Evgeny Epelbaum, Ruhr University Bochum

Workshop in celebration of James' 80th Jubilee
IMP CAS, Lanzhou, June 5, 2023

Nuclear chiral interactions: Recent developments

Picture taken by Peter Druck in Skiathos during Lightcone 2013+

on the occasion of James' 80th birthday

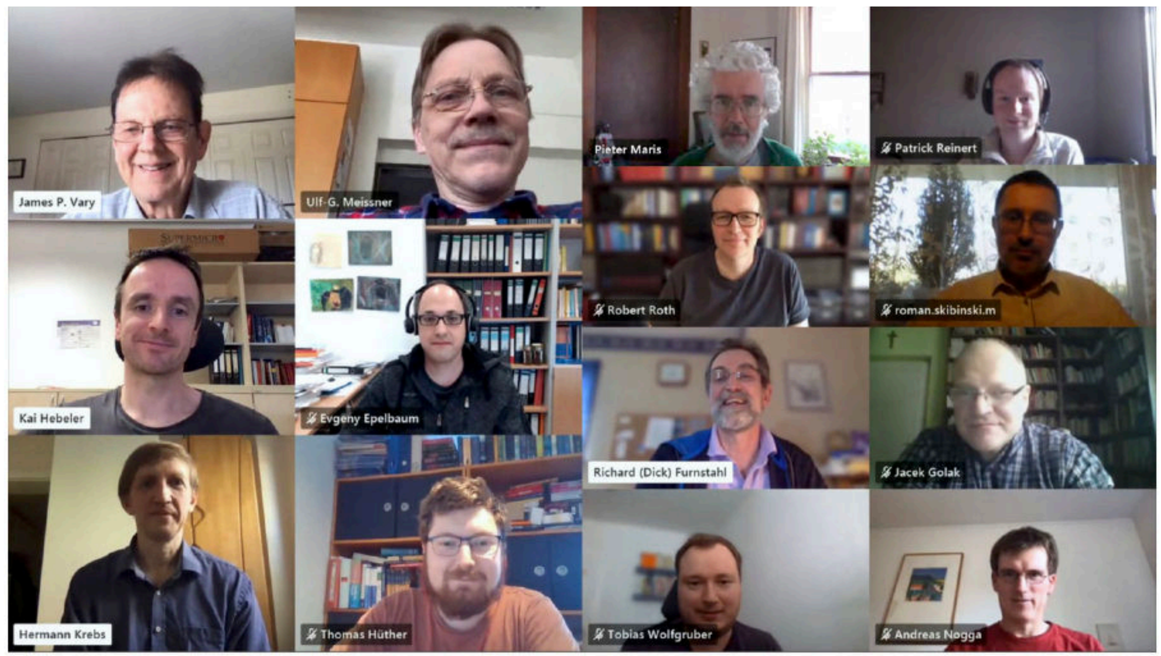
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council
NRW-FAIR

10th anniversary of the

Low-Energy Nuclear Physics International Collaboration

The first LENPIC meeting took place on July 4, 2013, at Ruhr University Bochum

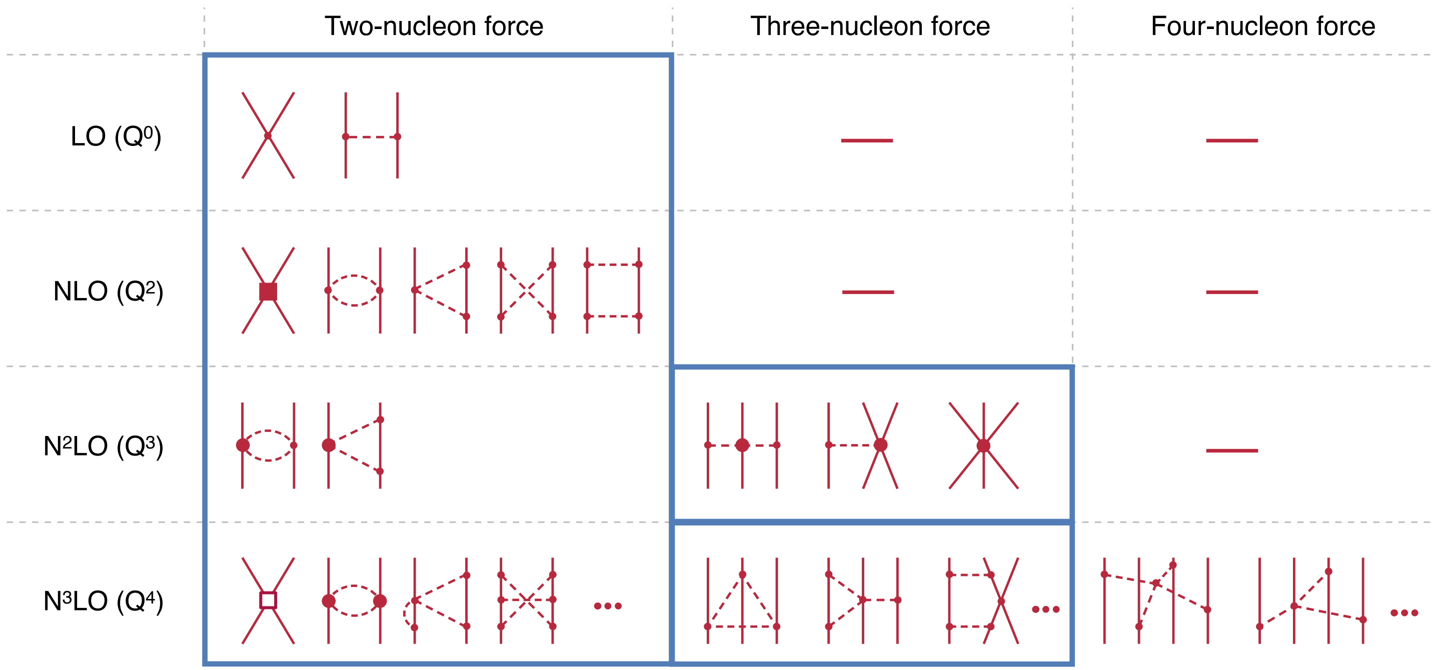
LENPIC



see: www.lenpic.org

„LENPIC aims to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding exchange currents.“

Chiral EFT for nuclear forces back in 2013…



- The Bochum-Bonn and Idaho NN chiral N³LO potentials: Nonlocal regulators, semi-quantitative description of NN data
- The leading 3NF (at N²LO) well established and implemented in the partial-wave basis
- Derivation of the N³LO contributions to the 3NF completed in 2011 Bernard, EE, Krebs, Meißner
  These corrections are parameter-free but involve complicated expressions and rich operator structure ⇒ manual partial wave decomposition not feasible…

Some milestones (LENPIC and beyond)

- Partial wave decomposition of the 3NF

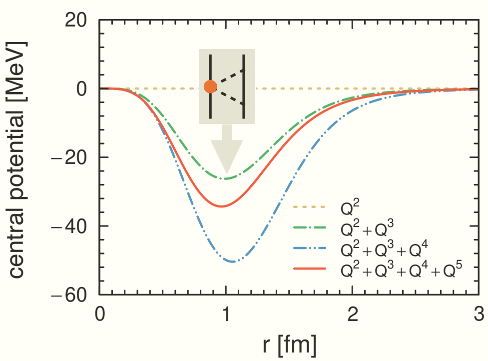
$$\underbrace{\langle p' q' \alpha' | V_{3N} | p q \alpha \rangle}_{|(l\,\lambda)L\,(s\,1/2)S\,(L\,S)J M_J\rangle} = \sum \int \underbrace{d\hat{p}'\, d\hat{q}'\, d\hat{p}\, d\hat{q}}_{\text{can be reduced to a 5-dimensional integration}}\, V_{3N}(\vec{p}\,', \vec{q}\,', \vec{p}, \vec{q})\ [\text{CG's}]\ Y^{\star}_{l'm'}(\hat{p}')\, Y^{\star}_{\lambda'\mu'}(\hat{q}')\, Y_{lm}(\hat{p})\, Y_{\lambda\mu}(\hat{q})$$

Golak et al., EPJA 43 (2010)

Still computationally involved (need $\sim 10^5 \times 10^5$ MEs); has been further optimized using the fact that (unregularized) 3NFs are either local (long-range) or polynomial **Hebeler et al., PRC 91 (15)**

- 3NF beyond N³LO

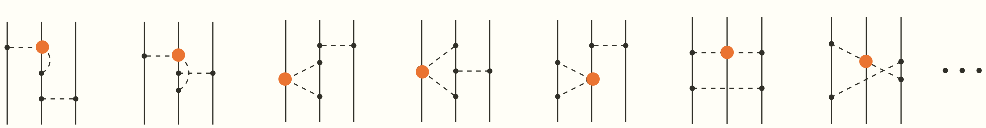
The strongest contribution to the 2π-exchange NN potential is generated from diagrams involving subleading πN vertices governed by the Δ:



This important physics is missing in the N³LO corrections to the 3NF

⇒ worked out N⁴LO corrections

Krebs, Gasparyan, EE PRC 85 (12), 88 (13)



- Nuclear current operators to N³LO

Worked out electromagnetic, weak and scalar nuclear currents completely up through N³LO

Kölling, EE, Krebs, Meißner, PRC 80 (2009), PRC 84 (2011); Krebs, EE, Meißner, Annals Phys. 378 (2017), Few Body Syst. 60 (2019); Eur. Phys. J. A 56 (2020)

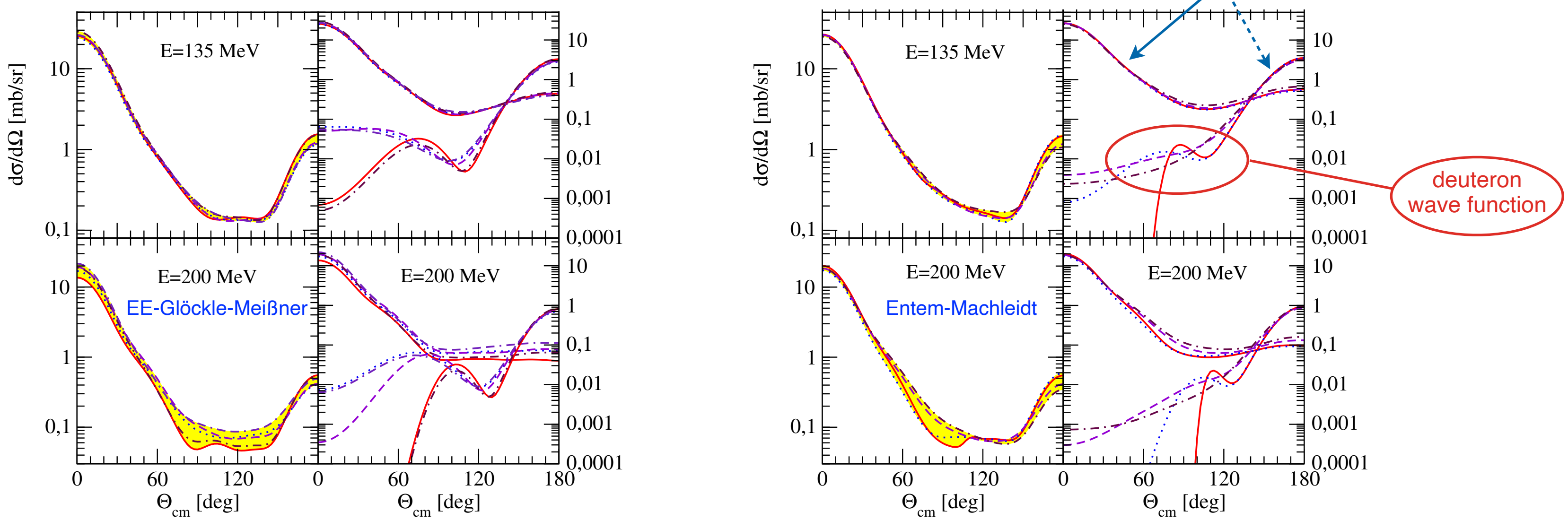
Regularization and cutoff artifacts

Non-locally regularized potentials (EE-Glöckle-Meißner ’05, Entem-Machleidt ’03, Entem-Machleidt-Nosyk ’17)

— long-range finite-cutoff artifacts: $V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\, e^{-\frac{p^4 + p'^4}{\Lambda^4}} \longrightarrow \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\left(1 - \frac{p^4 + p'^4}{\Lambda^4} + \dots\right)$

— elastic Nd scattering: $T\,|\phi\rangle = tP\,|\phi\rangle + tPG_0T\,|\phi\rangle, \quad \langle\phi'|\,U\,|\phi\rangle = \langle\phi'|\,PG_0^{-1}\,|\phi\rangle + \langle\phi'|\,PT\,|\phi\rangle$

Witala et al., J. Phys. G: Nucl. Part. Phys. 41 (14)



New LENPIC NN interactions (EE, Krebs, Meißner, EPJA 51 (15), PRL 115 (15), Reinert, Krebs, EE, EPJA 5(18))

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\, e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2}\, e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

Both issues solved!

SMS chiral NN potentials

Reinert, Krebs, EE, EPJA 54 (18) 86; PRL 126 (21) 092501

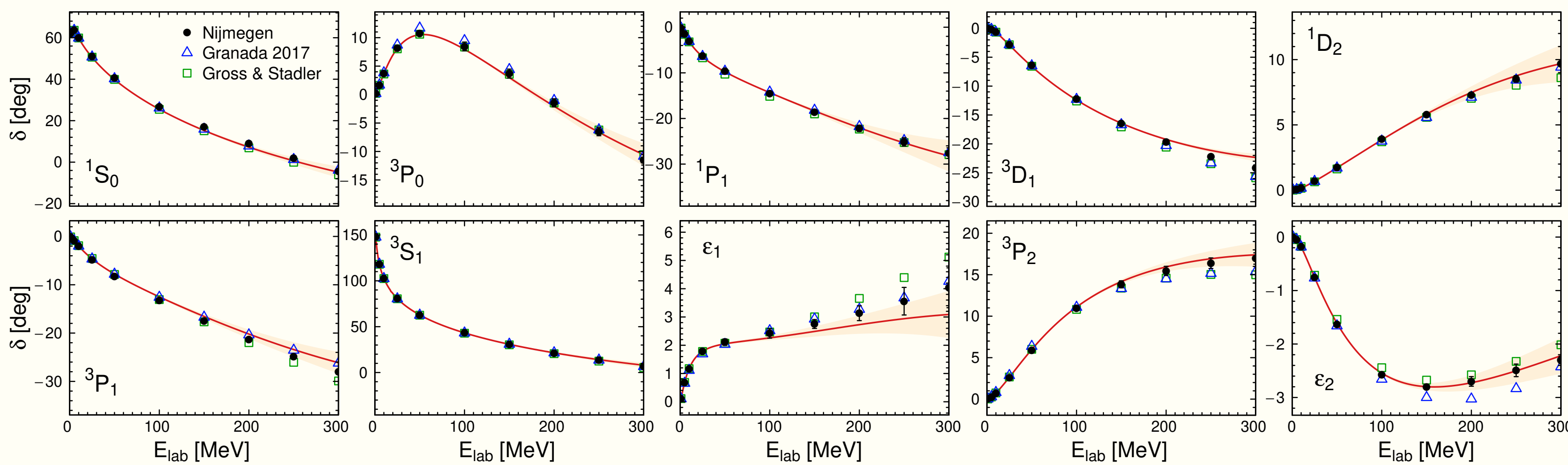
Statistically perfect description of mutually consistent NN scattering data

(own database of 2124 proton-proton + 2935 neutron-proton data below E_{lab} = 290 MeV)

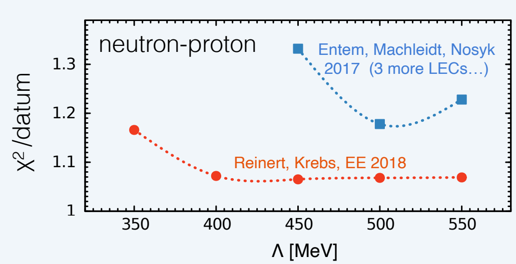
high-precision „realistic" potentials				*Idaho χEFT*		*Bochum SMS χEFT*	
Nijm I	Nijm II	Reid93	CD Bonn	$N^4LO^+_{450}$	$N^4LO^+_{500}$	$\mathbf{N^4LO^+_{450}}$	$\mathbf{N^4LO^+_{500}}$
1.061	1.070	1.078	1.042	2.019	1.203	**1.013**	**1.015**

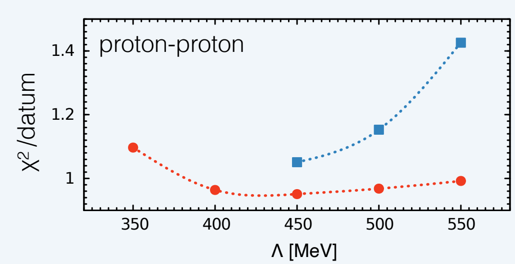
Reinert, Krebs, EE, 2021

Resulting neutron-proton phase shifts



Negligible residual cutoff dependence @N⁴LO⁺

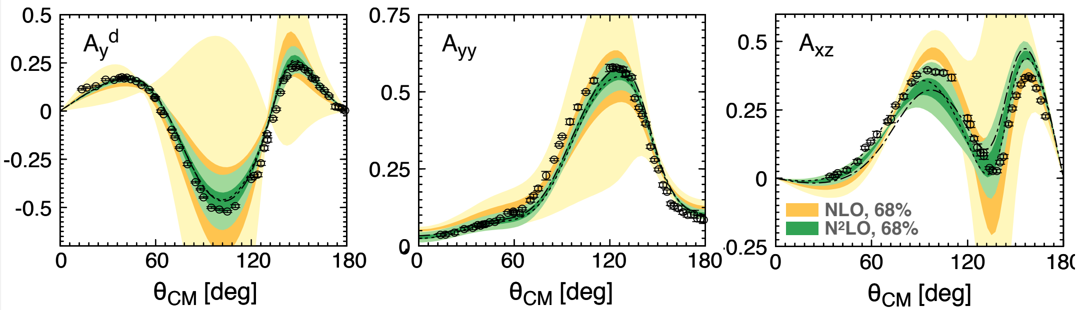




Few-N systems and light nuclei to N²LO

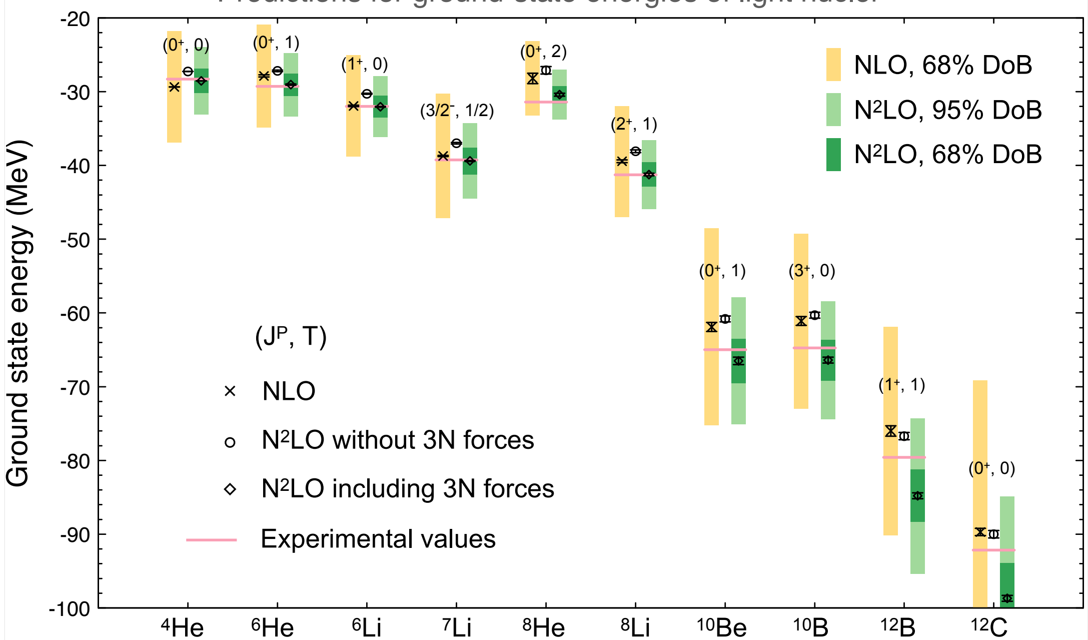
P. Maris et al. (LENPIC), Phys. Rev. C 103 (2021) 5, 054001

Tensor analyzing powers in Nd elastic scattering at 70 MeV

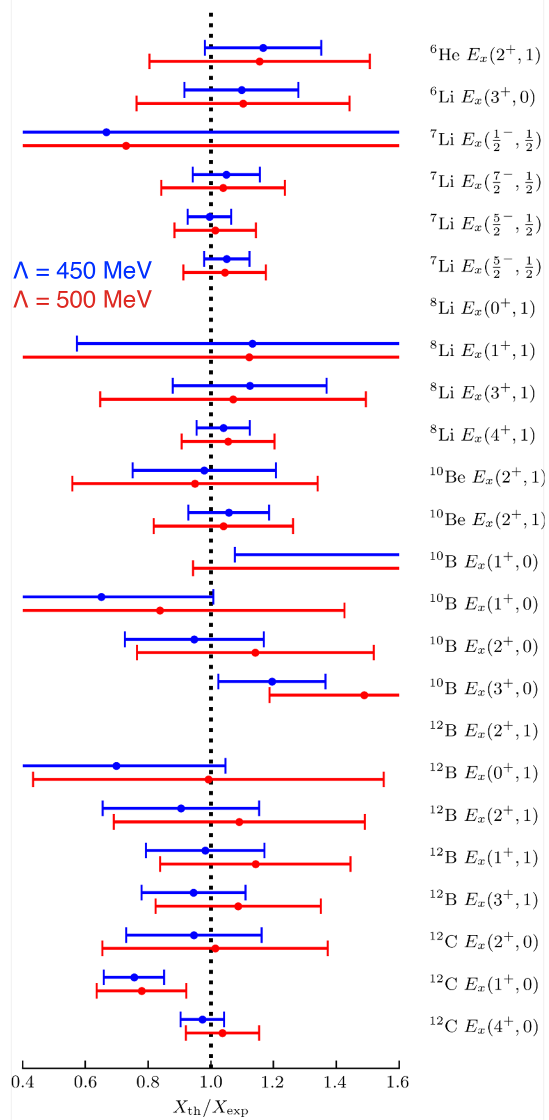


Bayesian approach to estimate truncation errors: BUQEYE (Furnstahl et al. ’15 - ’23), EE et al. ’20

Predictions for ground-state energies of light nuclei



Excitation energies

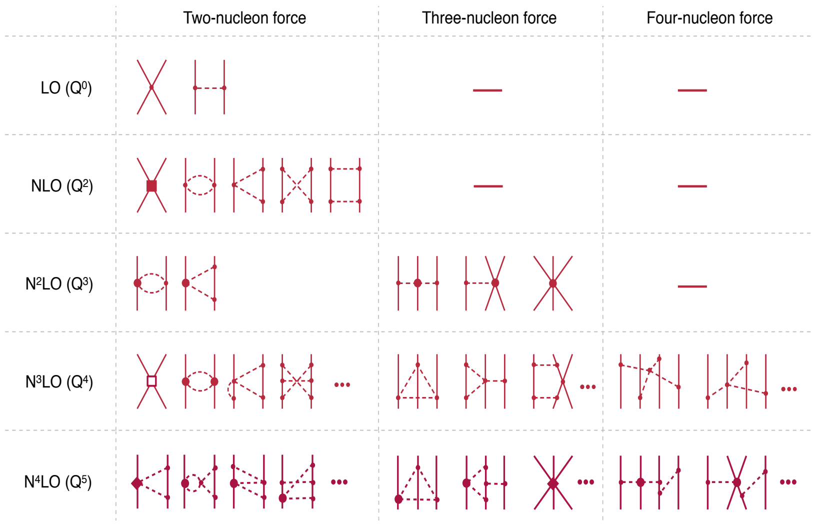


Regularization again…

— Nuclear potentials are derived using dimensional regularization

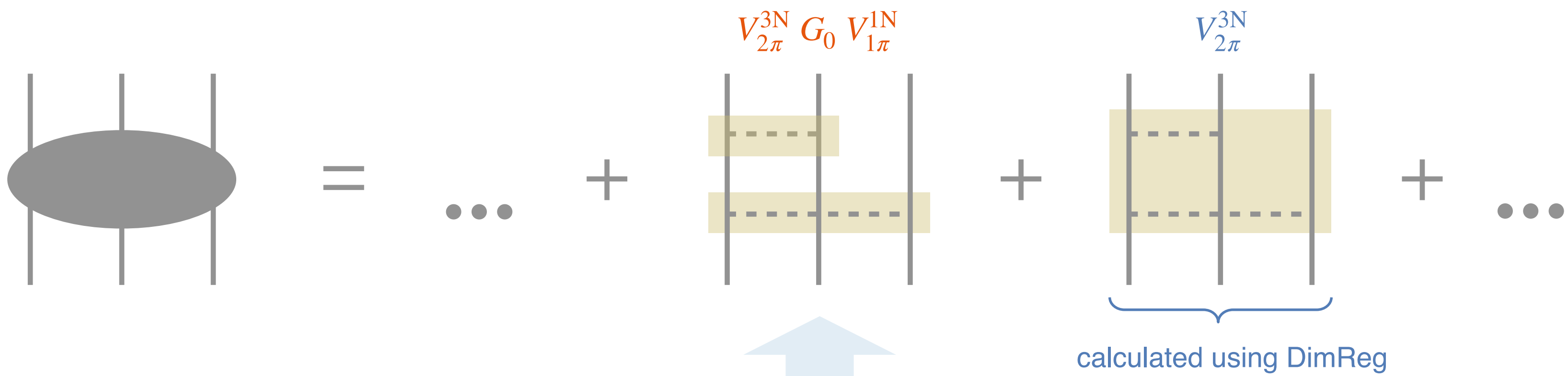
— Schrödinger equation is regularized using a cutoff

⇒ consistent procedure?



Regularization again…

Faddeev equation for 3N scattering:



$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\Big[\underbrace{\tau_1\cdot\tau_3\,(\vec q_3\cdot\vec\sigma_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec q_2\cdot\vec\sigma_3)}_{\text{violates chiral symmetry}}\Big]\frac{\vec q_3\cdot\vec\sigma_3}{q_3^3+M_\pi^2}+\ldots$$

If $V_{2\pi}^{3N}$ were calculated with a cutoff, the problematic divergence would cancel exactly. This issue affects all loop contributions beyond N²LO to 3NF and currents. In contrast, NN forces are not affected (at a fixed M_π).

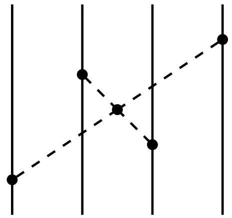
$\Rightarrow$ Re-derive nuclear forces & currents using **SYMMETRY PRESERVING** cutoff regularization

Hermann Krebs, EE: Work in progress using Gradient Flow Regularization

Example: gradient flow reg. of the 4NF

Hermann Krebs, EE, in preparation

Consider e.g. the contribution to the 4NF at N[3]LO involving a 4π-vertex:



Unregularized expression:

EE, PLB 639 (2006) 456; EPJA 34 (2007) 197

$$V_{4\mathrm{N}} = \frac{g_A^4}{2(2F_\pi)^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{\sigma}_3 \cdot \vec{q}_3 \; \vec{\sigma}_4 \cdot \vec{q}_4}{(\vec{q}_1^{\,2} + M_\pi^2)\,(\vec{q}_2^{\,2} + M_\pi^2)\,(\vec{q}_3^{\,2} + M_\pi^2)\,(\vec{q}_4^{\,2} + M_\pi^2)} \Big[(\vec{q}_1 + \vec{q}_2)^2 + M_\pi^2 \Big]$$

$$+ \text{ 3-pole terms } + \text{ all permutations}$$

Applying the gradient flow regularization method consistent with the 2NF yields:

Hermann Krebs, EE, preliminary

$$V_{4\mathrm{N}} = \frac{g_A^4}{2(2F_\pi)^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{\sigma}_3 \cdot \vec{q}_3 \; \vec{\sigma}_4 \cdot \vec{q}_4}{(\vec{q}_1^{\,2} + M_\pi^2)\,(\vec{q}_2^{\,2} + M_\pi^2)\,(\vec{q}_3^{\,2} + M_\pi^2)\,(\vec{q}_4^{\,2} + M_\pi^2)} \Big[(\vec{q}_1 + \vec{q}_2)^2 + M_\pi^2 \Big]$$

$$\times \left(4\, e^{-\frac{\vec{q}_2^{\,2} + M_\pi^2}{\Lambda^2}} \, e^{-\frac{\vec{q}_3^{\,2} + M_\pi^2}{\Lambda^2}} \, e^{-\frac{\vec{q}_4^{\,2} + M_\pi^2}{\Lambda^2}} - 3\, e^{-\frac{\vec{q}_1^{\,2} + M_\pi^2}{2\Lambda^2}} \, e^{-\frac{\vec{q}_2^{\,2} + M_\pi^2}{2\Lambda^2}} \, e^{-\frac{\vec{q}_3^{\,2} + M_\pi^2}{2\Lambda^2}} \, e^{-\frac{\vec{q}_4^{\,2} + M_\pi^2}{2\Lambda^2}} \right)$$

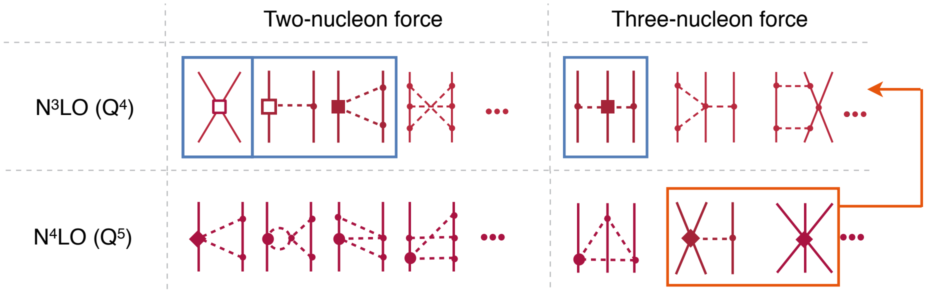
$$+ \text{ 3-pole terms } + \text{ all permutations}$$

Off-shell ambiguities of NN forces

Off-shell effects cannot be observed in QFT. How does this work out in χEFT?

Off-shell ambiguities of nuclear forces

Off-shell ambiguities first appear at N³LO:



— long-range 1/m corrections: β_8, β_9
(also in the current operators)

— NN contacts ~Q^4 ($^1S_0$, $^3S_1$, $^3S_1$-$^3D_1$)

For example: $\langle p', {}^1S_0 | V_{\rm cont} | p, {}^1S_0 \rangle = \underbrace{\tilde{C}_{1S0}}_{\text{tuned to the scatt. length}} + \underbrace{C_{1S0}(p'^2 + p^2)}_{\text{tuned to the effective range}} + \underbrace{D_{1S0}p^2p'^2 + D_{1S0}^{\rm off}(p'^2 - p^2)^2}_{\text{tuned to the first shape parameter}}$

$\Rightarrow D_{1S0}^{\rm off}$ cannot be fixed from NN data Hammer, Furnstahl '00; Beane, Savage '01; Reinert, Krebs, EE '18

Thus, it should be possible to eliminate the off-shell contacts via a suitable UT. Indeed:

$$U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3} \quad \text{with} \quad T_1 \propto (p_1'^2 + p_2'^2 - p_1^2 - p_2^2), \quad T_2 \propto (p_1'^2 + p_2'^2 - p_1^2 - p_2^2)\, \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad T_3 = \ldots$$

$$\Rightarrow \delta H = U^\dagger H U - H = \sum_i \gamma_i \Big[\Big(H_{\rm kin} + V_{1\pi}^{(0)} + V_{\rm cont}^{(0)} + \mathcal{O}(Q^2) \Big), T_i \Big] + \mathcal{O}(\gamma_i^2)$$

$V_{1\pi}^{(0)} + V_{\rm cont}^{(0)}$: induce N⁴LO 3NFs with enhanced LECs ($m_N \sim \Lambda_b^2/Q \gg \Lambda_b$) $\Rightarrow$ contribute at N³LO

$H_{\rm kin}$: induce off-shell NN contact interactions; setting $D_{1S0}^{\rm off} = 0$ requires choosing $\gamma_i \sim m_N/\Lambda_b$

Implications

see also Girlanda, Marcucci, Kiebsky, Viviani, PRC 102 (20)

— if power counting works, taking any natural-size $D_i^{\rm off}$ should yield similar NN fits

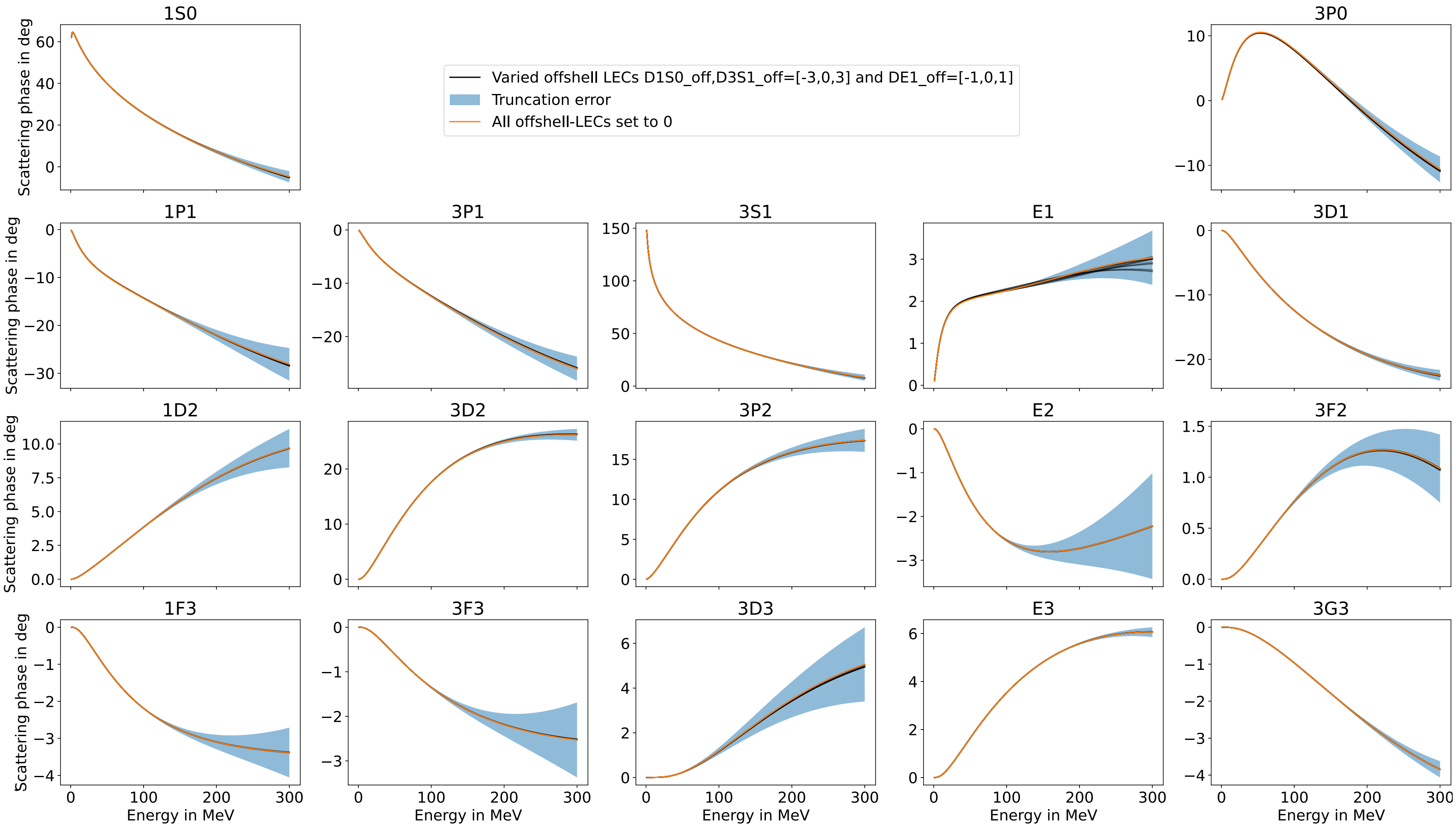
— if setting $D_i^{\rm off} = 0$, some N⁴LO 3NFs depending on 3 LECs must be promoted to N³LO

— alternatively, $D_i^{\rm off}$ in $V_{\rm NN}$ can be fixed from 3N data at N³LO (they become redundant at N⁴LO)

Off-shell ambiguities of nuclear forces

Sven Heihoff et al., work in progress

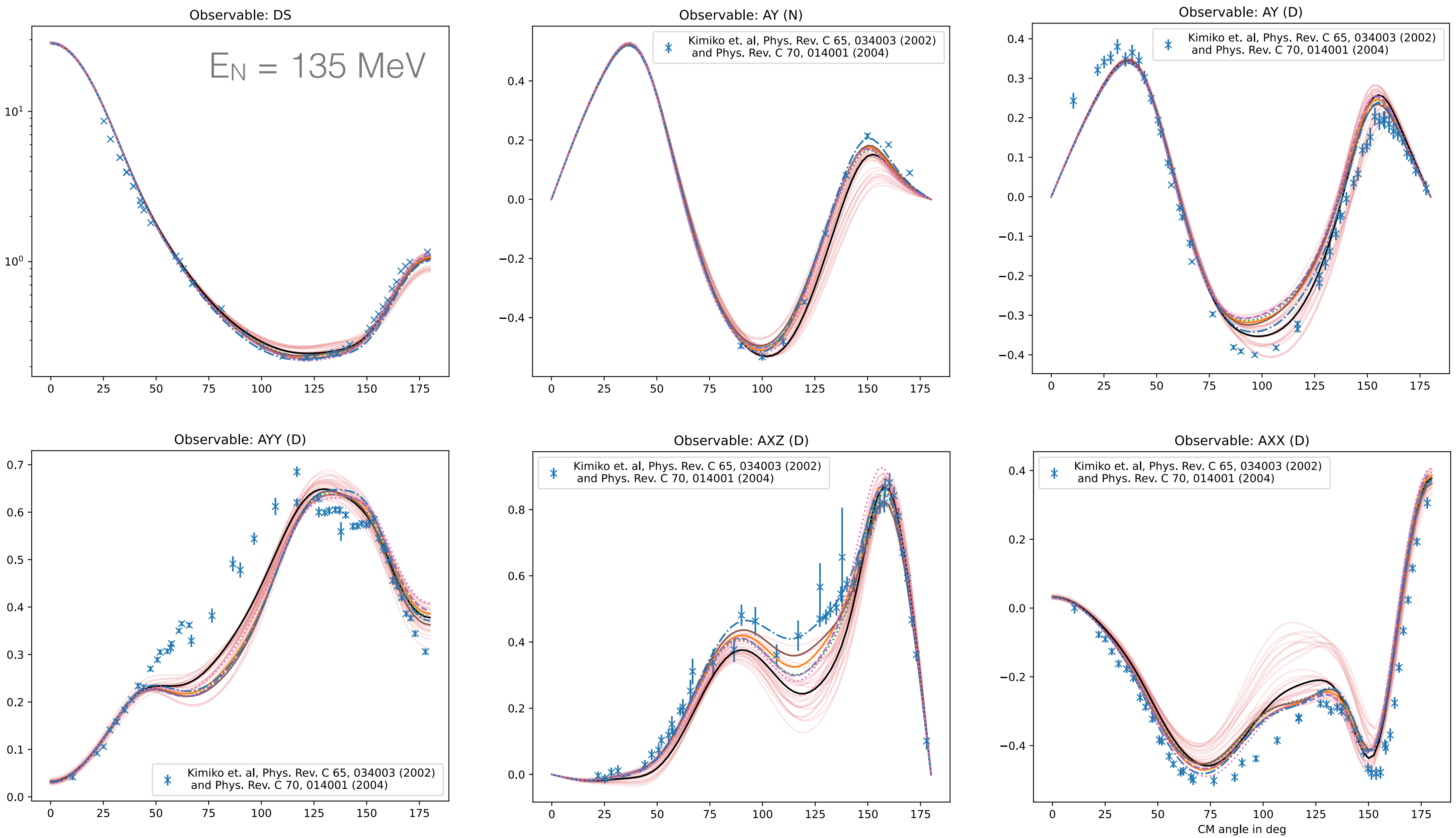
Extended the SMS N⁴LO⁺ potential ($D_i^{\rm off} = 0$) with 26 potentials: $D_S^{\rm off} = \{-3,0,3\}$, $D_{\epsilon 1}^{\rm off} = \{-1,0,1\}$



Consistently with power counting, nearly phase shift equivalent: $\chi^2_{\rm datum} = 1.010\ldots1.014$

Sensitivity of Nd elastic scattering observables

Sven Heihoff et al., work in progress



— emulating 3N scattering results works efficiently using interpolation in D_i^{off}

— getting ready to incorporate consistently regularized 3NFs@N^{3}LO once available

LENPIC: Productive decade & exciting future ahead

Happy Birthday, James!