Diffractive Production of Vector Mesons: A BLFQ Perspective

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Outlines

- ❑Basis Light Front Quantization
- ❑QED in Basis Light Front Quantization
- ❑QCD in Basis Light Front Quantization
- ❑Small-Basis Light Front Quantization
- ❑Diffractive Heavy Quarkonium production

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BLFQ Introduced

PHYSICAL REVIEW C **81**, 035205 (2010)

Hamiltonian light-front field theory in a basis function approach

J. P. Vary,[1] H. Honkanen,[1] Jun Li,[1] P. Maris,[1] S. J. Brodsky,[2] A. Harindranath,[3] G. F. de Teramond,[4] P. Sternberg,[5,*] E. G. Ng,[5] and C. Yang[5]

Discretized Light Cone Quantization

Pauli%&%Brodsky%1985%

⇩

Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_{\alpha}(\vec{x}) a_{\alpha}^{+} + f_{\alpha}^{*}(\vec{x}) a_{\alpha} \right]$$

where $\{a_{\alpha}\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_{\alpha}(\vec{x})$ are arbitrary except for conditions :

Orthonormal: $\int f_{\alpha}(\vec{x})\, f_{\alpha'}^{*}(\vec{x}) d^3 x = \delta_{\alpha\alpha'}$

Complete: $\sum_{\alpha} f_{\alpha}(\vec{x})\, f_{\alpha}^{*}(\vec{x}') = \delta^3(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_{a}(\vec{x})$ and our initial choice is

$$f_{\alpha}(\vec{x}) = N e^{ik^{+}x^{-}}\, \Psi_{n,m}(\rho,\varphi) = N e^{ik^{+}x^{-}}\, f_{n,m}(\rho)\chi_{m}(\varphi)$$

Basis Light-front Quantization

- ❑ Finding spectrum using light-front Hamiltonian

$$H_{LF}|\psi_h\rangle = M_h^2|\psi_h\rangle, \quad (H_{LF} \equiv P^+\hat{P}^-_{LF} - \vec{P}^2_\perp)$$

- ❑ Adopting basis according to the symmetry of system
- ❑ Advantages:
  - Boost Invariant Amplitude
  - Parton Interpretation
  - Fully relativistic
  - Moore's Law

General Procedures of BLFQ

Vary et al '10

- Derive LF-Hamiltonian from Lagrangian
- Construct basis states $|\alpha\rangle$, and truncation scheme
- Evaluate Hamiltonian in the basis
- Diagonalize Hamiltonian and obtain its eigen states and their LF-amplitudes
- Evaluate observables using LF-amplitudes
- Extrapolate to continuum limit

Basis Functions

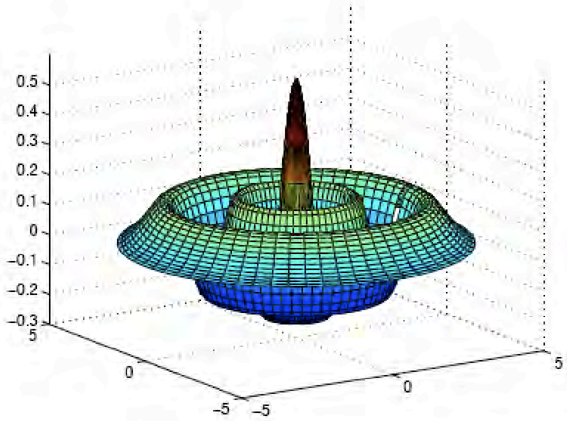
- Fermion

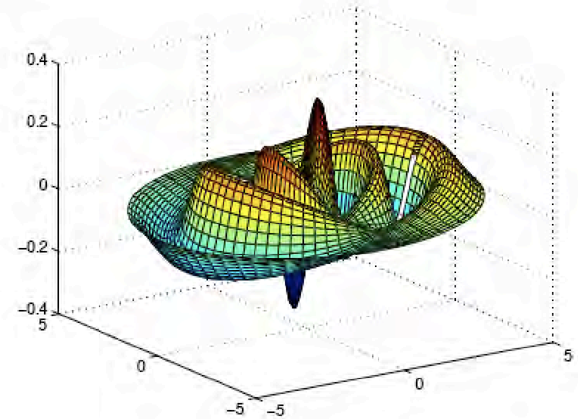
$$\Psi(x) = \sum_{\bar{\alpha}} \frac{1}{\sqrt{2L}} \int \frac{d^2 p^\perp}{(2\pi)^2} \Big[b_{\bar{\alpha}} \tilde{\Phi}_{nm}(p^\perp) u(p,\lambda) e^{-i\mathsf{p}\cdot\mathsf{x}} + d^\dagger_{\bar{\alpha}} \tilde{\Phi}^*_{nm}(p^\perp) v(p,\lambda) e^{i\mathsf{p}\cdot\mathsf{x}} \Big]$$

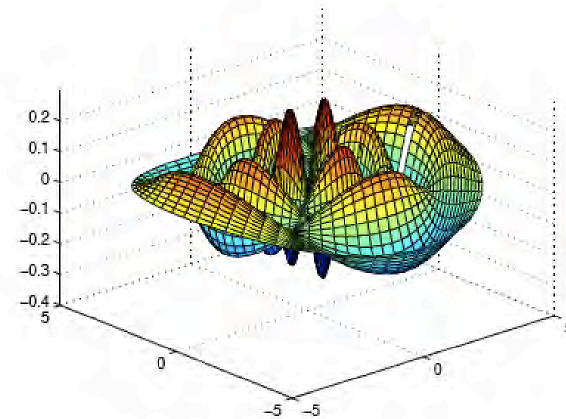
- 2D HO wavefunction in transverse plane

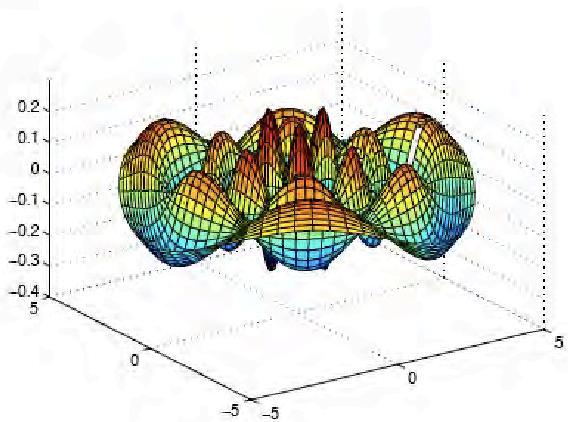
$$\tilde{\Phi}^b_{nm}(p^\perp) = (2\pi) \frac{\sqrt{2}}{b} \sqrt{\frac{n!}{(n+|m|)!}} e^{-p^2/(2b^2)} \left(\frac{p}{b}\right)^{|m|} \times L_n^{|m|}\left(\frac{p^2}{b^2}\right) \frac{1}{\sqrt{2\pi}} e^{im\phi}$$

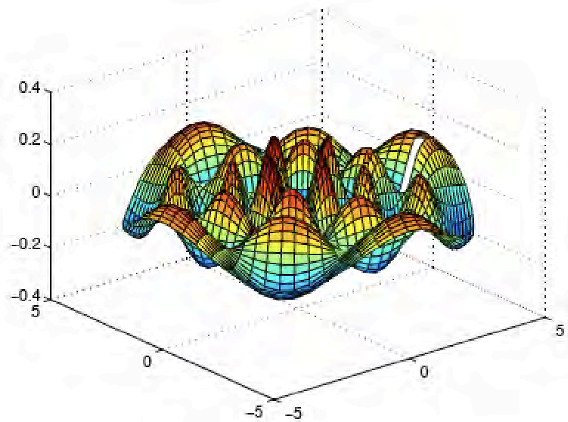
2D HO modes for n=4











Outlines

- ❑Basis Light Front Quantization
- ❑QED in Basis Light Front Quantization
- ❑Heavy Quarkonium in Basis Light Front Quantization
- ❑Diffractive Heavy Quarkonium Production

LF QED Hamiltonian

- QED Lagrangian $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$
- Derived Light-front Hamiltonian

$$P^- = \int d^2x^\perp dx^- \; F^{\mu+}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L} \qquad (A^+ = 0)$$

$$= \int d^2x^\perp dx^- \; \frac{1}{2}\bar{\Psi}\gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j (i\partial^\perp)^2 A^j$$

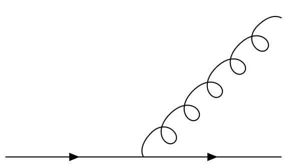
kinetic energy terms

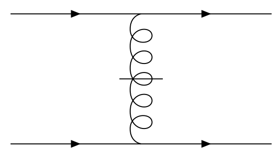
$$+ e j^\mu A_\mu + \frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+ + \frac{e^2}{2} \bar{\Psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu \Psi$$

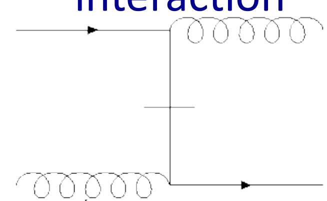
vertex interaction

instantaneous photon interaction

instantaneous fermion interaction



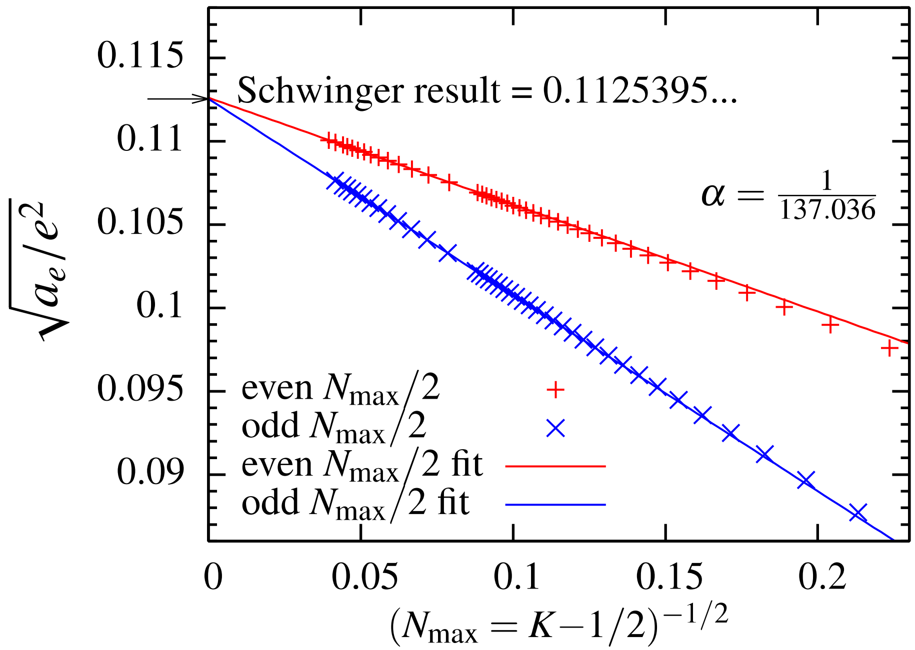




1st Application: Electron g-2

X. Zhao et al., PLB 737, (2014) 65, H. Honkanen et al., PRL 106, (2011), 061603

- With wavefunction renormalization

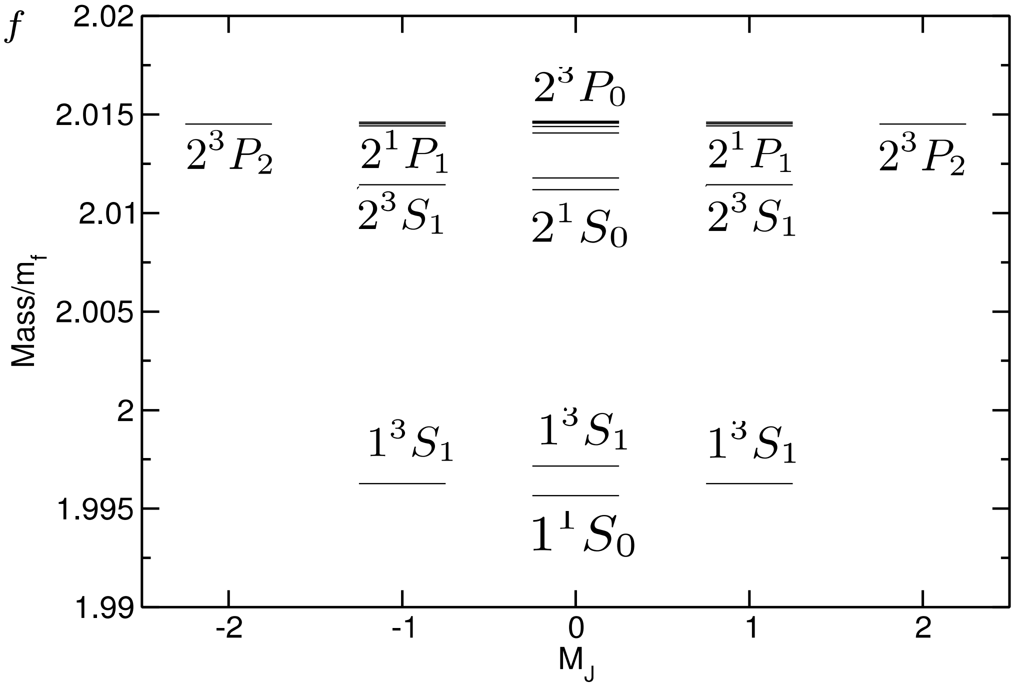


- Less than 0.1% deviation from Schwinger result
- Largest calculation with basis dim > 28 billion

2nd Application: Positronium with strong coupling

- Positronium spectrum at N_{max}=K=19,

$\mu = 0.1 m_f$

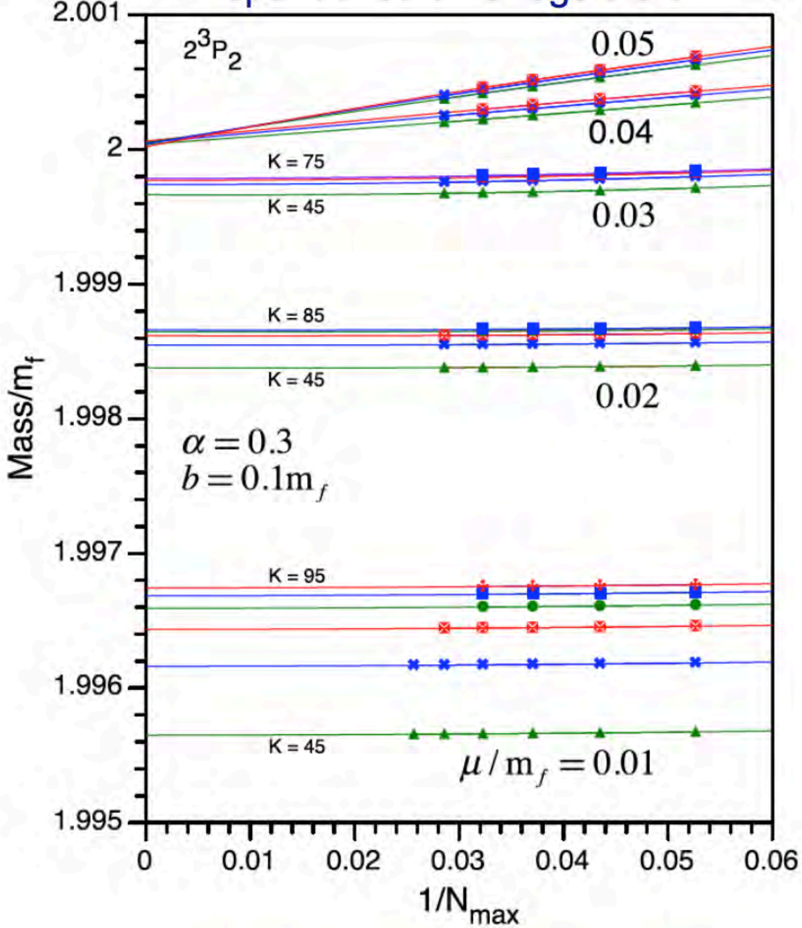


μ is a fictitious photon mass to regulate Coulomb singularity

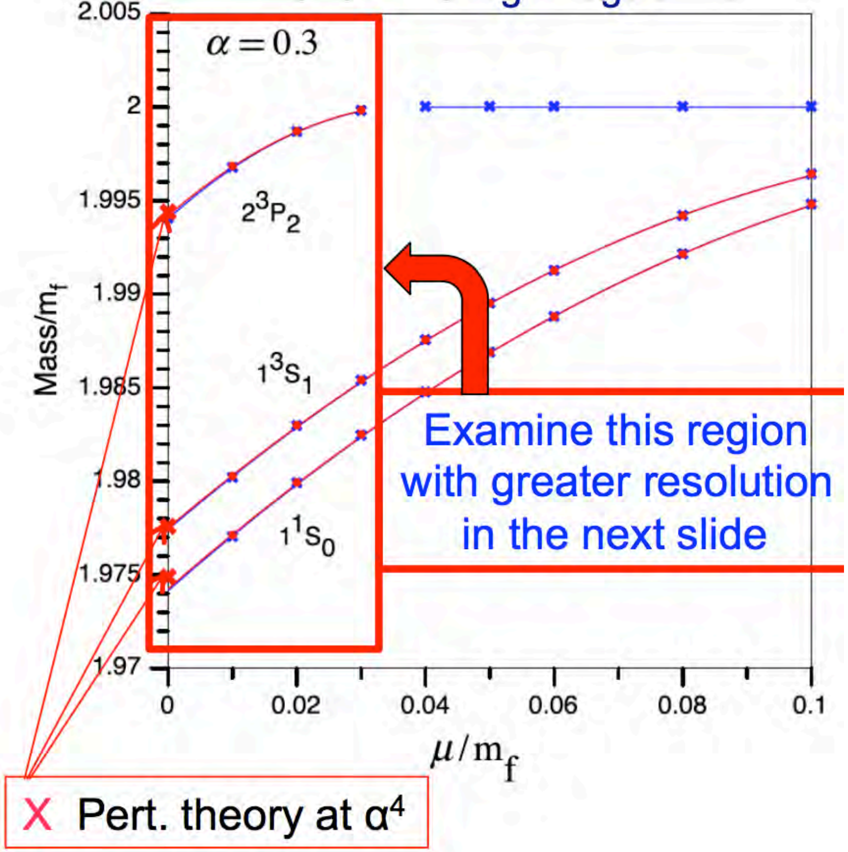
[P. Wiecki et al.,PRD 91, 105009, 2015]

Basis Light-Front Quantization (BLFQ)
Positronium in QED at Strong Coupling (α = 0.3)
Systematic removal of regulators (b = HO momentum scale)

Dependence on 3 regulators

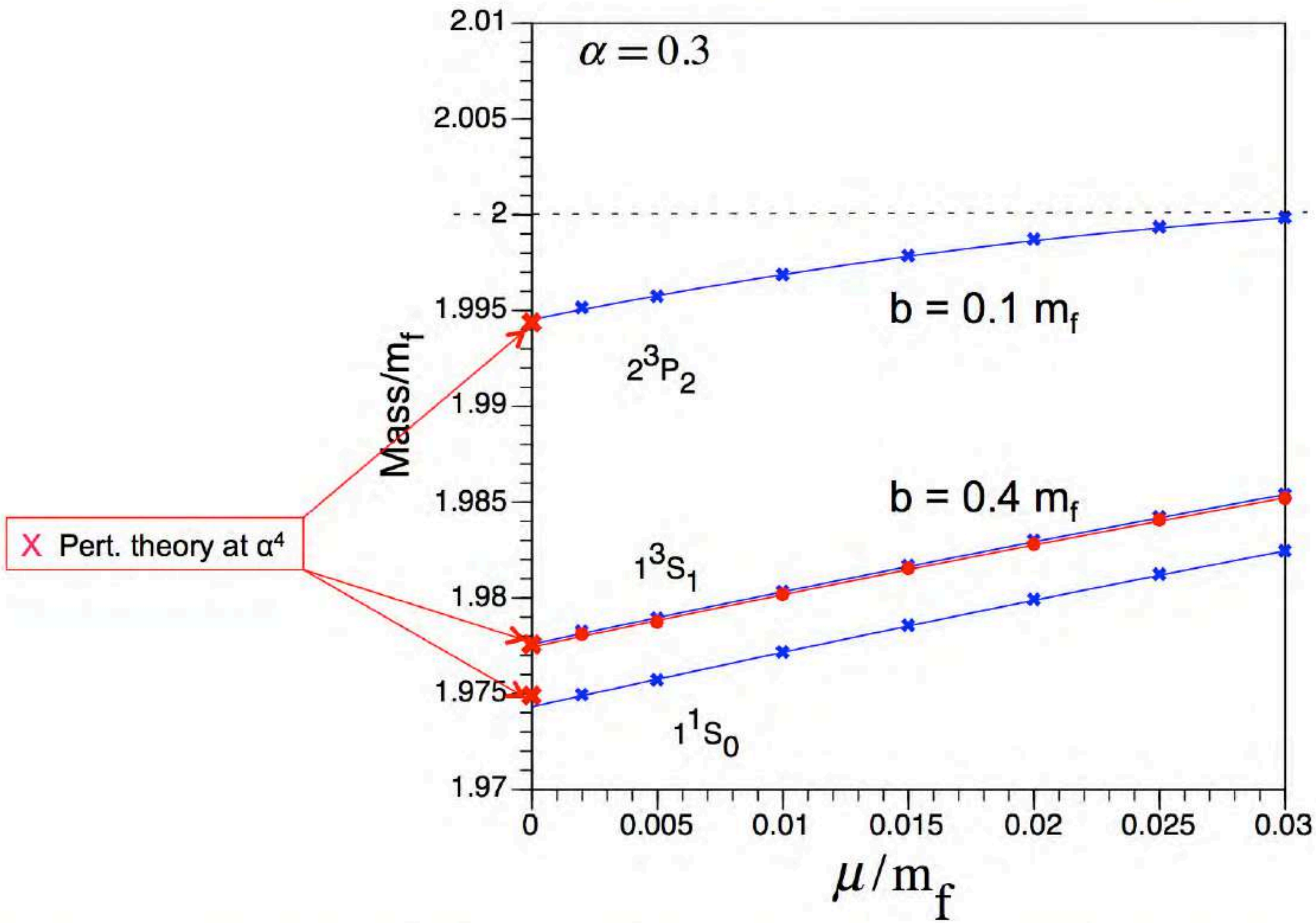


After eliminating 2 regulators



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

Positronium in QED at Strong Coupling
Covariant Basis Light-Front Quantization (BLFQ)



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015); & to be published

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- ❑ QCD in Basis Light Front Quantization
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1st QCD Application: Heavy Quarkonium

- Effective Hamiltonian in the $q\bar{q}$ sector   Y. Li et al., PLB 758,118, 2016

$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

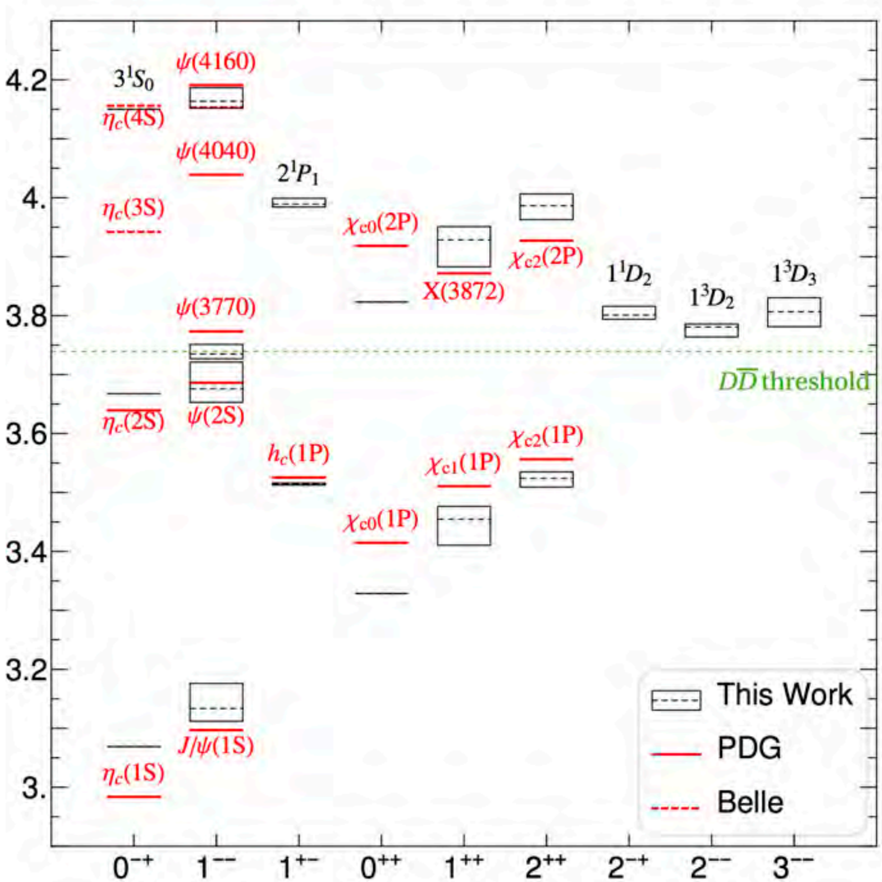
where $x = p_q^+/P^+$, $\vec{k}_\perp = \vec{p}_{q\perp} - x\vec{P}_\perp$, $\vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}$.

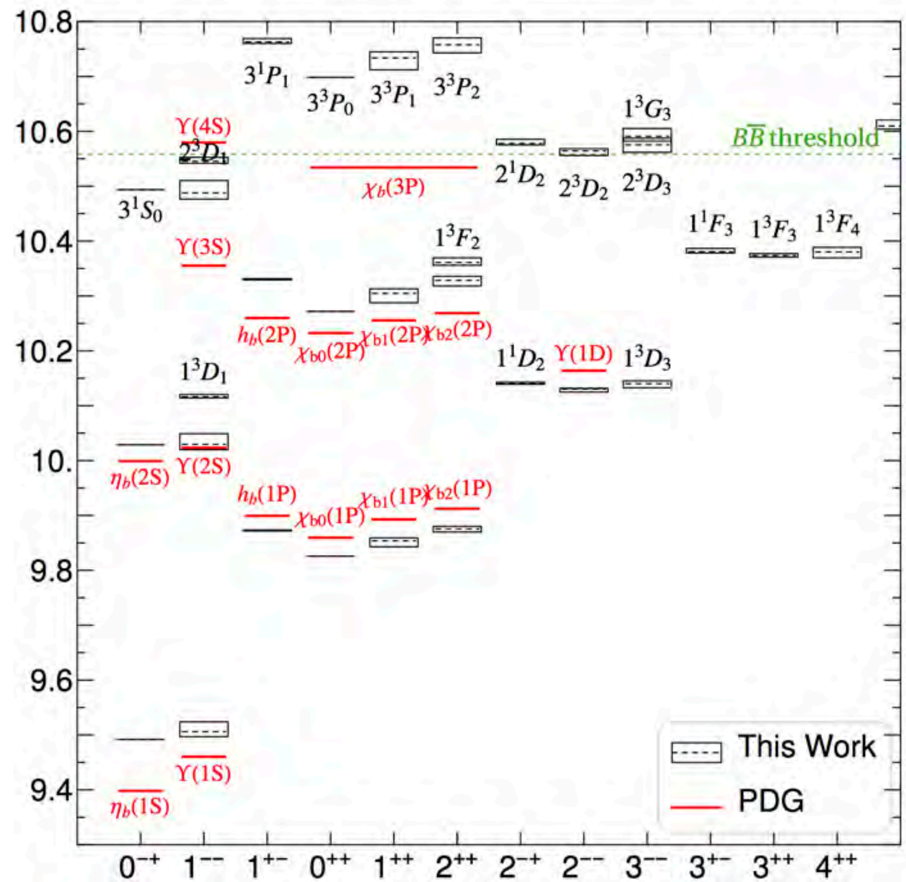
  - Confinement
    transverse holographic confinement [S.J.Brodsky,PR584,2015]
    longitudinal confinement [Y.Li,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$

- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0 (LF kinetic energy+ confinement)

Mass Spectroscopy – fixed α

Y. Li et al., PLB 758,118, 2016



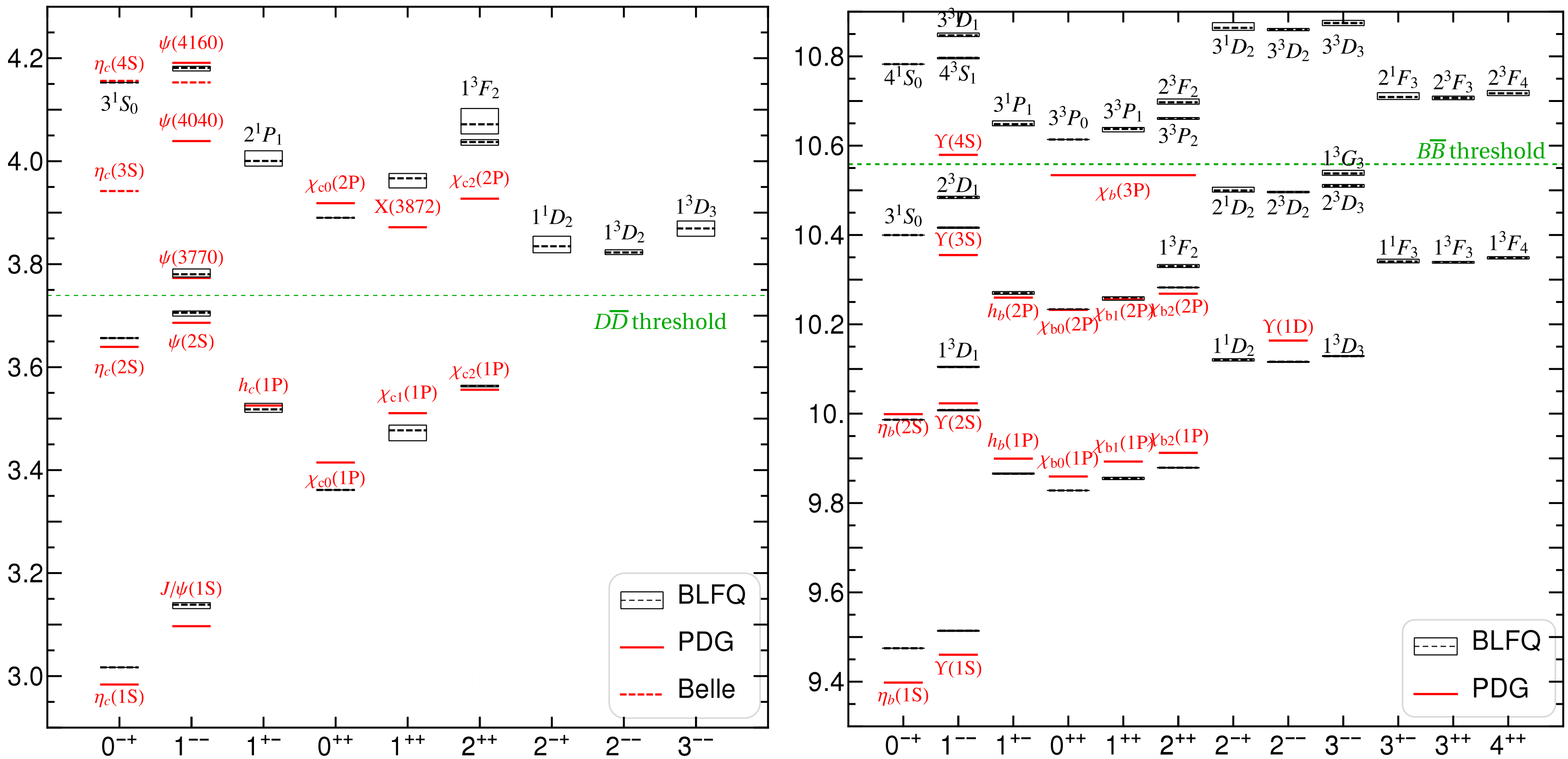


$$\delta\overline{M} = 52\ \mathrm{MeV}$$

$$\delta\overline{M} = 50\ \mathrm{MeV}$$

Mass Spectroscopy – running α

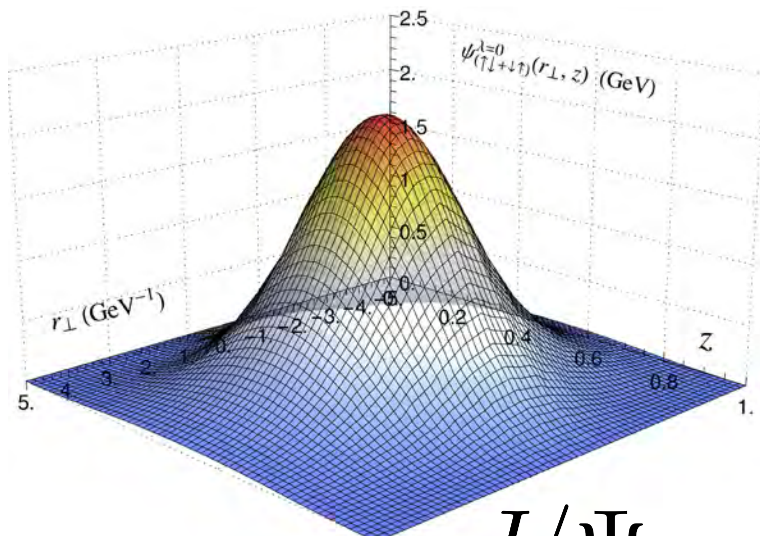
Y. Li et al., PRD 96, 016022, 2017

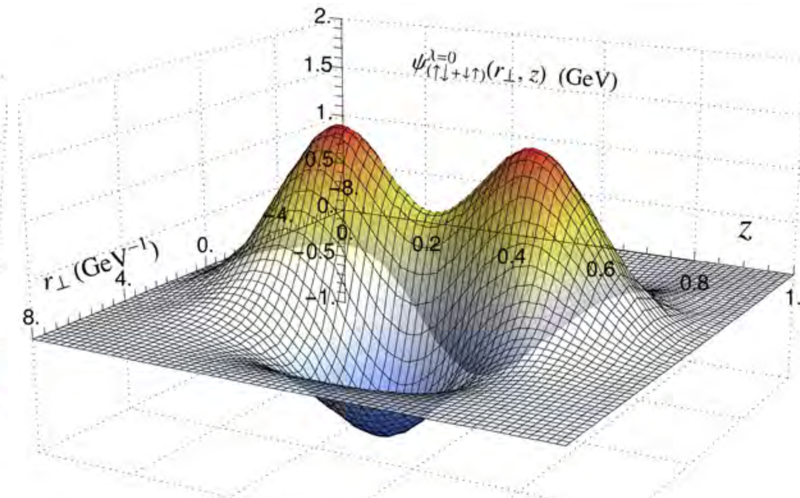


rms deviations: 31 – 38 MeV

Light-front Wavefunction: a complete solution

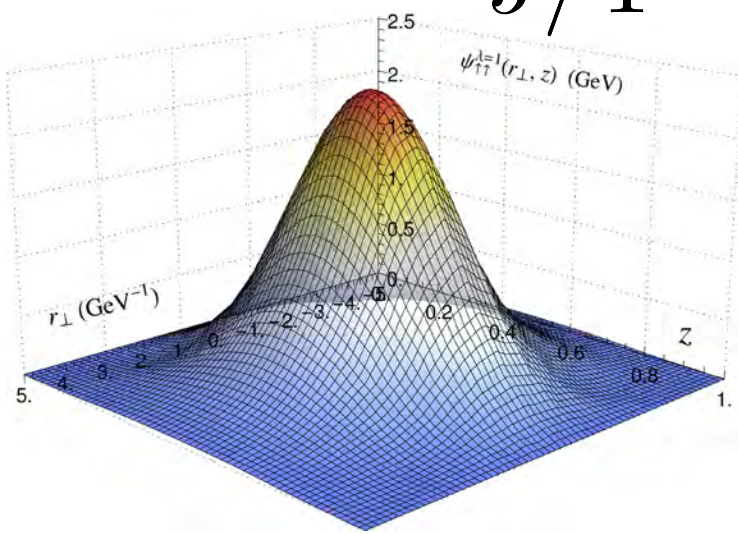
Y. Li et al., PLB, 2016, PRD 2017

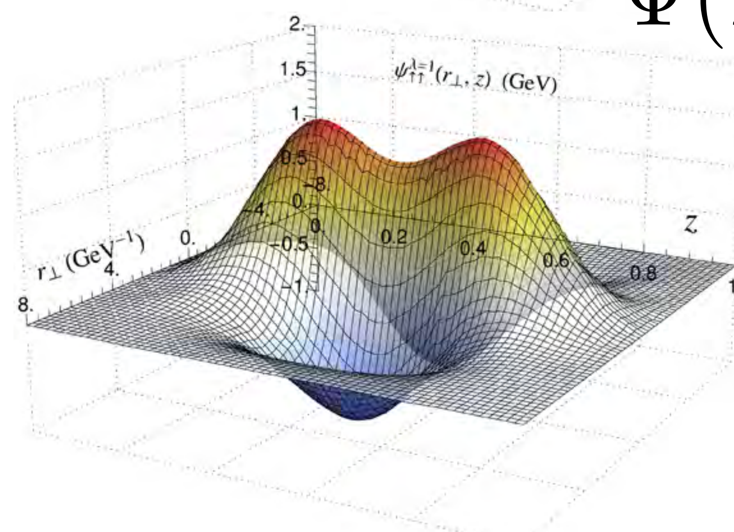




J/Ψ

$\Psi(2S)$

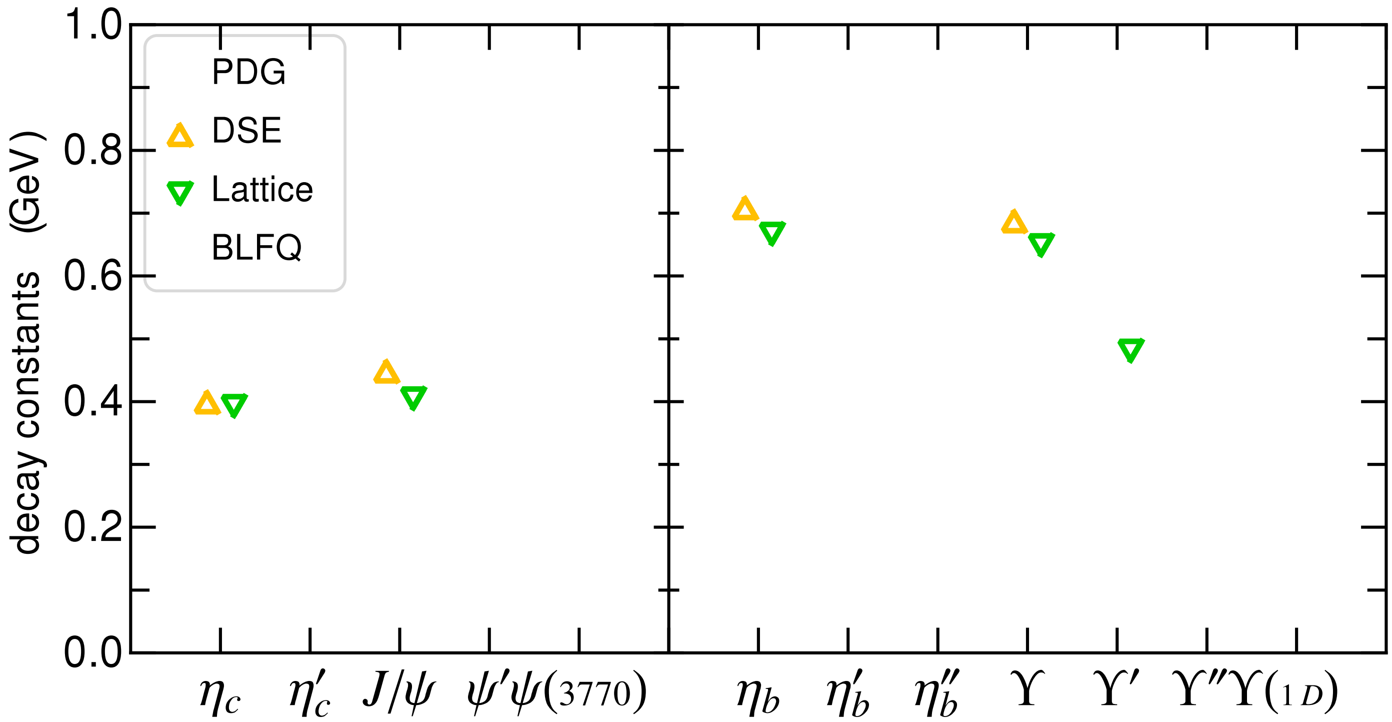




Decay Constants

Y. Li et al., PRD 96, 016022, 2017

- Derived from obtained LFWFs (no fitting)



Radiative Transitions

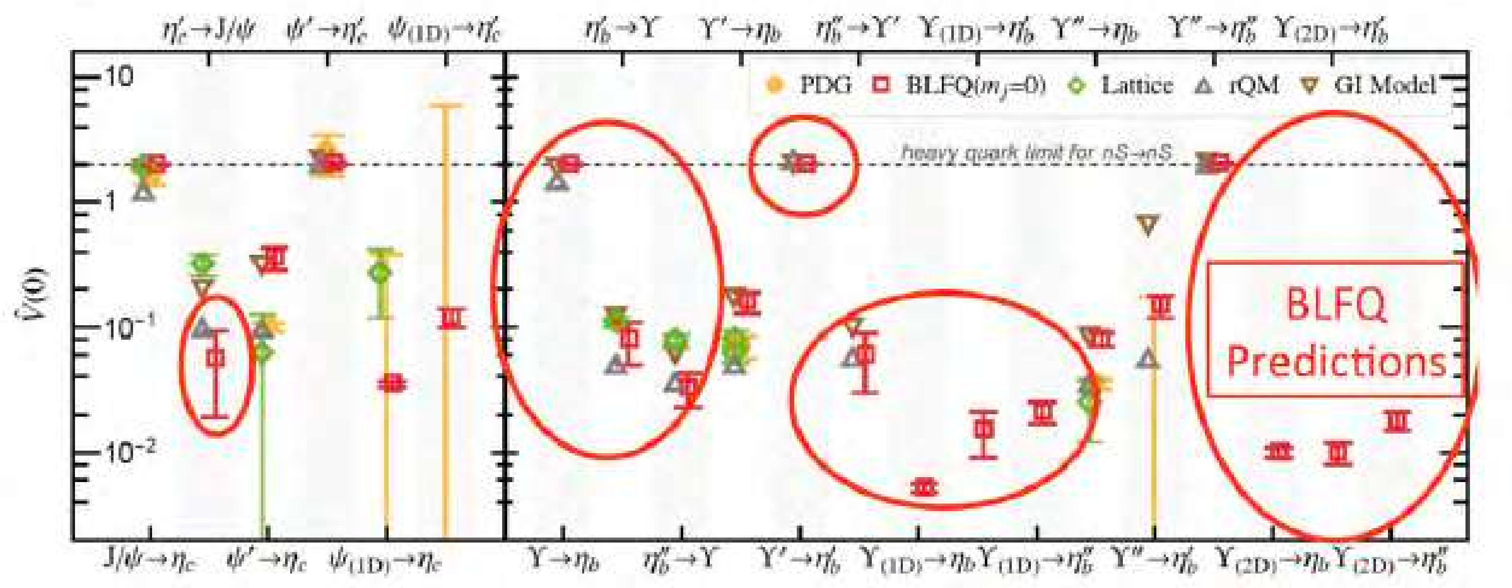
Y. Li et al., PRD 98, 034024, 2018

- Derived from obtained LFWFs (no fitting)

Y. Li et al., PRD 96, 016022, 2017

Decay width:

$$\Gamma(\mathcal{V} \to \mathcal{P} + \gamma) = \int \mathrm{d}\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{m_{\mathcal{V}}^2} \frac{1}{2J_{\mathcal{V}}+1} \sum_{m_j,\lambda} |\mathcal{M}_{m_j,\lambda}|^2 = \frac{(m_{\mathcal{V}}^2 - m_{\mathcal{P}}^2)^3}{(2m_{\mathcal{V}})^3 (m_{\mathcal{P}} + m_{\mathcal{V}})^2} \frac{|V(0)|^2}{(2J_{\mathcal{V}}+1)\pi}$$



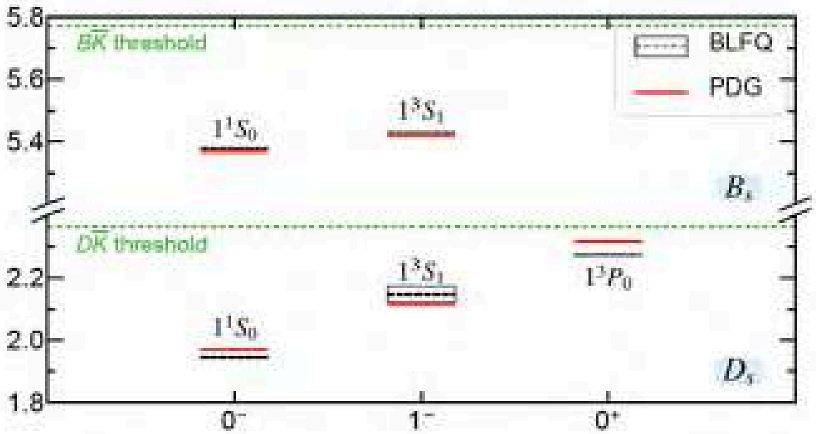
Heavy-light system

S. Tang, et al., PRD, 104, 016002, 2020

Heavy-Light systems

	N_f	κ (GeV)	m_c (GeV)	m_b (GeV)	m_s (GeV)	rms (MeV)	$N_{\max} = L_{\max}$
D_s ($c\bar{s}$)	3	0.800	1.603	–	0.647	40 (9 states)	32
B_s ($b\bar{s}$)	4	1.067	–	4.902	0.647	37 (4 states)	32

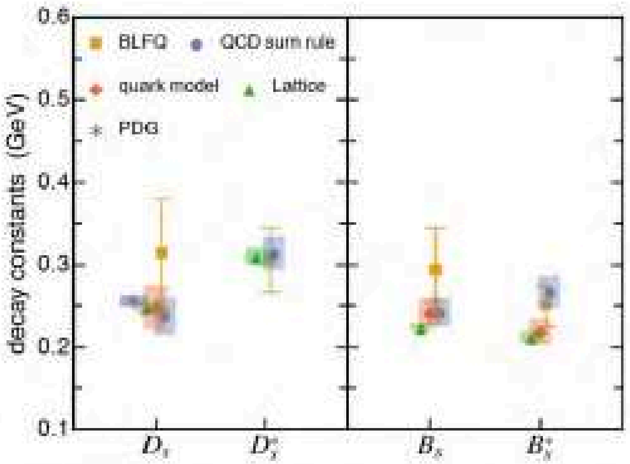
$$\kappa = \sqrt{(\kappa^2_{b\bar{b}/c\bar{c}} + \kappa^2_{s\bar{s}})/2}, \text{ with } \kappa_{ss} = 0.54 \text{ GeV}$$



Mass spectrum of heavy-light systems. States under open flavor threshold confirmed by experiments.

Decay constants calculated with $N_{\max}$ = 8 for D_s, $N_{\max}$ = 32 for B_s, corresponding to UV cutoffs:

$$\Lambda_{UV} \triangleq \kappa\sqrt{N_{\max}} \approx m_q + m_{\bar{q}}$$

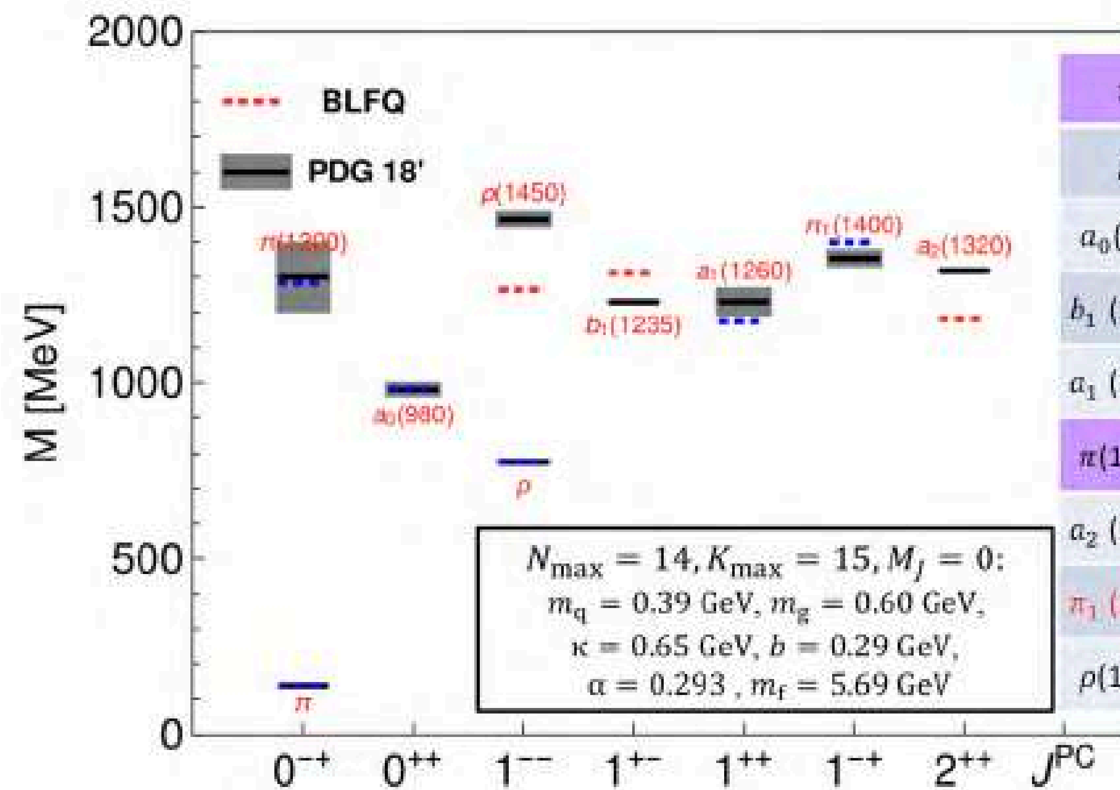


Light Mesons

W. Qian, et al., PRC, 102, 055207, 2020

J. Lan, et al., PLB, 825, 136890, 2022

Light Meson Mass Spectrum Including One Dynamical Gluon



	Norm$q\bar{q}$	DC[MeV]
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

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Motivation

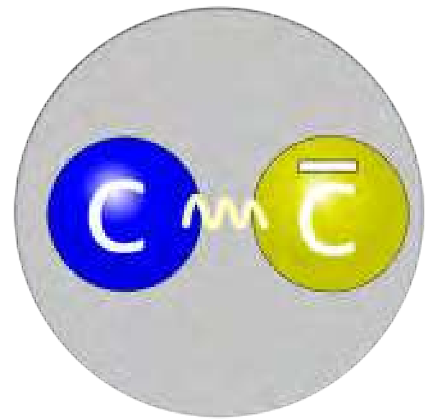
- Goals: simple-function charmonium LFWFs with few parameters!

i. Approximation to QCD.

ii. Retain more symetries.

iii. Matching the NR limit.

iv. Emphsis on decay width.



- We designed LFWFs for η_c, J/ψ, ψ' and $\psi(3770)$.

Basis functions

- LF holography/Basis LF Quantization Hamiltonian.

$$H_0 = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) r_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x)\partial_x) ,$$

i. Two parameters: m_q and κ.

ii. One-gluon interactions were treated perturbatively.

- The basis function representation.

$$\tilde{\psi}^{(m_j)}_{ss'/h}(\vec{r}_\perp, x) = \sqrt{x(1-x)} \sum_{n,m,l} \psi_h^{(m_j)}(n,m,l,s,s') \tilde{\phi}_{nm}(\sqrt{x(1-x)}\vec{r}_\perp)\chi_l(x) ,$$

Teramond and Brodsky, '09

Li et al., PLB 758, 118 (2016)

Basis functions

- Small basis for charmonium states:

$$\psi_{\mathrm{LF}-1S} = \psi_{0,0,0} \ .$$

$$\psi_{\mathrm{LF}-1P0} = \psi_{0,0,1} \ ,$$

$$\psi_{\mathrm{LF}-1P\pm1} = -\psi_{0,\pm1,0} \ .$$

$$\psi_{\mathrm{LF}-2S} = \sqrt{\frac{2}{3}}\psi_{1,0,0} - \sqrt{\frac{1}{3}}\psi_{0,0,2} \ .$$

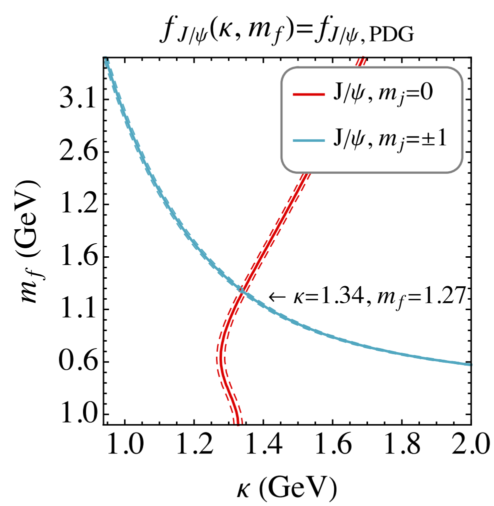
$$\psi_{\mathrm{LF}-1D0} = \sqrt{\frac{1}{3}}\psi_{1,0,0} + \sqrt{\frac{2}{3}}\psi_{0,0,2} \ ,$$

$$\psi_{\mathrm{LF}-1D\pm1} = -\psi_{0,\pm1,1} \ ,$$

$$\psi_{\mathrm{LF}-1D\pm2} = \psi_{0,\pm2,0} \ .$$

J/ψ as a 1^{--} state

- We assume a 100% LF-1S state for J/ψ.
- Matching J/ψ decay constant to the PDG value:



- We fix m_c and κ using the J/ψ decay constant.

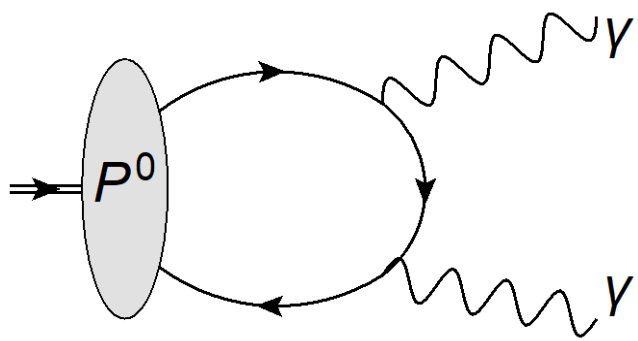
M. Li et al., EPJC 82, 1045, 2022

η_c as a 0^{-+} state

- η_c predominantly LF-1S+LF-2S and LF-1P.

$$\psi_{\eta_c} = C_{\eta_c,1S}\,\psi_{\mathrm{LF-1}S,0-+} + C_{\eta_c,2S}\,\psi_{\mathrm{LF-2}S,0-+} + C_{\eta_c,1P}\,\psi_{\mathrm{LF-1}P,0-+}\,.$$

- Basis coeffecients are determined using the diphoton decay width $\Gamma(\eta_c \to \gamma\gamma)$.

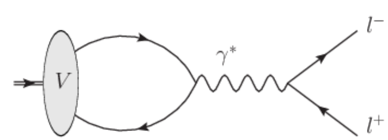


ψ' as a 1^{--} state

- A mix of LF-1S and LF-2S states for ψ'.

$$
\begin{aligned}
\psi_{\psi'}^{(m_j=0)} &= C_{\psi',1S}^{(m_j=0)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=0)} + C_{\psi',2S}^{(m_j=0)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=0)} , \\
\psi_{\psi'}^{(m_j=1)} &= C_{\psi',1S}^{(m_j=1)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=1)} + C_{\psi',2S}^{(m_j=1)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=1)} , \\
\psi_{\psi'}^{(m_j=-1)} &= C_{\psi',1S}^{(m_j=-1)} \psi_{\mathrm{LF}-1S,1--}^{(m_j=-1)} + C_{\psi',2S}^{(m_j=-1)} \psi_{\mathrm{LF}-2S,1--}^{(m_j=-1)} .
\end{aligned}
$$

- Basis coeffecients are determined using the dilepton decay constant.



$$
\left| f_{\mathcal{V}}|_{m_j=0} \right| = \left| f_{\mathcal{V}}|_{m_j=\pm 1} \right| = f_{\mathcal{V},\mathrm{experiment}} .
$$

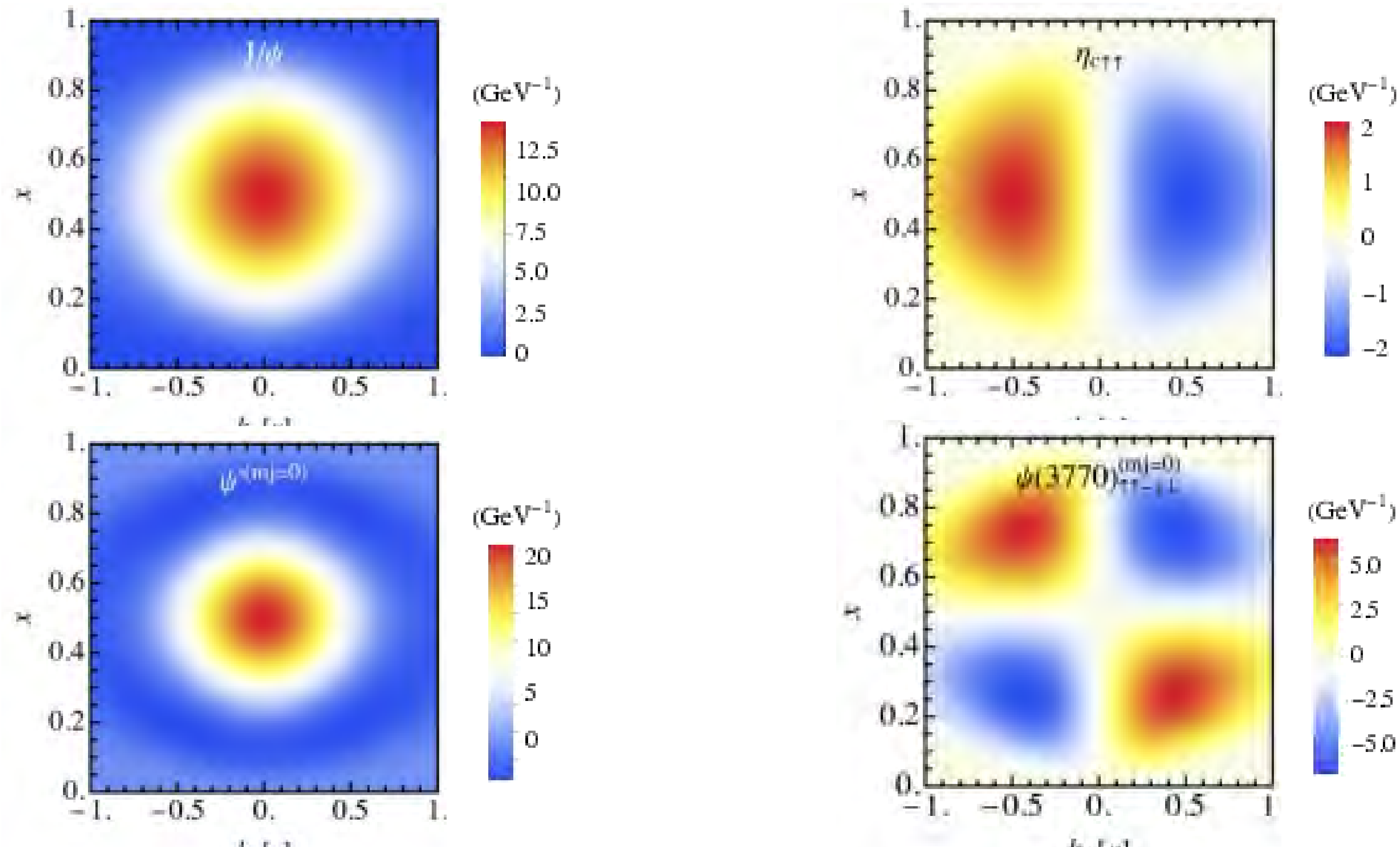
$\psi(3770)$ as a 1^{--} state

- A mix of LF-1S, LF-2S, LF-1D states for $\psi(3770)$, LF-1D is dominating.

$$
\begin{aligned}
\psi_{\psi(3770)}^{(m_j=0)} =& C_{\psi(3770),1S}^{(m_j=0)} \psi_{\mathrm{LF-1}S,1--}^{(m_j=0)} \\
&+ C_{\psi(3770),2S}^{(m_j=0)} \psi_{\mathrm{LF-2}S,1--}^{(m_j=0)} \\
&+ C_{\psi(3770),1D}^{(m_j=0)} \psi_{\mathrm{LF-1}D,1--}^{(m_j=0)} ,
\end{aligned}
$$

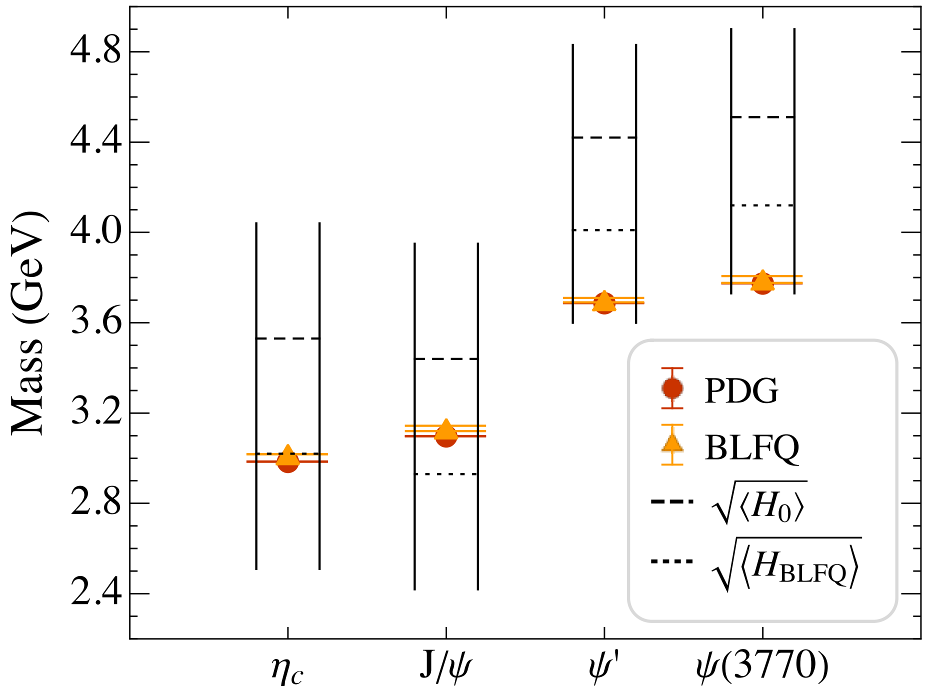
- Basis coeffecients are determined by requiring orthogonality between ψ' and $\psi(3770)$.

LFWFs by design



M. Li et al., EPJC 82, 1045, 2022

The mass spectrum



$$\left(\tilde{M}_h^{(m_j)}\right)^2 = \sum_{n,m,l,s,\bar{s}} \sum_{n',m',l',s',\bar{s}'} \psi_h^{(m_j)}(n,m,l,s,\bar{s}) \times \psi_h^{(m_j)*}(n',m',l',s',\bar{s}') \times \Big[M^2_{n,m,l}\delta_{n,n'}\delta_{m,m'}\delta_{l,l'}\delta_{s,s'}\delta_{\bar{s},\bar{s}'} + \langle \beta_{n',m',l',s',\bar{s}'} | \Delta H | \beta_{n,m,l,s,\bar{s}} \rangle \Big] .$$

$$V_{\mathrm{OGE}} = -\frac{C_F 4\pi\alpha_s(q^2)}{q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}') ,$$

Masses calculated from small-basis LFWFs should be regarded as Estimated!

M. Li et al., EPJC 82, 1045, 2022

Y. Li et al., PLB 758, 118 (2016)

The charge radii

- Defined in terms of the slope of the charge form factor at zero momentum transfer.

$$\langle r_h^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_0(Q^2)\Big|_{Q\to 0} .$$

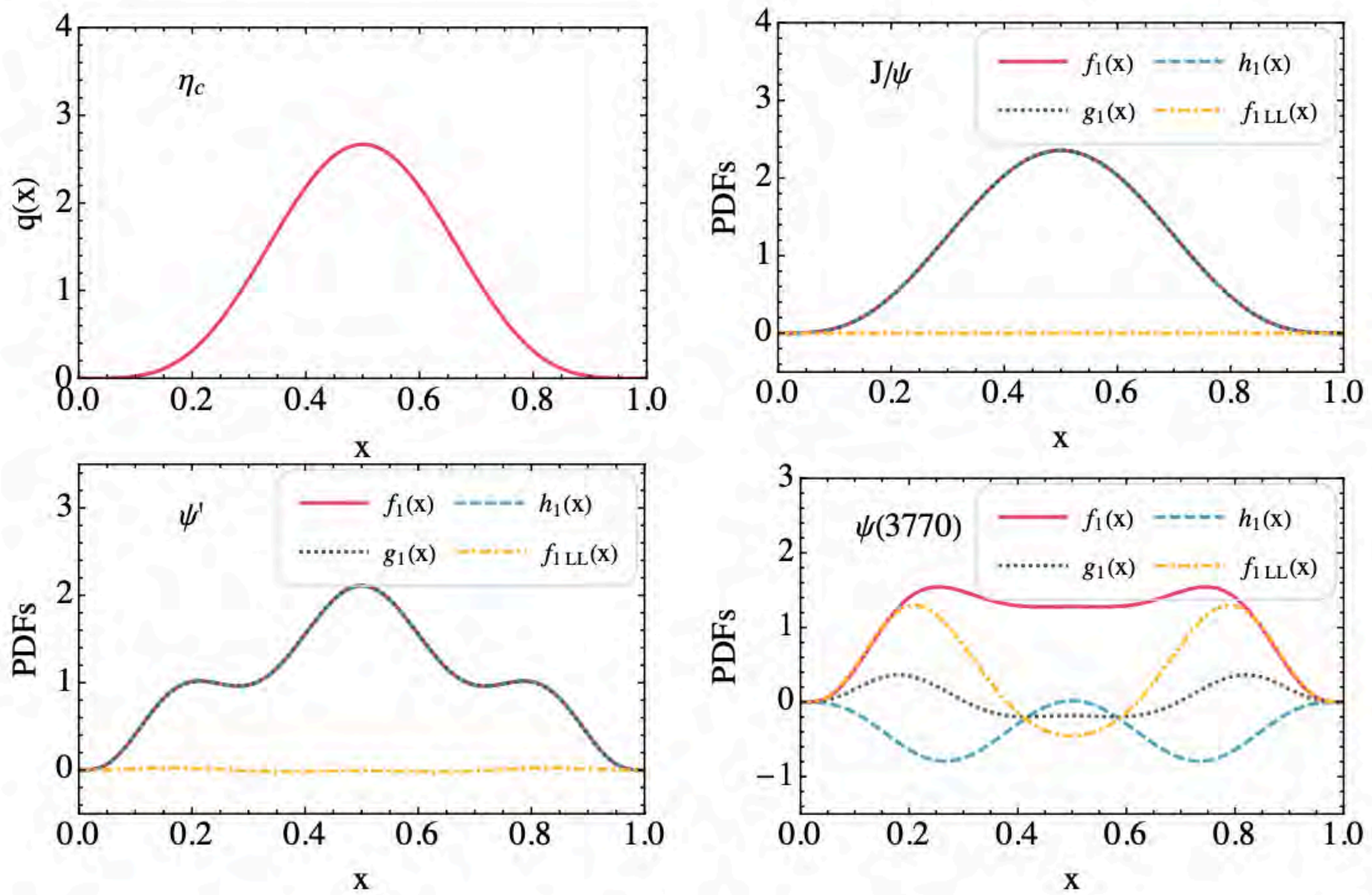
(fm^2)	$\langle r^2_{\eta_c} \rangle$	$\langle r^2_{J/\psi} \rangle$	$\langle r^2_{\psi'} \rangle$	$\langle r^2_{\psi(3770)} \rangle$
this work	0.098	0.046	0.154	0.138
BLFQ [27]	0.029(1)	0.0402(2)	0.13(0)	0.13(0)

- J/ψ, ψ' and ψ(3770) radii consistent with BLFQ calculations.
- A large size η_c!

M. Li et al., EPJC 82, 1045, 2022

Li et al., PLB 758, 118 (2016)

Parton Distribution Functions (PDFs)



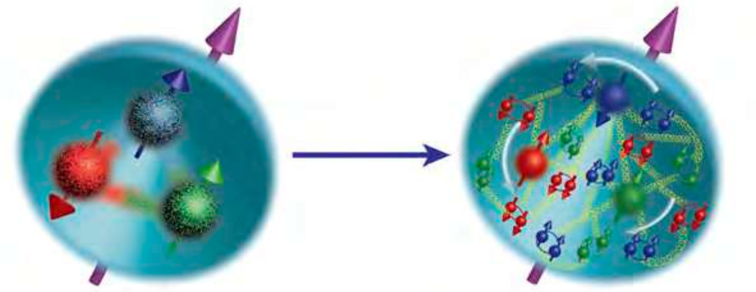
M. Li et al., EPJC 82, 1045, 2022

Outlines

- ❑Basis Light Front Quantization
- ❑QED in Basis Light Front Quantization
- ❑QCD in Basis Light Front Quantization
- ❑Small-Basis Light Front Quantization
- ❑Diffractive Heavy Quarkonium production

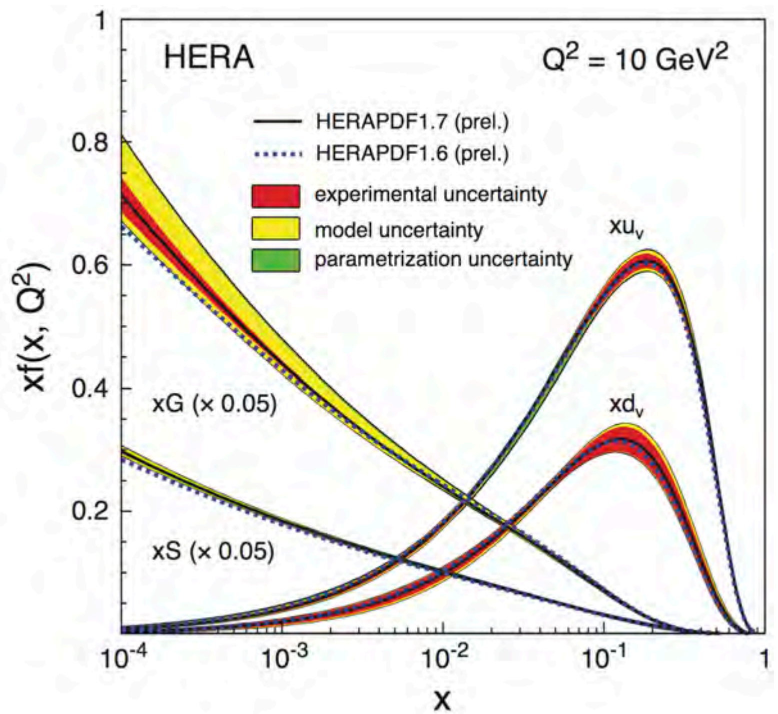
Proton at High Energy

❑Our knowledge of proton is rapidly evolving



❑Proton at high energy?

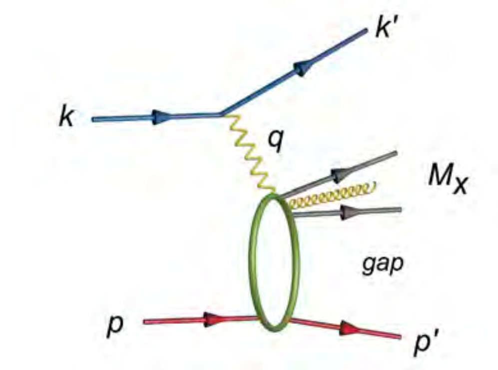
- 3D tomography
- Saturation
- Spin



Exclusive Diffractive Processes

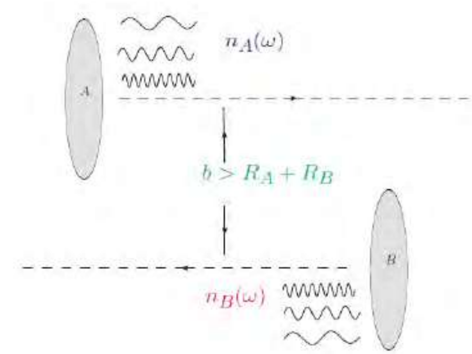
❑First observed at HERA

- 15% events are diffractive
- Characterized by rapidity gap
- $x_{\text{probed}} \quad (Q^2 + M_X^2)/W^2$

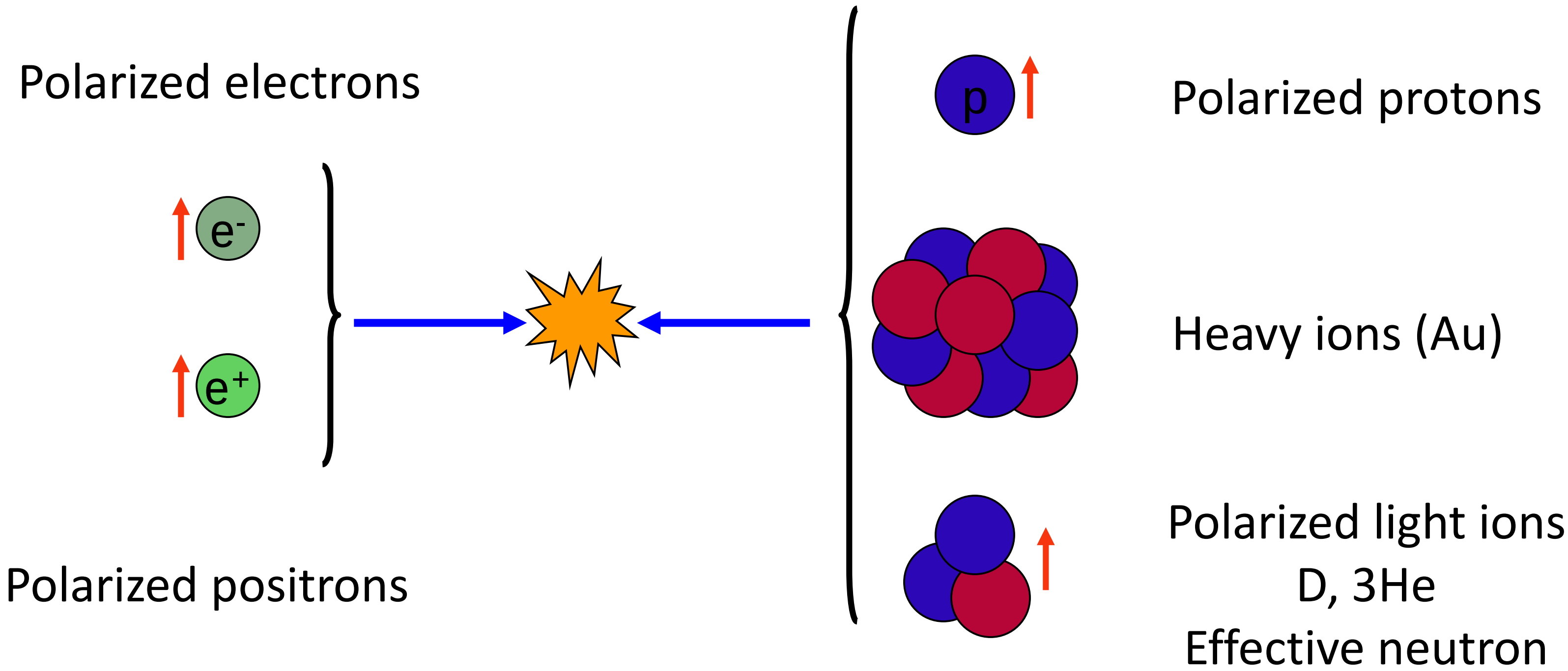


❑Ultra-Peripheral Collisions

- Photon-nuclear interaction
- WW formalism for
  Photon flux ~ Z^2
- x_{probed} up to 10^{-5} at LHC



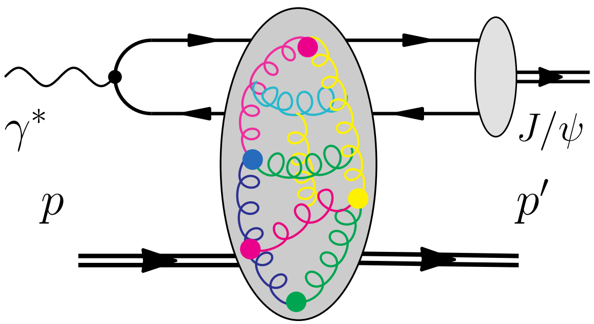
Electron-Ion Collider



A collider with versatile range of kinematics and beam polarizations, as well as beam species, wide energy variability and high luminosity, the next QCD frontier:
3D tomography of proton, spin, saturation, etc.

Exclusive process in the dipole picture

- Photon LFWF: pQED
- Dipole cross section
- Vector meson LFWFs



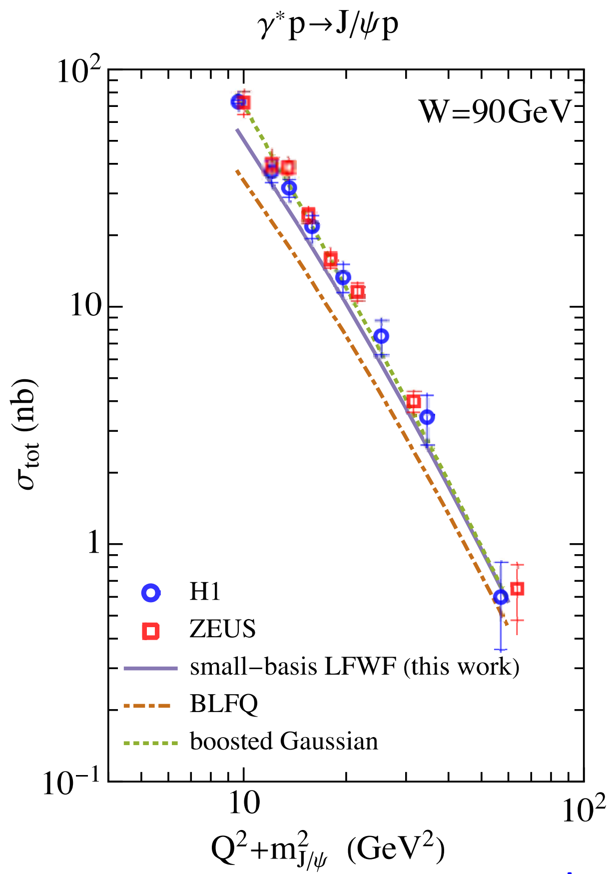
- $$\mathcal{A}_{T,L}^{\gamma^* p\rightarrow Ep}(x,Q,\Delta) = \mathrm{i}\int d^2\vec{r}\int_0^1 \frac{dz}{4\pi}\int d^2\vec{b}\,({\Psi_E}^*\Psi)_{T,L} \times \mathrm{e}^{-\mathrm{i}[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}}\,\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}$$
- Probing gluon density at small-x

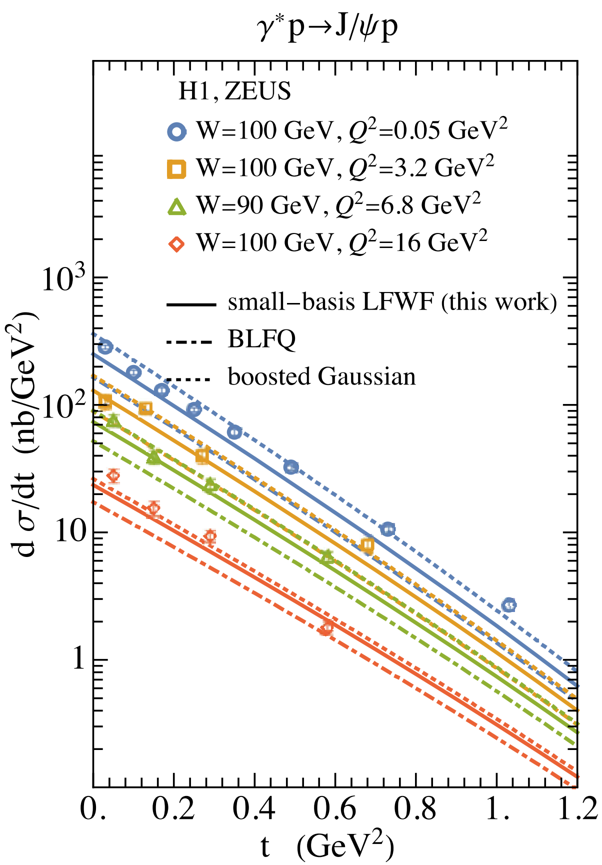
$$\sigma \sim [xg(x_{IP},Q^2)]^2$$

A. Mueller, '90
N. Nikolaev, '91
K. Golec-Biernat el al., '99

J/ψ production at HERA

ZEUS, 2004.
H1, 2006.



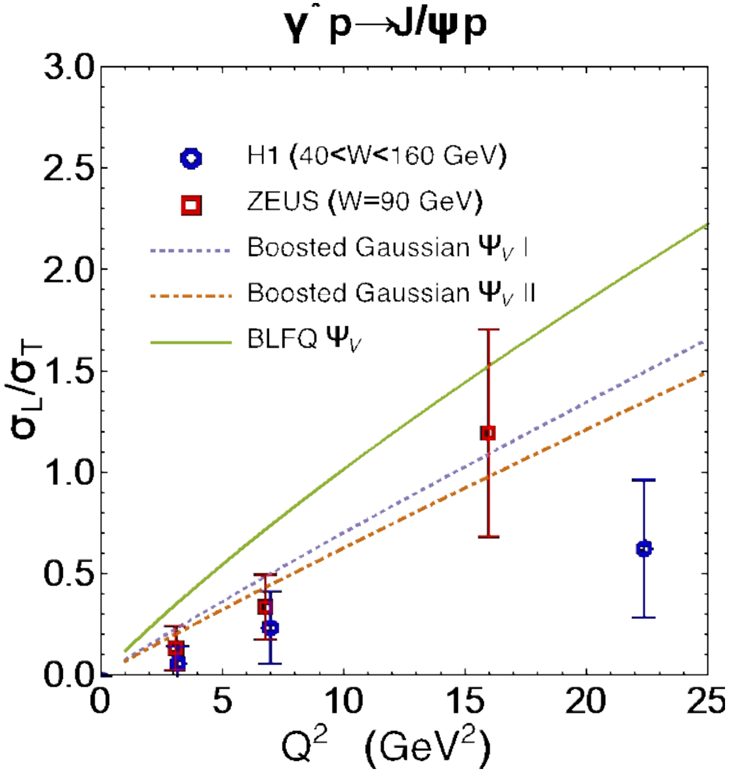


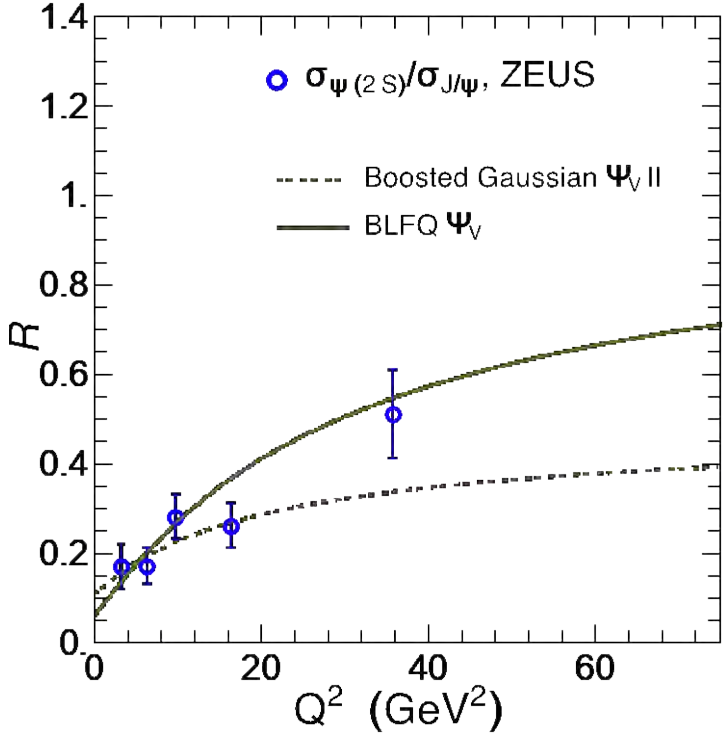
GC et al., PLB 769, 477, 2017
GC et al., PRC 100, 025208, 2019
M. Li et al., EPJC, 82, 1045, 2022

HERA: cross-section ratio

ZEUS, 2016.
H1, 2006.

GC et al., PLB 769, 477, 2017

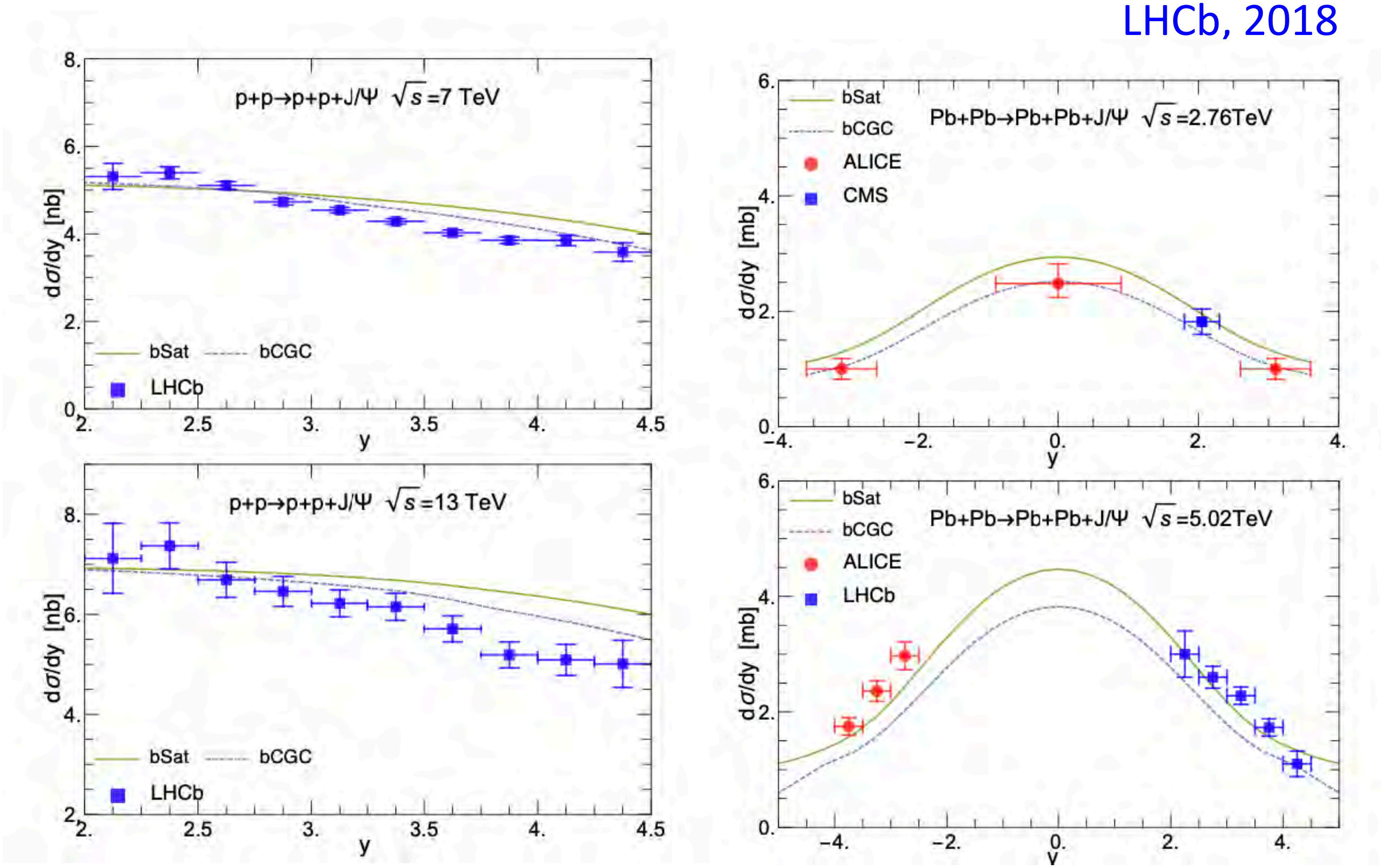




J/Ψ from pp/Pb-Pb UPC

GC et al., PLB 769, 477, 2017

ALICE, 2013.
CMS, 2016.
LHCb, 2018



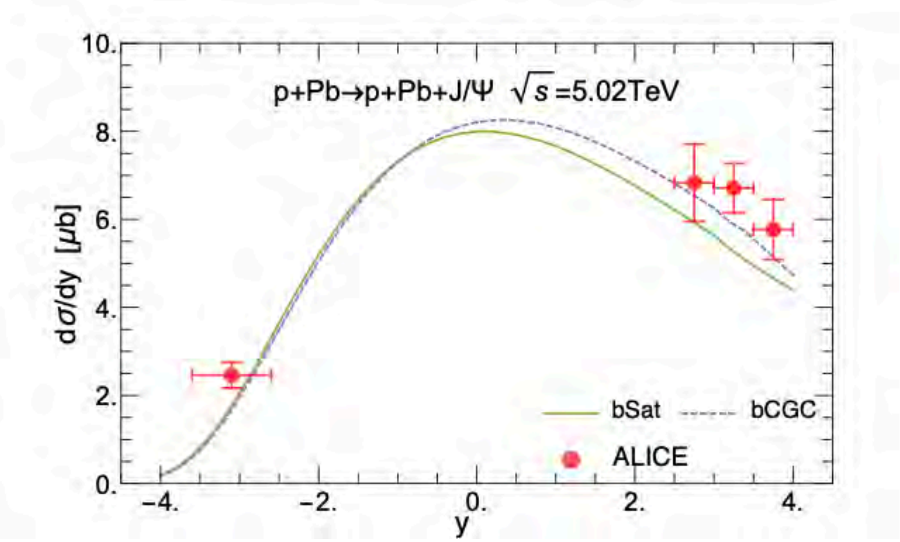
J/Ψ from UPC

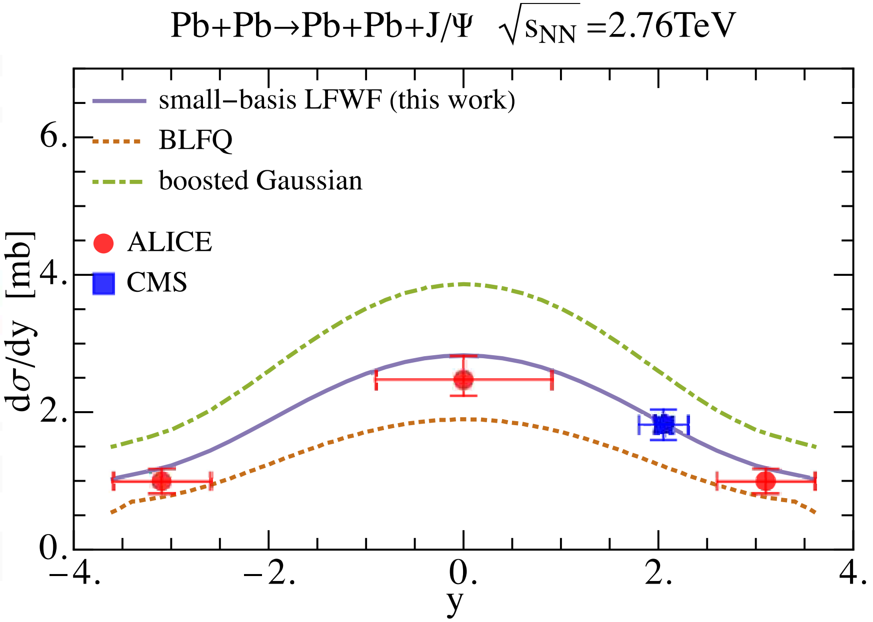
GC et al., PLB 769, 477, 2017

M. Li et al., EPJC, 82, 1045, 2022

ALICE, 2013.

CMS, 2016.

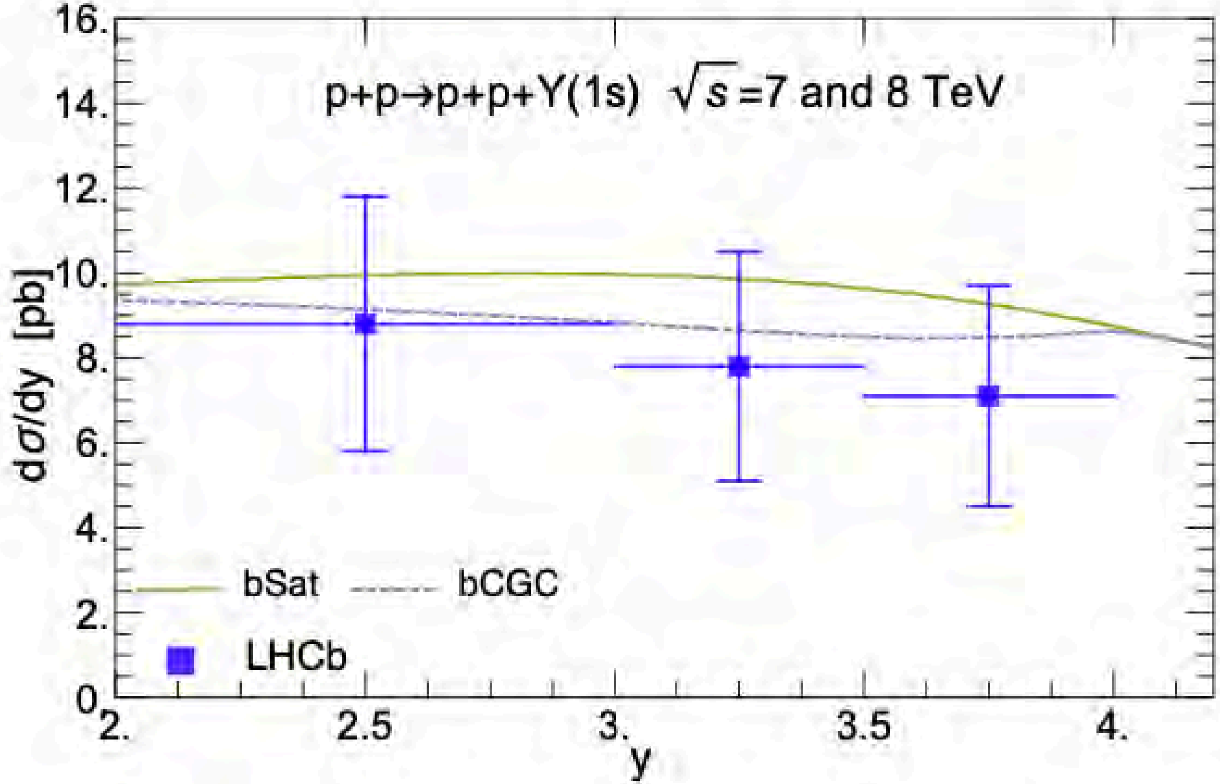




$\Upsilon(1s)$ from pp UPC at LHC

GC et al., PRC 100, 025208, 2019

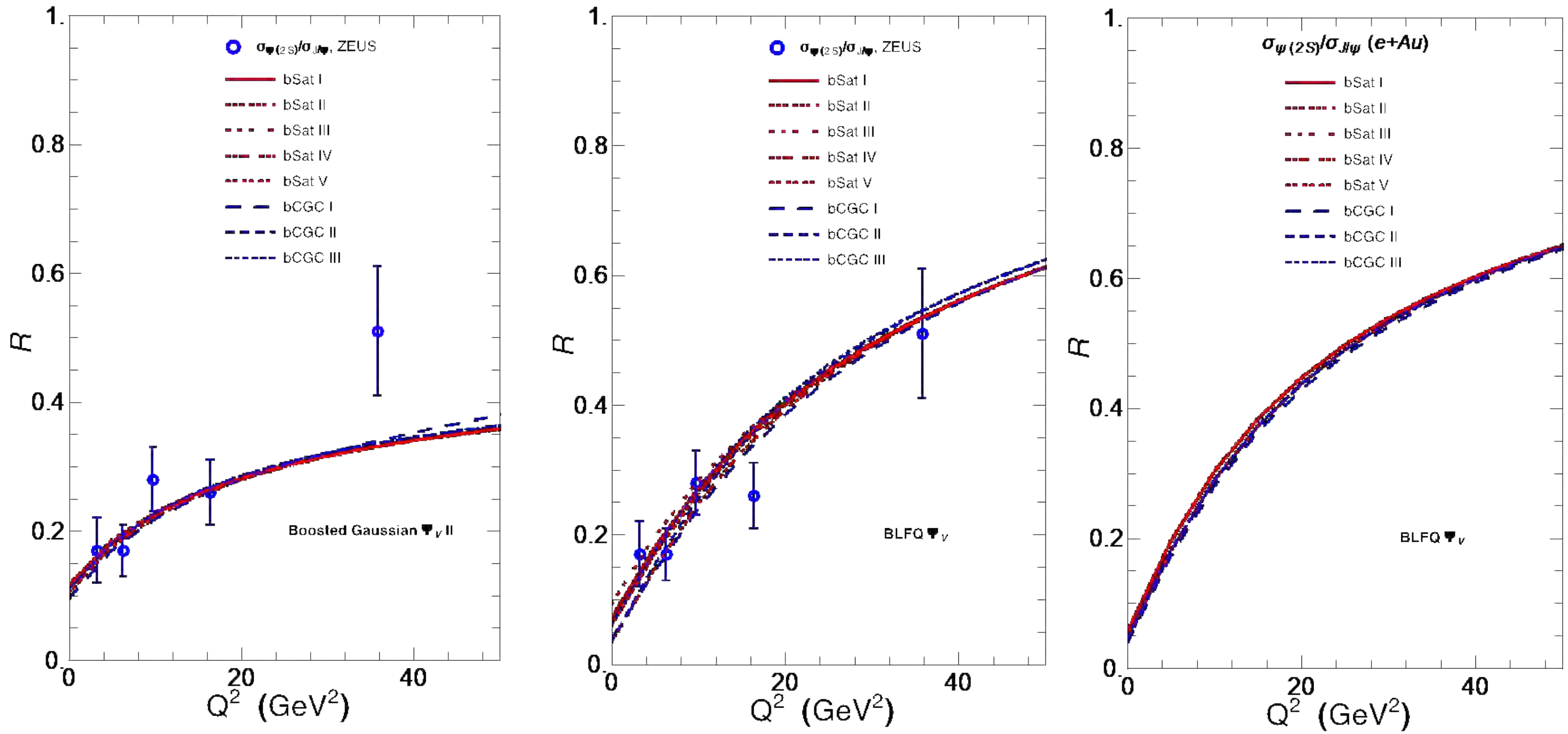
LHCb, 2015



Cross section ratio

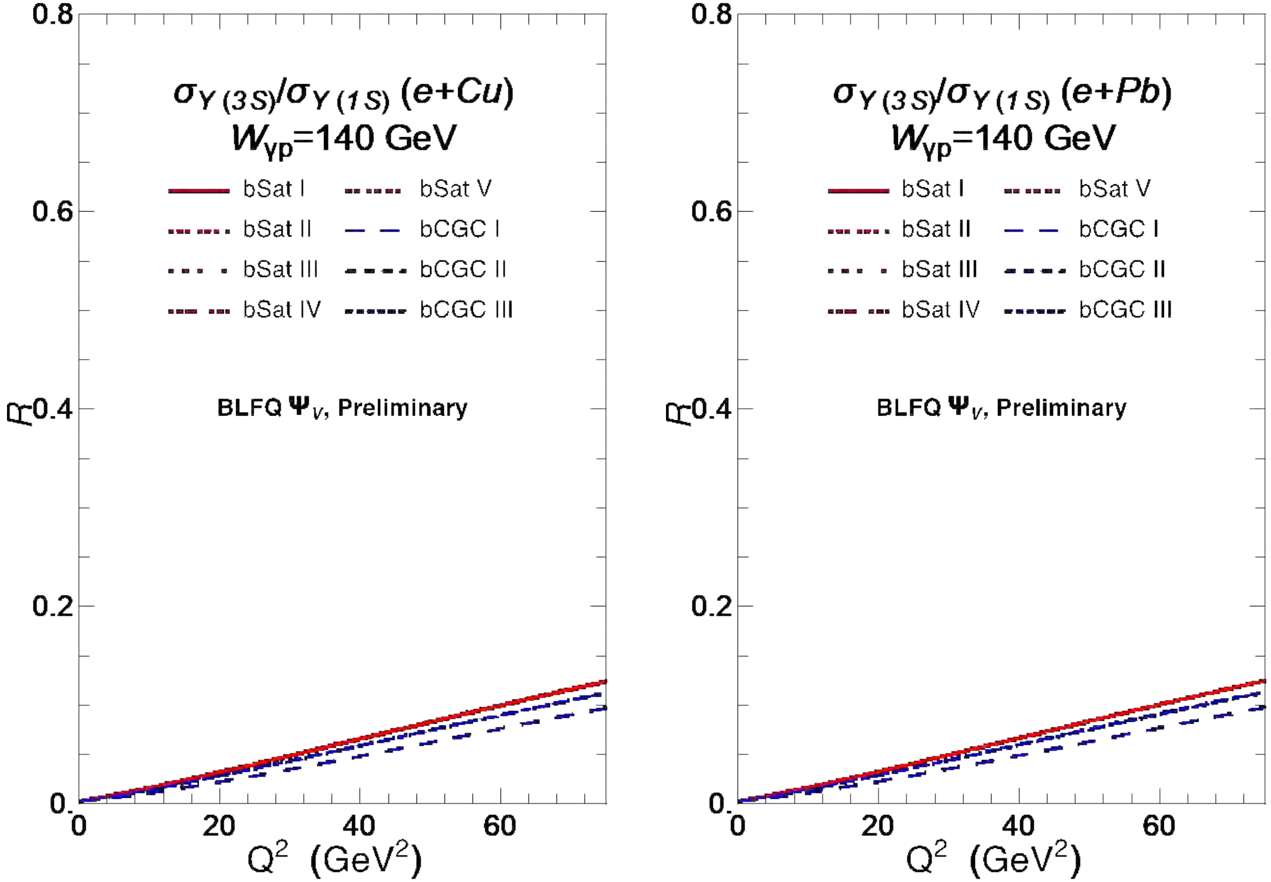
GC et al., PLB 769, 477, 2017

ZEUS, 2016.



Cross section ratio, Upsilons

GC et al., PRC 100, 025208, 2019

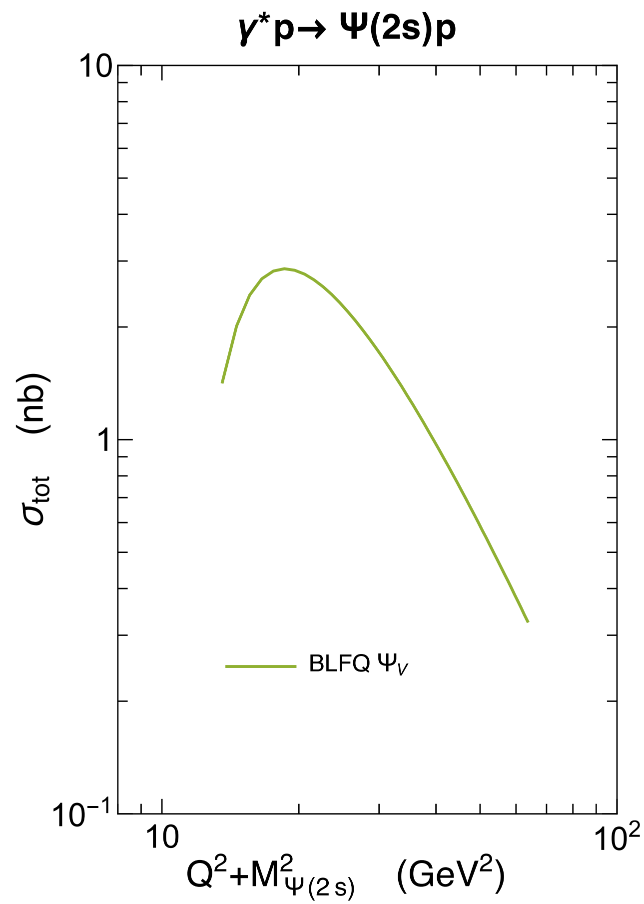


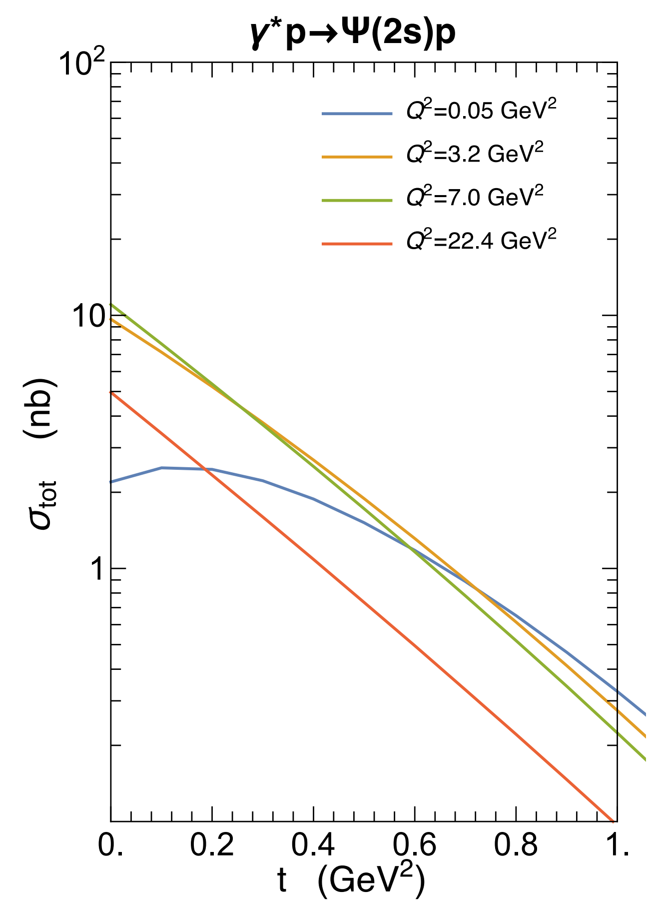
Future Opportunities

- Extended to higher Fock Sector LFWFs, with increasing computing power.
- Higher excited states available at EIC, LHeC, Electron-ion collider in China (EicC).

Higher Excited States at EIC

Preliminary





Summary

- ❑The BLFQ framework can solve the hadron system efficiently.
- ❑The BLFQ approach naturally provides LFWFs of mesons.
- ❑We can check BLFQ LFWFs at Future nuclear collision facilities, such as EIC.

Happy Birthday, James!

Backup Slides

Basis Function

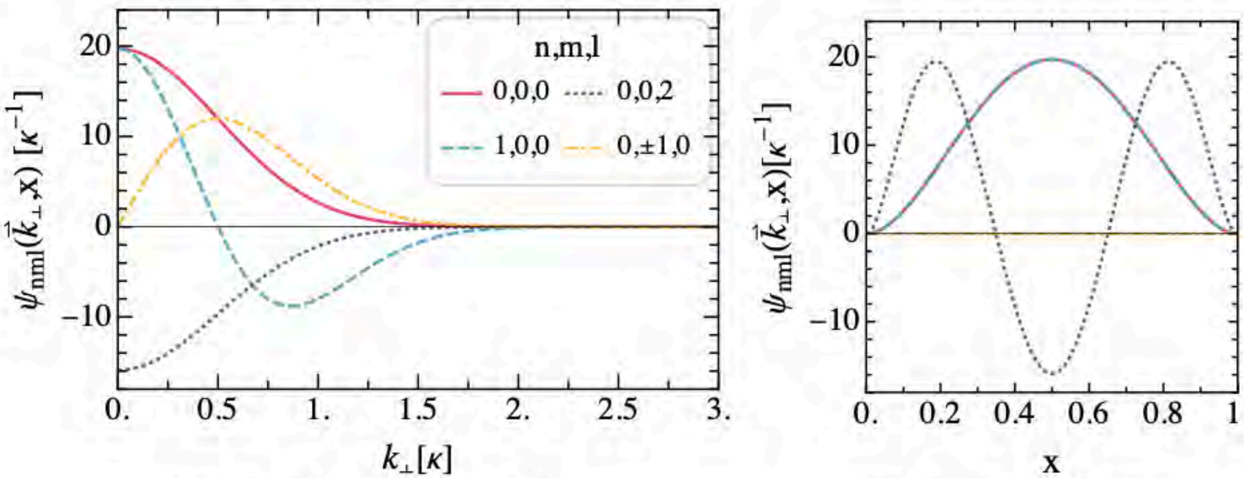
- Transverse:

$$\phi_{nm}(\vec{k}_\perp) = \kappa^{-1} \sqrt{\frac{4\pi n!}{(n+|m|)!}} \left(\frac{k_\perp}{\kappa}\right)^{|m|} \exp(-k_\perp^2/(2\kappa^2))$$
$$L_n^{|m|}({k_\perp}^2/\kappa^2) \exp(im\theta_k) \, ,$$

- Longitudinal:

$$\chi_l(x) = \sqrt{4\pi(2l+\alpha+\beta+1)} \sqrt{\frac{\Gamma(l+1)\Gamma(l+\alpha+\beta+1)}{\Gamma(l+\alpha+1)\Gamma(l+\beta+1)}}$$
$$x^{\beta/2}(1-x)^{\alpha/2} P_l^{(\alpha,\beta)}(2x-1) \, ,$$

Sample Basis Function



(a) $\psi_{nml}(k_\perp, \theta_k = 0, x = 0.5)$ (b) $\psi_{nml}(\vec{k}_\perp = \vec{0}_\perp, x)$

Mirror Parity and Chargeconjugation

TABLE I. Basis states $\psi_{nml}\sigma_{s\bar{s}}$ categorized by the mirror parity m_{P} according to Eq. (17).

m_j	m	$m_{\mathsf{P}} = 1$	$m_{\mathsf{P}} = -1$
0	0	$\psi_{n,0,l}\sigma_+$	$\psi_{n,0,l}\sigma_-$
	± 1	$\frac{1}{\sqrt{2}}(\psi_{n,-1,l}\sigma_{\uparrow\uparrow} - \psi_{n,1,l}\sigma_{\downarrow\downarrow})$	$\frac{1}{\sqrt{2}}(\psi_{n,-1,l}\sigma_{\uparrow\uparrow} + \psi_{n,1,l}\sigma_{\downarrow\downarrow})$
$1, -1$	0	$\psi_{n,0,l}\sigma_{\uparrow\uparrow}, \psi_{n,0,l}\sigma_{\downarrow\downarrow}$	$\psi_{n,0,l}\sigma_{\uparrow\uparrow}, -\psi_{n,0,l}\sigma_{\downarrow\downarrow}$
	± 1	$\psi_{n,1,l}\sigma_\pm, \mp\psi_{n,-1,l}\sigma_\pm$	$\psi_{n,1,l}\sigma_\pm, \pm\psi_{n,-1,l}\sigma_\pm$
	± 2	$\psi_{n,2,l}\sigma_{\downarrow\downarrow}, \psi_{n,-2,l}\sigma_{\uparrow\uparrow}$	$\psi_{n,2,l}\sigma_{\downarrow\downarrow}, -\psi_{n,-2,l}\sigma_{\uparrow\uparrow}$

TABLE II. Basis states $\psi_{nml}\sigma_{s\bar{s}}$ categorized by the charge conjugation C according to Eq. (18).

$m + l$	$\mathsf{C} = 1$	$\mathsf{C} = -1$
even	$\psi_{n,m,l}\sigma_-$	$\psi_{n,m,l}\sigma_+, \psi_{n,m,l}\sigma_{\uparrow\uparrow}, \psi_{n,m,l}\sigma_{\downarrow\downarrow}$
odd	$\psi_{n,m,l}\sigma_+, \psi_{n,m,l}\sigma_{\uparrow\uparrow}, \psi_{n,m,l}\sigma_{\downarrow\downarrow}$	$\psi_{n,m,l}\sigma_-$

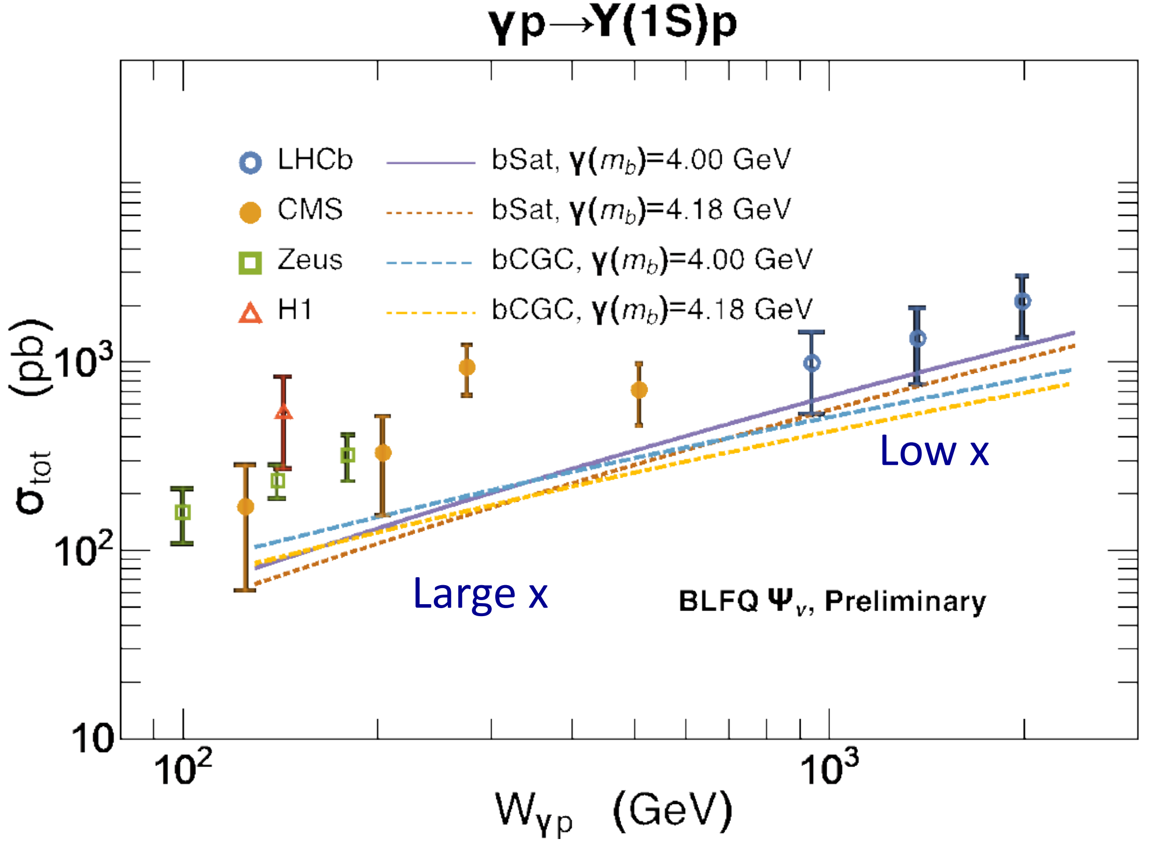
J/ψ Decay Constant

$$f_{\mathcal{V}}|_{m_j=0} = \sqrt{2N_c} \int_0^1 \frac{\mathrm{d}x}{\sqrt{x(1-x)}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \psi_{+/\mathcal{V}}^{(m_j=0)}(\vec{k}_\perp, x) \, ,$$

$$f_{\mathcal{V}}|_{m_j=1} = \frac{\sqrt{N_c}}{2m_{\mathcal{V}}} \int_0^1 \frac{\mathrm{d}x}{[x(1-x)]^{3/2}} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \Big\{ k^L [(1-2x)\psi_{+/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x) - \psi_{-/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x)] - \sqrt{2} m_f \psi_{\uparrow\uparrow/\mathcal{V}}^{(m_j=1)}(\vec{k}_\perp, x) \Big\} \, ,$$

$\Upsilon(1s)$ in γp at LHC

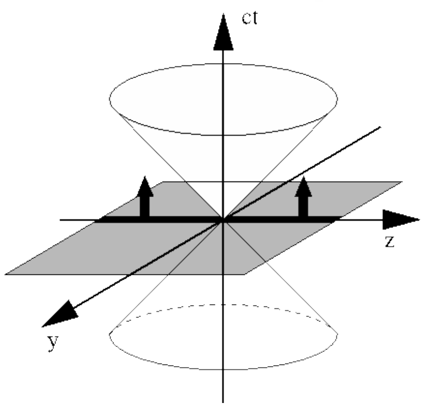
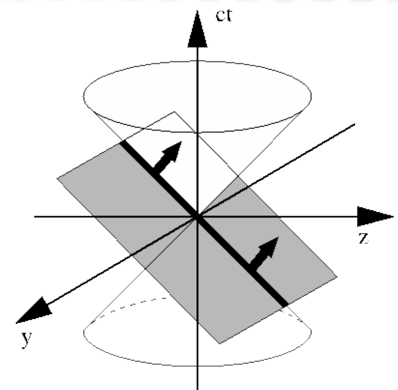
GC et al., in preparation



ZEUS, 2009.
H1, 2012.
CMS, 2016.
LHCb, 2016.

$$x \sim \frac{M_V^2}{W_{\gamma p}^2}$$

Equal time vs. Light-front Quantization

$t \square x^0$	$t \square x^+ = x^0 + x^3$
	 Dirac (1949)
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$H = P^0$	$P^- = P^0 - P^3$