|---|---|
| $\eta_c$ | PDG | - | 5.15(35) | | |
| | BLFQ | - | 3.7(6) | | |
| $J/\psi$ | PDG | 5.53(10) | - | 1.6(4) | |
| | BLFQ | 5.7(1.9) | - | 2.6(1) | |
| $\chi_{c0}$ | PDG | - | 2.1(1.6) | - | $15(1)\times10^3$ |
| | BLFQ | - | 1.9(4) | - | in progress |
| $\chi_{c1}$ | PDG | - | - | - | 288(16) |
| | BLFQ | - | - | - | in progress |
| $\vdots$ | | | | | |

[YL, PRD '17; Li, PRD '18; Wang, in progress]



# Application to nucleon structures

[Mondal:2019jdg, Xu:2021wwj, Xu:2022abw, Liu:2022fvl]

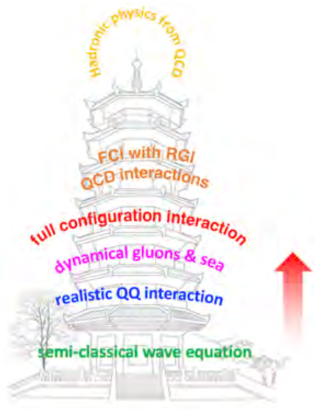


# Basis Light-front Quantization

[Review: 50 years of QCD Sect. 5.4, 2212.11107]

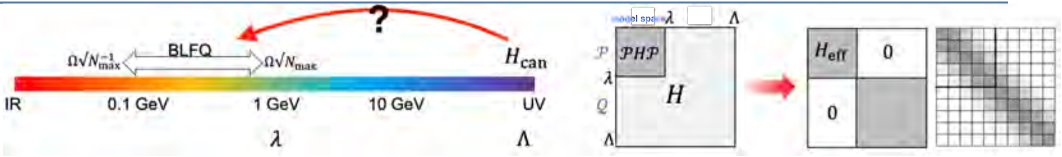
- Semi-classical LF wave equations: [Li:2021jqb, Li:2022izo, Li:2022ytx, cf. deTeramond:2021yyi, Ahmady:2021lsh, Ahmady:2021yzh, Lyubovitskij:2022rod, Ahmady:2022dfv]
- Realistic QQ interaction:
  - QED: [Honkanen:2010rc, Zhao:2014xaa, Wiecki:2014ola, Hu:2020arv, Nair:2022evk, Nair:2023lir]
  - heavy flavors [Li:2015zda, Li:2017mlw, Leitao:2017esb, Li:2018uif, Adhikari:2018umb, Tang:2018myz, Lan:2019img, Tang:2019gvn, Tang:2020org, Li:2021ejv, Li:2021cwv, Shuryak:2021fsu, Shuryak:2021hng, Shuryak:2021mlh, Shuryak:2022thi, Shuryak:2022wtk]
  - light mesons [Jia:2018ary, Lan:2019vui, Lan:2019rba, Qian:2020utg, Mondal:2021czk, Adhikari:2021jrh, Li:2022mlg]
  - nucleons [Mondal:2019jdg, Xu:2021wwj, Liu:2022fvl, Hu:2022ctr, Peng:2022lte, Zhu:2023lst]
  - tetraquarks [Kuang:2022vdy]
- Dynamical gluons & sea: [Lan:2021wok, Xu:2022abw]
- Time-dependent problems (tBLFQ): [Zhao:2013cma, Zhao:2013jia, Chen:2017uuq, Li:2020uhl, Li:2023jeh]
- Hadronic physics on quantum computer: [Kreshchuk:2020kcz, Kreshchuk:2020aiq, Qian:2021jxp]

# Jacob's tower




“更上一层楼” ("take it to the next level")

# Renormalization in non-perturbative Hamiltonian formalism




- Standard renormalization (Weinbergian renormalization):

$$H_R = \underbrace{H_{\text{can}}}_{\text{divergence}} + \underbrace{H_{\text{ct}}}_{\text{divergence}} = H_{\text{can}} + \sum_i c_i O_i$$

- Wilsonian renormalization: low-energy effective Hamiltonian with similarity renormalization group (SRG) transformations [Glazek:1993rc, Wilson:1994fk, Glazek:2012qj]

$$H_s = \underbrace{\mathcal{U}_s^{-1} H_\infty \mathcal{U}_s}_{\text{divergence free}}$$

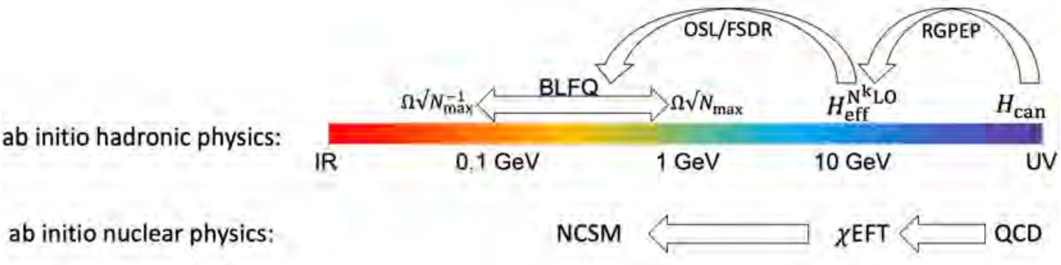
  - Renormalized realistic QQ interactions: Ôkubo-Suzuki-Lee (OSL) [Suzuki:1980yp]

$$H_{\text{eff}} = \mathcal{P} X H_0 X^{-1} \mathcal{P}$$

- Fock sector dependent renormalization [Karmanov:2008br, Li:2015iaw]

# Towards ab initio LFQCD

- $H_\infty \to H_s$: generate a renormalized Hamiltonian $H_s$ at some finite pQCD scale $s$ with RGPEP
- $H_s \to H_{\text{eff}}^{(\Omega)}$: generate a low-energy few-body Hamiltonian $H_{\text{eff}}^{(\Omega)}$ at some hadron scale $\Omega\sqrt{N_{\max}}$ with OSL
- Diagonalize $H_{\text{eff}}^{(\Omega)}$ with BLFQ



# Summary

- Light-front Hamiltonian formalism is a natural framework to describe hadrons as relativistic quantum many-body problems
- Holographic LFQCD provides an adequate first approximation to QCD
- Basis light-front quantization provides an avenue to solving QCD from first principles

Thank you!

Happy birthday, Prof. James Vary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Take it to the next level