Towards Nuclear Physics as Precision Science

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by CAS, PIFI

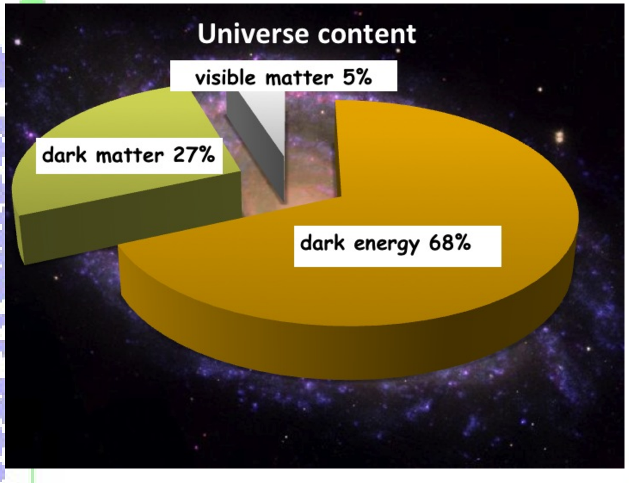
by VolkswagenStftung

CONTENTS

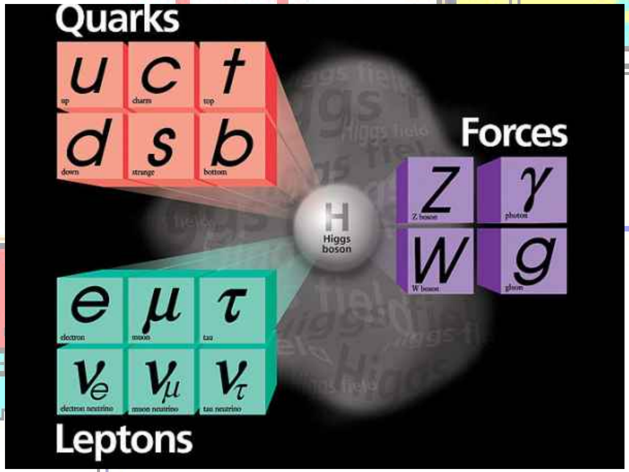
The BIG Picture

WHY NUCLEAR PHYSICS?

- **The matter we are made off**



- **The last frontier of the SM**



- **Access to the Multiverse**

⇒ Precision mandatory

AB INITIO NUCLEAR STRUCTURE and SCATTERING

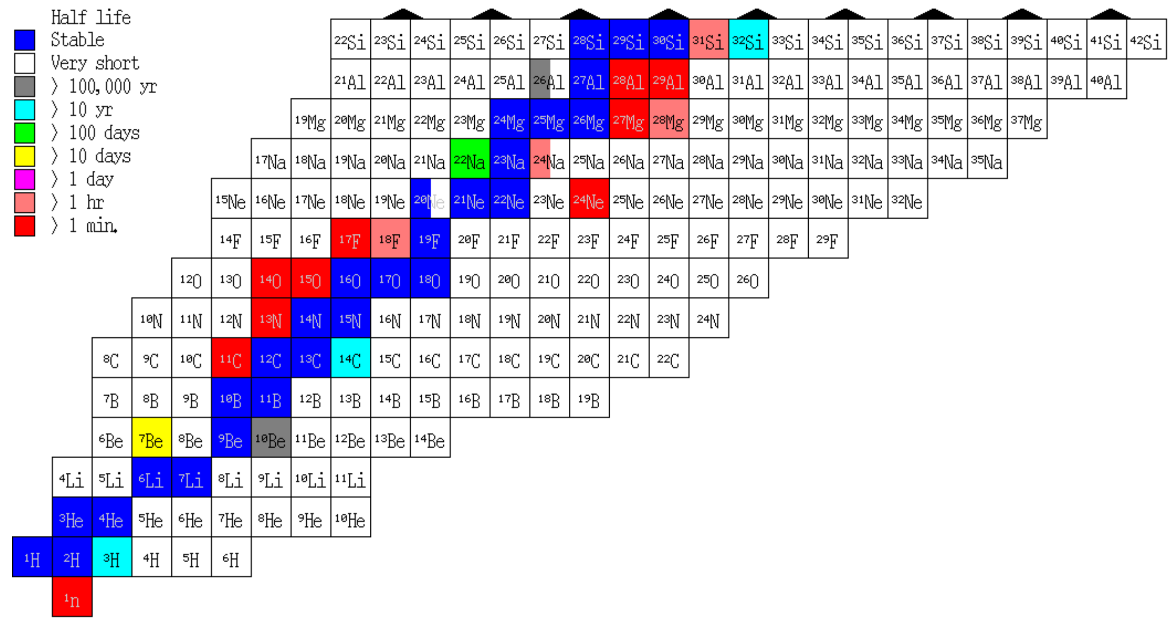
- Nuclear structure:

  ⋆ limits of stability

  ⋆ 3-nucleon forces

  ⋆ alpha-clustering

  ⋮



© National Nuclear Data Center

- Nuclear scattering: processes relevant for nuclear astrophysics

  ⋆ alpha-particle scattering: $^4\text{He} + {}^4\text{He} \to {}^4\text{He} + {}^4\text{He}$

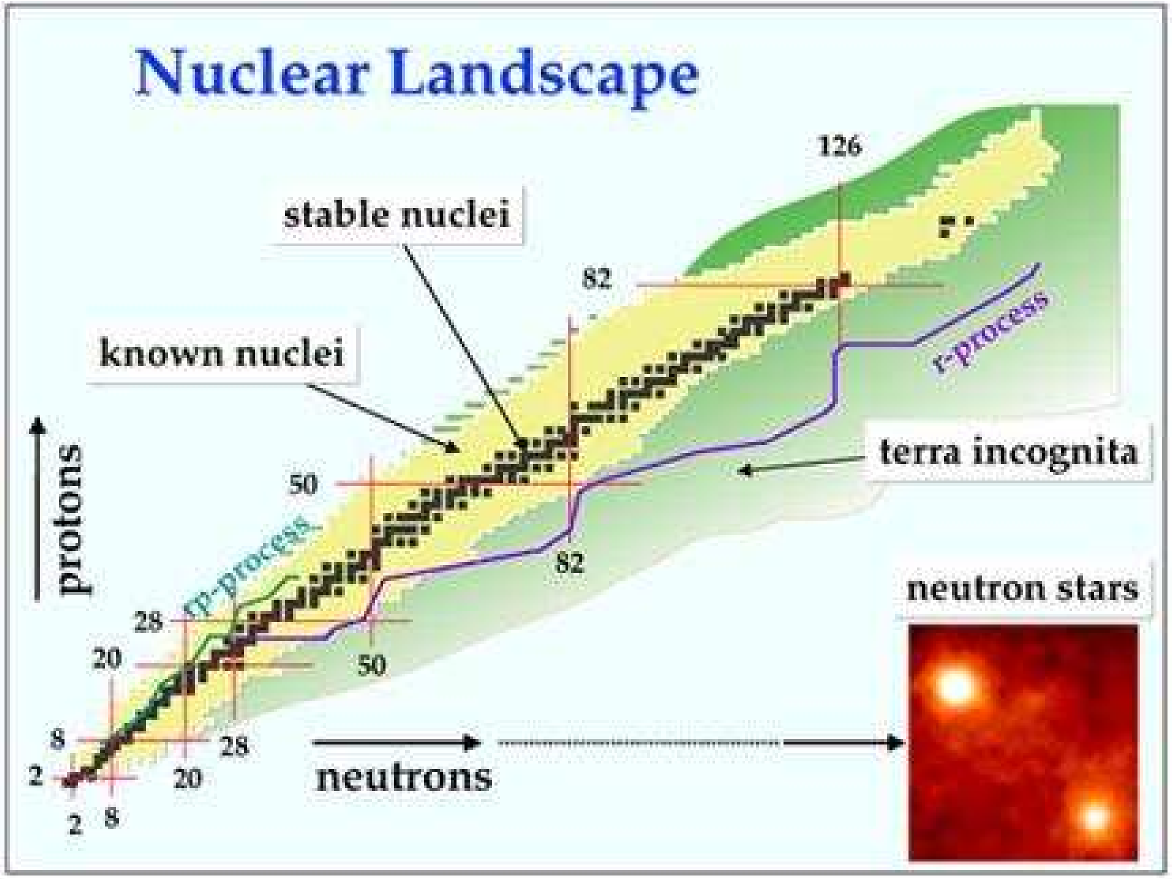
  ⋆ triple-alpha reaction: $^4\text{He} + {}^4\text{He} + {}^4\text{He} \to {}^{12}\text{C} + \gamma$

  ⋆ alpha-capture on carbon: $^4\text{He} + {}^{12}\text{C} \to {}^{16}\text{O} + \gamma$

  ⋮

THE NUCLEAR LANDSCAPE: AIMS & METHODS

- Theoretical methods:
  - Lattice QCD: *A = 0,1,2, . . .*
  - NCSM, Faddeev-Yakubowsky, GFMC, ... : *A = 3 - 16*
  - coupled cluster, . . .: *A = 16 -100*
  - density functional theory, . . .: $A \geq 100$



- Chiral EFT:
  - provides **accurate 2N, 3N and 4N forces**
  - successfully applied in light nuclei with *A = 2, 3, 4*
  - combine with known many-body methods
    $\hookrightarrow$ various talks to come/ LENPIC
  - combine with simulation methods to get to larger A

$\Rightarrow$ Nuclear Lattice Effective Field Theory

Basics of nuclear lattice simulations

for an easy intro, see: UGM, Nucl. Phys. News **24** (2014) 11

for an early review, see: D. Lee, Prog. Part. Nucl. Phys. **63** (2009) 117

upcoming textbook, see: T. Lähde, UGM, Springer Lecture Notes in Physics

NUCLEAR LATTICE EFFECTIVE FIELD THEORY

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem

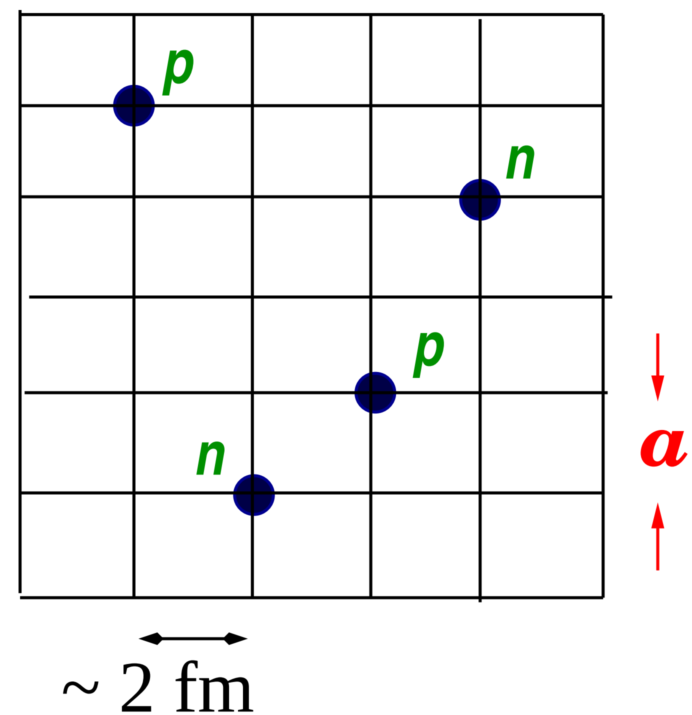
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\max} = \frac{\pi}{a} \simeq 314 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

  [or a more sophisticated (correlated) initial/final state]

Euclidean time

- Transient energy

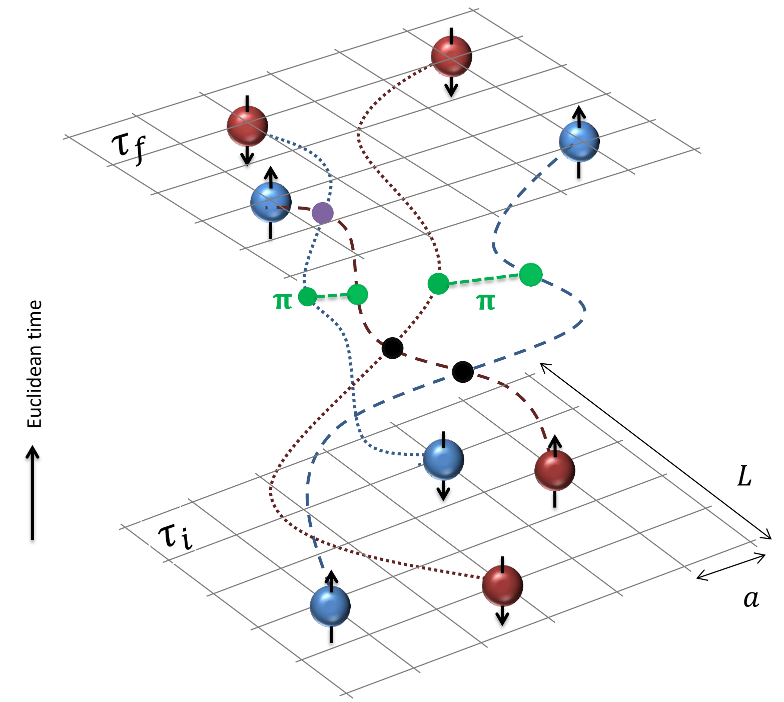
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

→ ground state: $E_A^0 = \lim_{\tau\to\infty} E_A(\tau)$

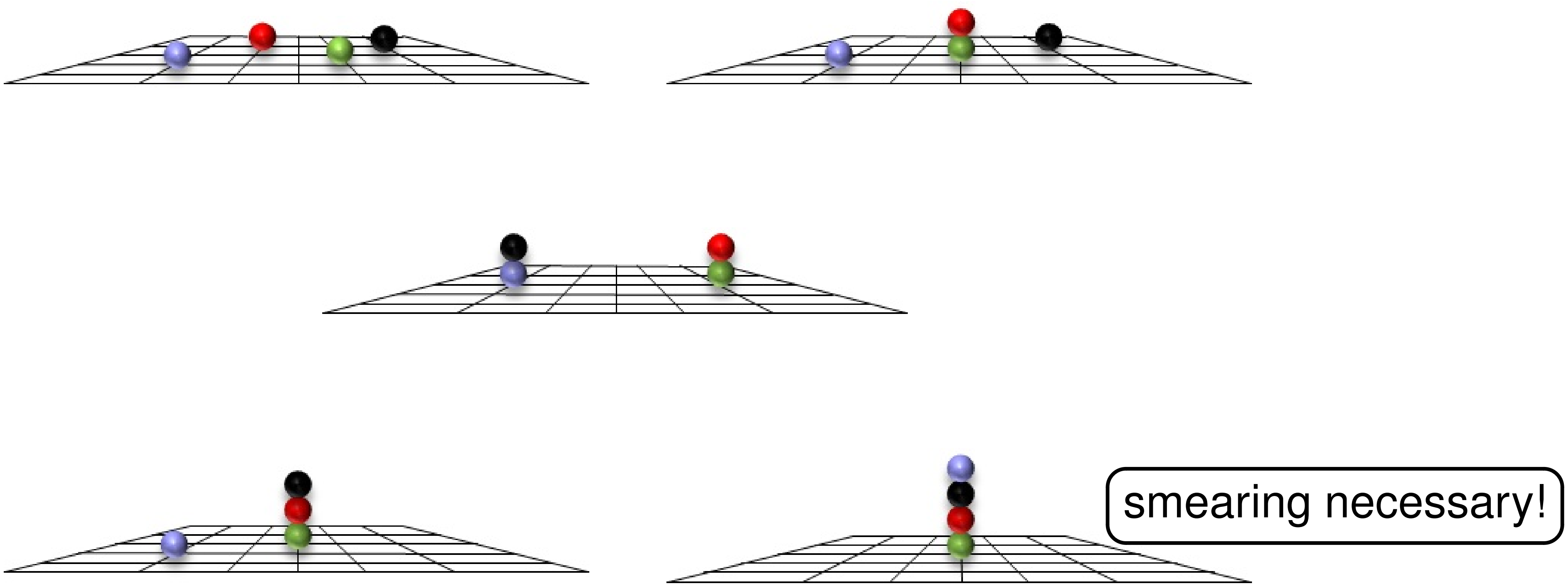
- Exp. value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A \rangle$$

$$\lim_{\tau\to\infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} | \Psi_A \rangle$$



CONFIGURATIONS



⇒ all *possible* configurations are sampled

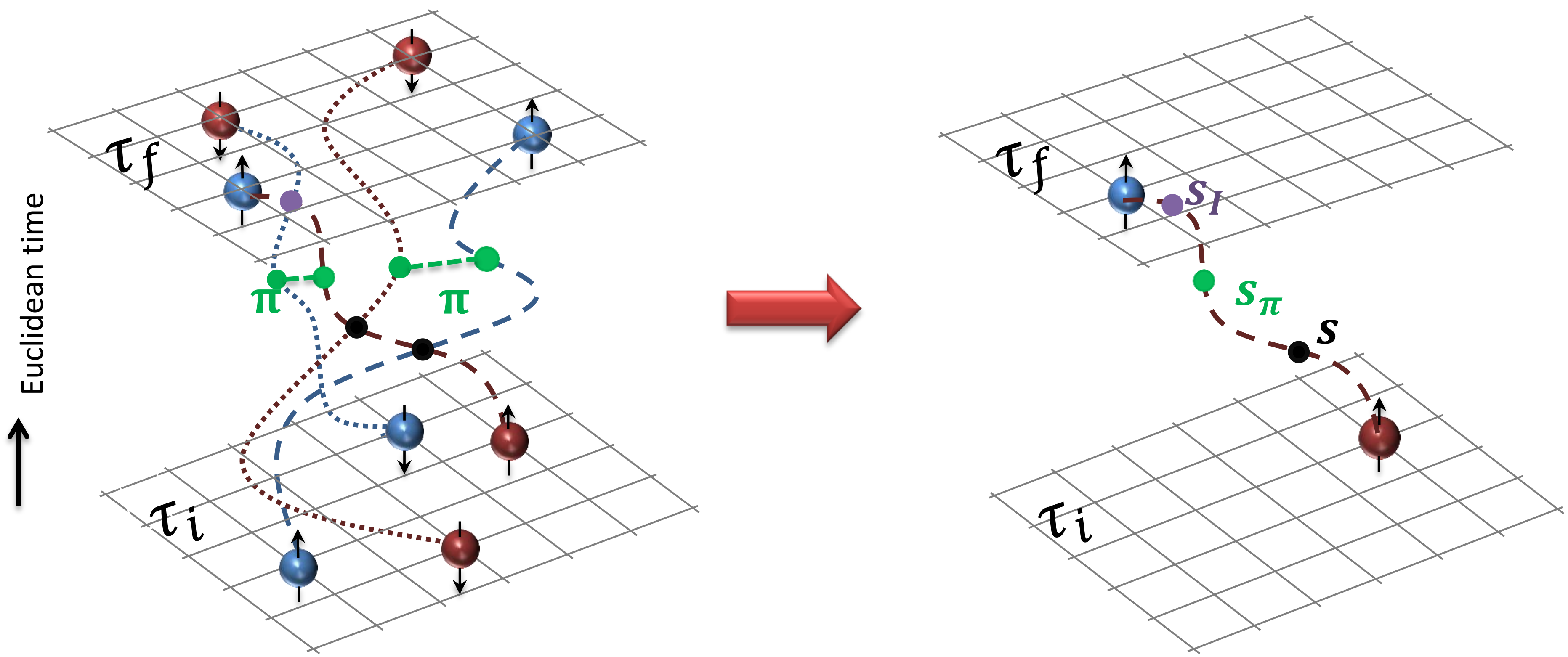
⇒ preparation of *all possible* initial/final states

⇒ *clustering* emerges *naturally*

AUXILIARY FIELD METHOD

- Represent interactions by auxiliary fields:

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds\, \exp\left[-\frac{s^2}{2} + \sqrt{C}\, s\left(N^\dagger N\right)\right]$$

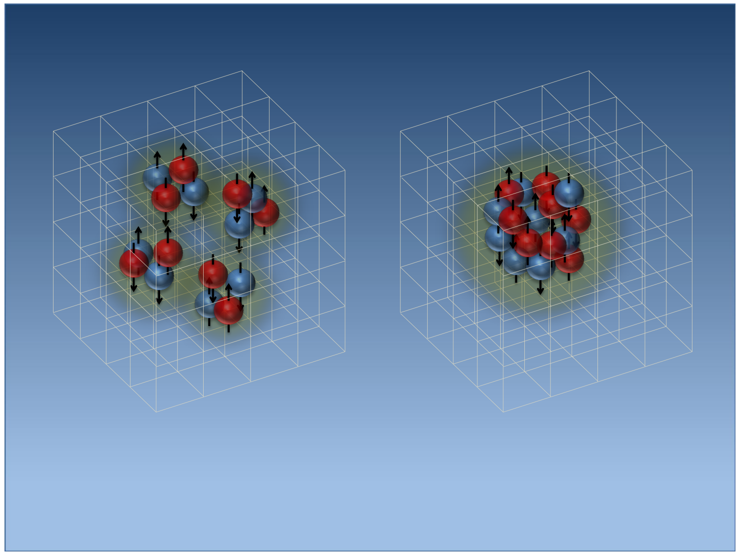


COMPUTATIONAL EQUIPMENT

- Past = JUQUEEN (BlueGene/Q)
- Present = JUWELS (modular system)

12 Pflops

Lattice: some results



NLEFT

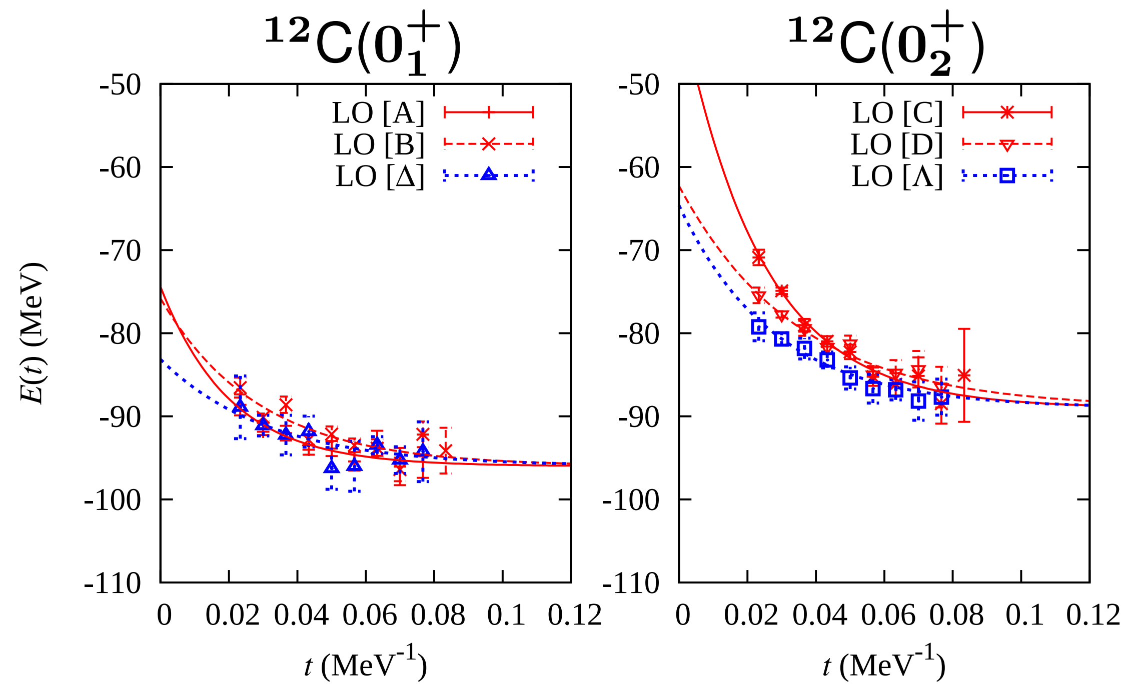
Epelbaum, Krebs, Lähde, Lee, Luu, UGM, Rupak + post-docs + students

FIXING PARAMETERS and FIRST RESULTS

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A **45** (2010) 335; . . .

- some groundstate energies and differences [NNLO, 11+2 LECs]

	E [MeV]	NLEFT	Exp.
old algorithm	^{3}He - ^{3}H	0.78(5)	0.76
	^{4}He	−28.3(6)	−28.3
	^{8}Be	−55(2)	−56.5
	^{12}C	−92(3)	−92.2
new algorithm	^{16}O	−131(1)	−127.6
	^{20}Ne	−166(1)	−160.6
	^{24}Mg	−198(2)	−198.3
	^{28}Si	−234(3)	−236.5



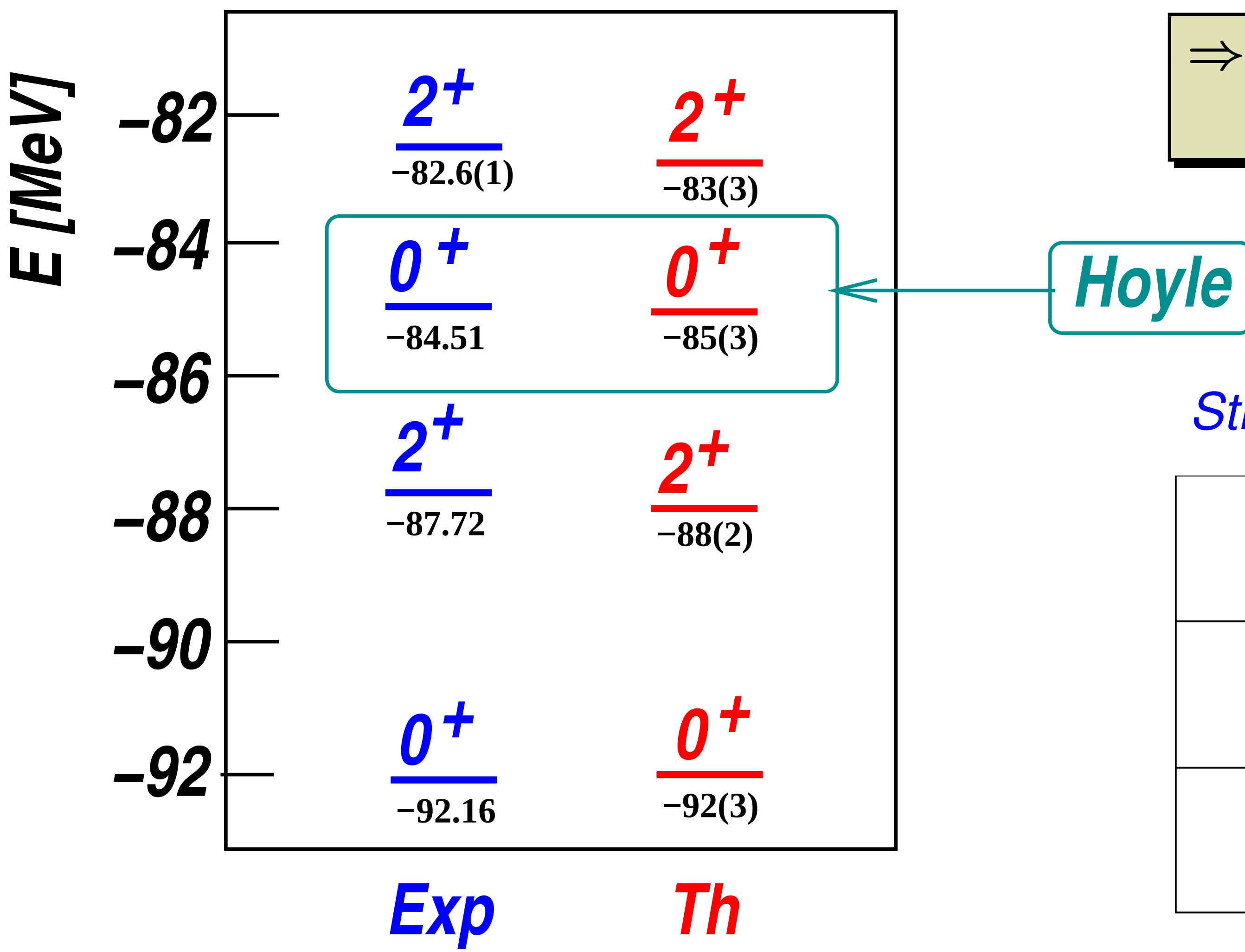
- promising results ⇒ uncertainties down to the 1% level
- excited states more difficult ⇒ projection MC method + triangulation

BREAKTHROUGH: SPECTRUM of CARBON-12

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 106 (2011) 192501

Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. 109 (2012) 252501

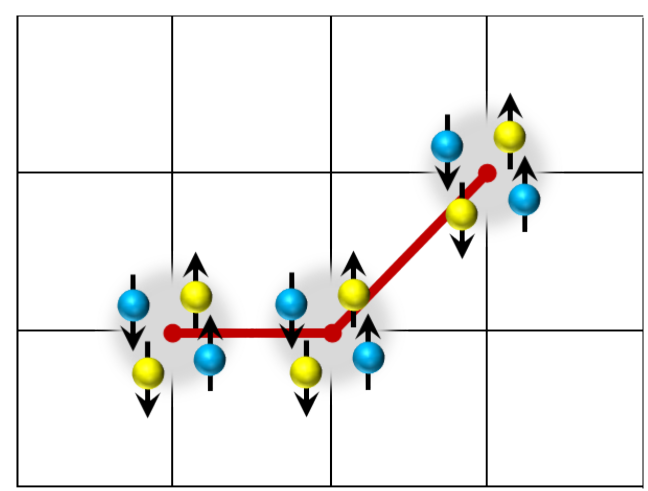
- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and “some” human work)



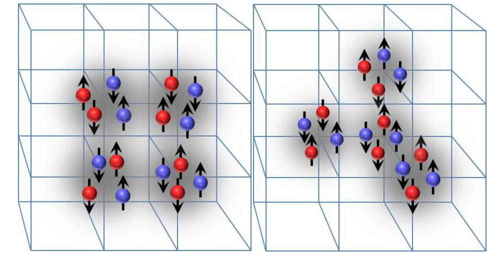
$\Rightarrow$ First ab initio calculation of the Hoyle state $\surd$

[see also Feldmeier & Neff, FMD]

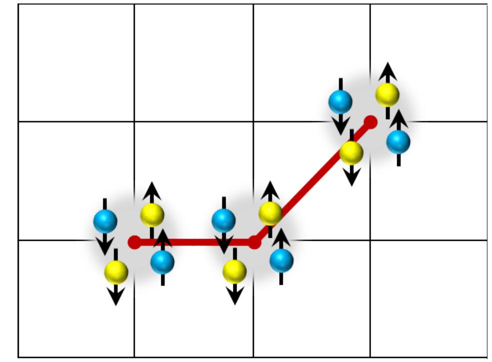
Structure of the Hoyle state:

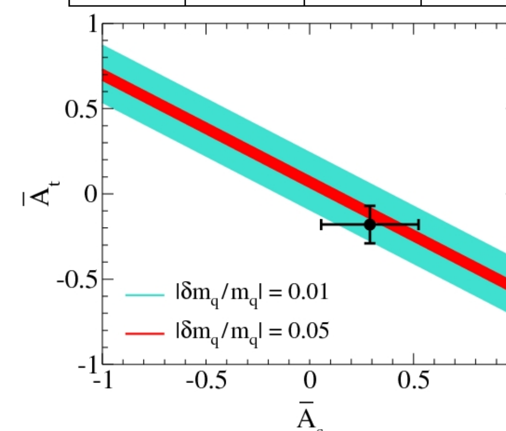


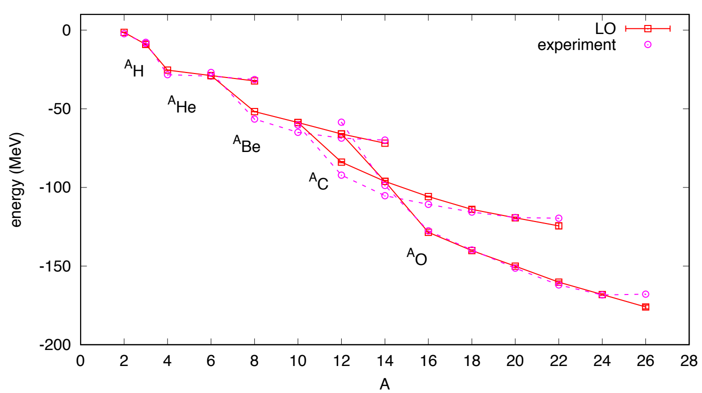
RESULTS from LATTICE NUCLEAR EFT

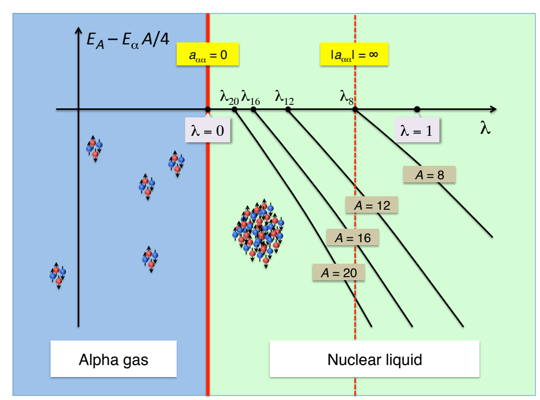


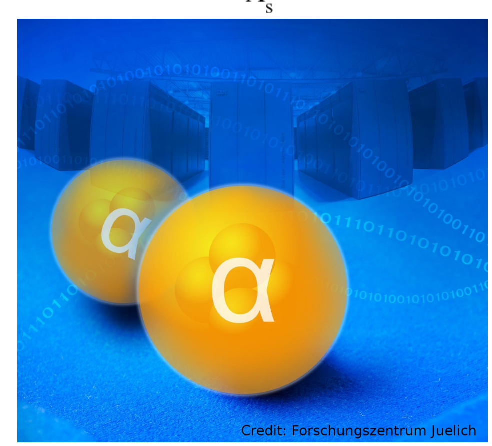
- ☐ Lattice EFT calculations for A=3,4,6,12 nuclei, PRL 104 (2010) 142501
- ☐ *Ab initio* calculation of the Hoyle state, PRL 106 (2011) 192501
- ☐ Structure and rotations of the Hoyle state, PRL 109 (2012) 142501
- ☐ Validity of Carbon-Based Life as a Function of the Light Quark Mass PRL 110 (2013) 142501
- ☐ *Ab initio* calculation of the Spectrum and Structure of ^{16}O, PRL 112 (2014) 142501
- ☐ *Ab initio* alpha-alpha scattering, Nature 528 (2015) 111
- ☐ Nuclear Binding Near a Quantum Phase Transition, PRL 117 (2016) 132501
- ☐ *Ab initio* calculations of the isotopic dependence of nuclear clustering, PRL 119 (2017) 222505



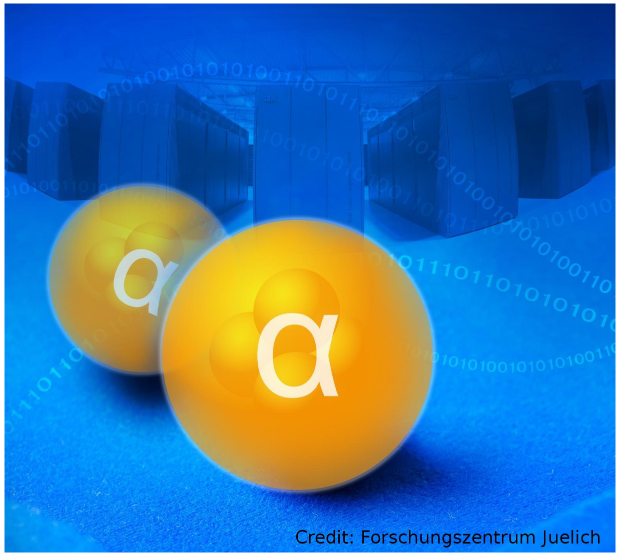








Ab initio calculation of α-α scattering

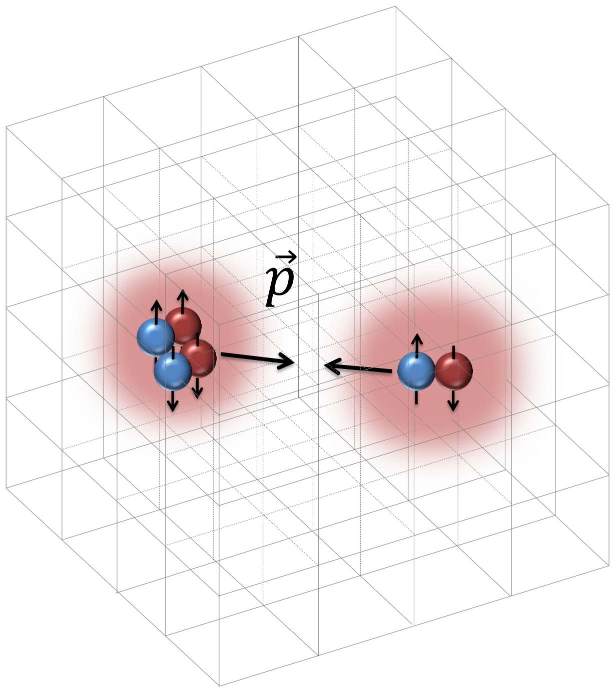


Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, UGM,
Nature **528** (2015) 111 [arXiv:1506.03513]

NUCLEUS–NUCLEUS SCATTERING on the LATTICE

- Processes involving α-particles and α-type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars

- Ab initio calculations of scattering and reactions suffer from computational scaling with the number of nucleons in the clusters



Lattice EFT computational scaling $\Rightarrow$ $(A_1 + A_2)^2$

Rupak, Lee, Phys. Rev. Lett. **111** (2013) 032502
Pine, Lee, Rupak, Eur. Phys. J. A **49** (2013) 151
Elhatisari, Lee, Phys. Rev. C **90** (2014) 064001
Elhatisari et al., Phys.Rev. C **92** (2015) 054612
Elhatisari, Lee, UGM, Rupak, Eur. Phys. J. A **52** (2016) 174

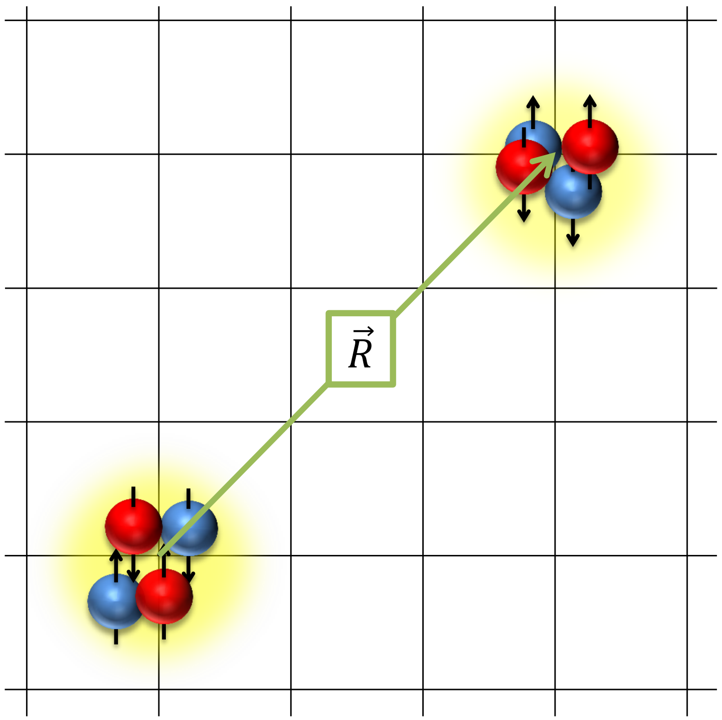
ADIABATIC PROJECTION METHOD

- Basic idea to treat scattering and inelastic reactions:
  split the problem into two parts

First part:
use Euclidean time projection to construct an *ab initio* low-energy cluster Hamiltonian, called the **adiabatic Hamiltonian**

Second part:
compute the two-cluster scattering phase shifts or reaction amplitudes using the adiabatic Hamiltonian

ADIABATIC PROJECTION METHOD II



- Construct a low-energy effective theory for clusters

- Use initial states parameterized by the relative separation between clusters

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r}+\vec{R}\rangle \otimes \vec{r}$$

- project them in Euclidean time with the chiral EFT Hamiltonian H

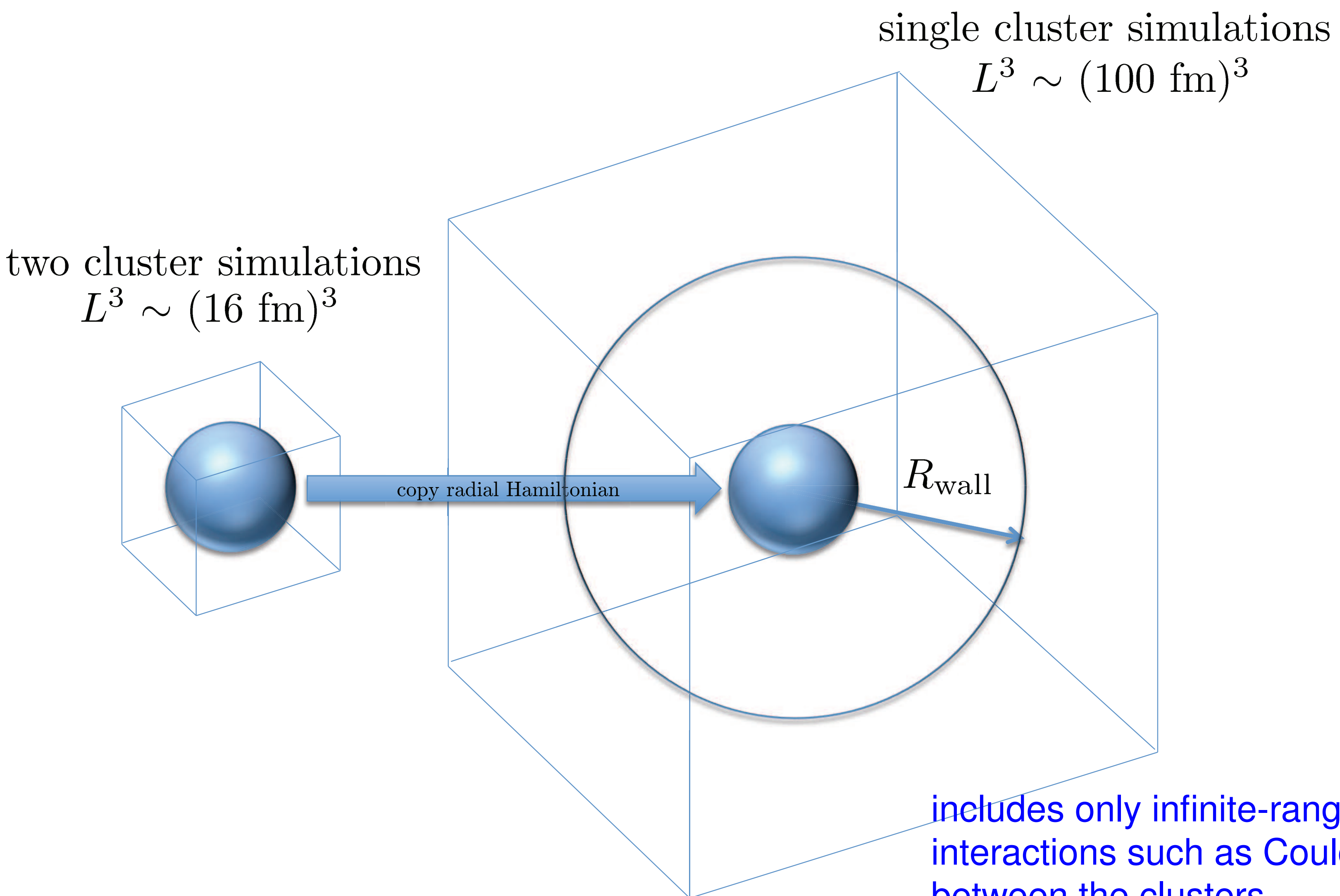
$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

$\to$ "dressed cluster states" (polarization, deformation, Pauli)

- Adiabatic Hamiltonian (requires norm matrices)

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

ADIABATIC HAMILTONIAN plus COULOMB



includes only infinite-range interactions such as Coulomb between the clusters

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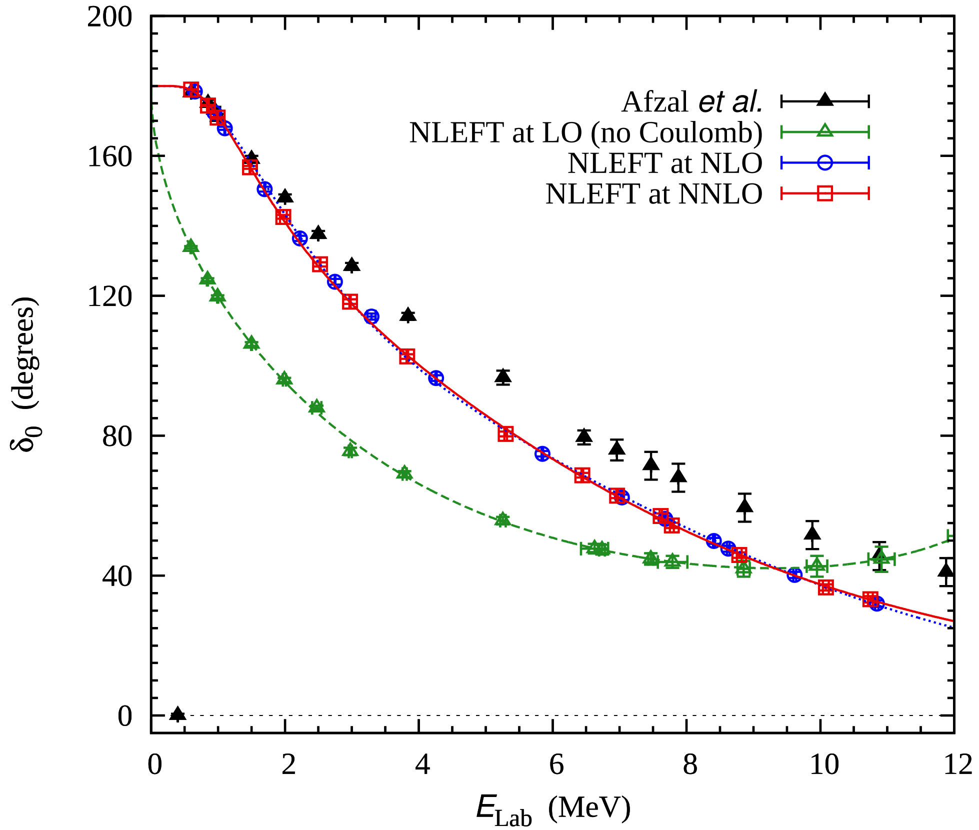
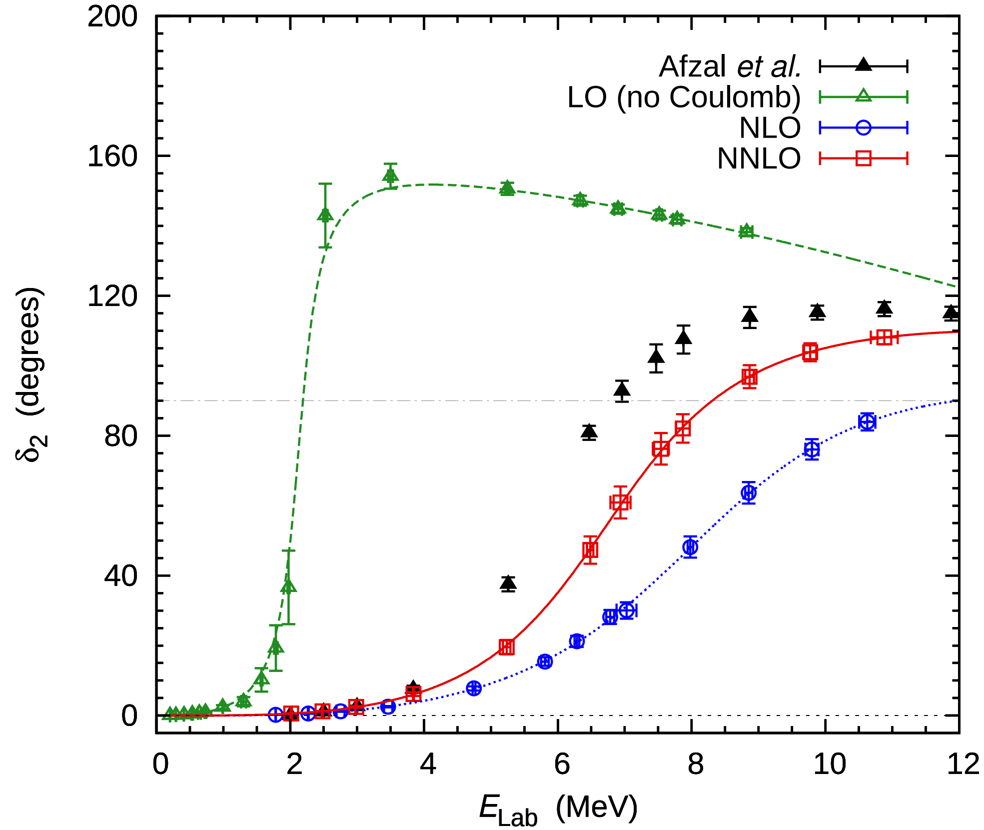
PHASE SHIFTS

- Same NNLO Lagrangian as used for the study of ^{12}C and ^{16}O





$E_R^{\rm NNLO} = -0.11(1)$ MeV [$+0.09$ MeV]

$E_R^{\rm NNLO} = 3.27(12)$ MeV [$2.92(18)$ MeV]

$\Gamma_R^{\rm NNLO} = 2.09(16)$ MeV [$1.35(50)$ MeV]

Data: Afzal et al., Rev. Mod. Phys. **41** (1969) 247

New insights into nuclear clustering

Elhatisari, Epelbaum, Krebs, Lähde, Lee, Li, Lu, UGM, Rupak
Phys. Rev. Lett. **119** (2017) 222505 [arXiv:1702.05177]

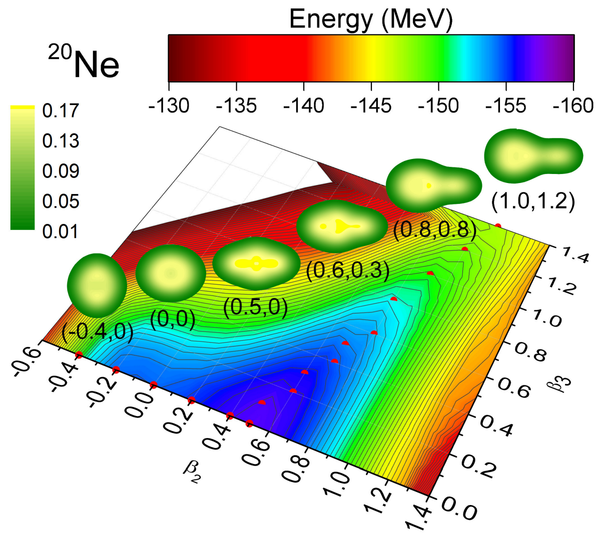
for a review: Freer, Horiuchi, Kanada-En'yo, Lee, UGM
Rev. Mod. Phys. **90** (2018) 035004 [arXiv:1705.06192]

CLUSTERING in NUCLEI

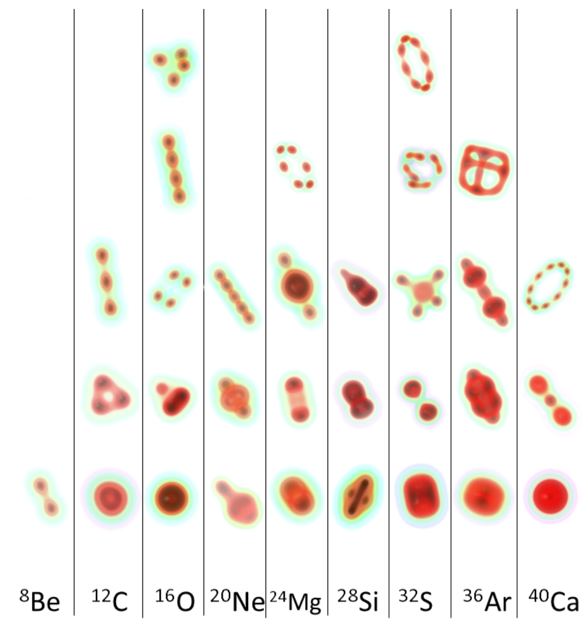
- Introduced theoretically by Wheeler already in 1937:

> John Archibald Wheeler, "Molecular Viewpoints in Nuclear Structure," Physical Review **52** (1937) 1083

- many works since then... Ikeda, Horiuchi, Freer, Ring, Schuck, Röpke, Khan, Zhou, Iachello, . . .



Zhou, Yao, Li, Ring, Meng (2015)



Ebran, Khan, Niksic, Vretenar (2014)

α-clusters

$\Rightarrow$ can we understand this phenomenon from *ab initio* calculations?

EARLIER RESULTS on NUCLEAR CLUSTERING

- Already a number of intriguing results on clustering:

  Ab initio calculation of the spectrum and structure of ^{12}C (esp. the Hoyle state)

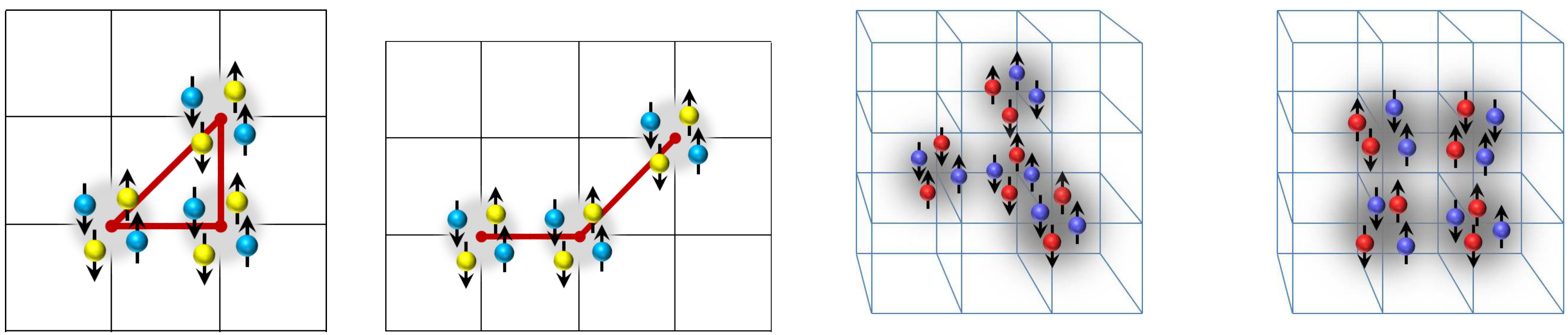
  Ab initio calculation of the spectrum and structure of ^{16}O

  Ground state energies of α-type nuclei up to ^{28}Si within 1%

  Ab initio calculation of α-α scattering

  Quantum phase transition from Bose gas of α's to nuclear liquid for α-type nuclei

- However: clustering probed through overlap with initial states



$\hookrightarrow$ Need a more **quantitative**/ regulator-independent measure

PROBING NUCLEAR CLUSTERING

- Local densities on the lattice: $\rho(\mathbf{n})\,,\quad \rho_p(\mathbf{n})\,,\quad \rho_n(\mathbf{n})$

- Probe of alpha clusters: $\rho_4 = \sum_{\mathbf{n}} : \rho^4(\mathbf{n})/4! :$

- Another probe for $Z = N =$ even nuclei: $\rho_3 = \sum_{\mathbf{n}} : \rho^3(\mathbf{n})/3! :$

- ρ_4 couples to the center of the α-cluster while ρ_3 gets contributions from a wider portion of the alpha-particle wave function

- Both ρ_3 and ρ_4 depend on the regulator, a, but not on the nucleus

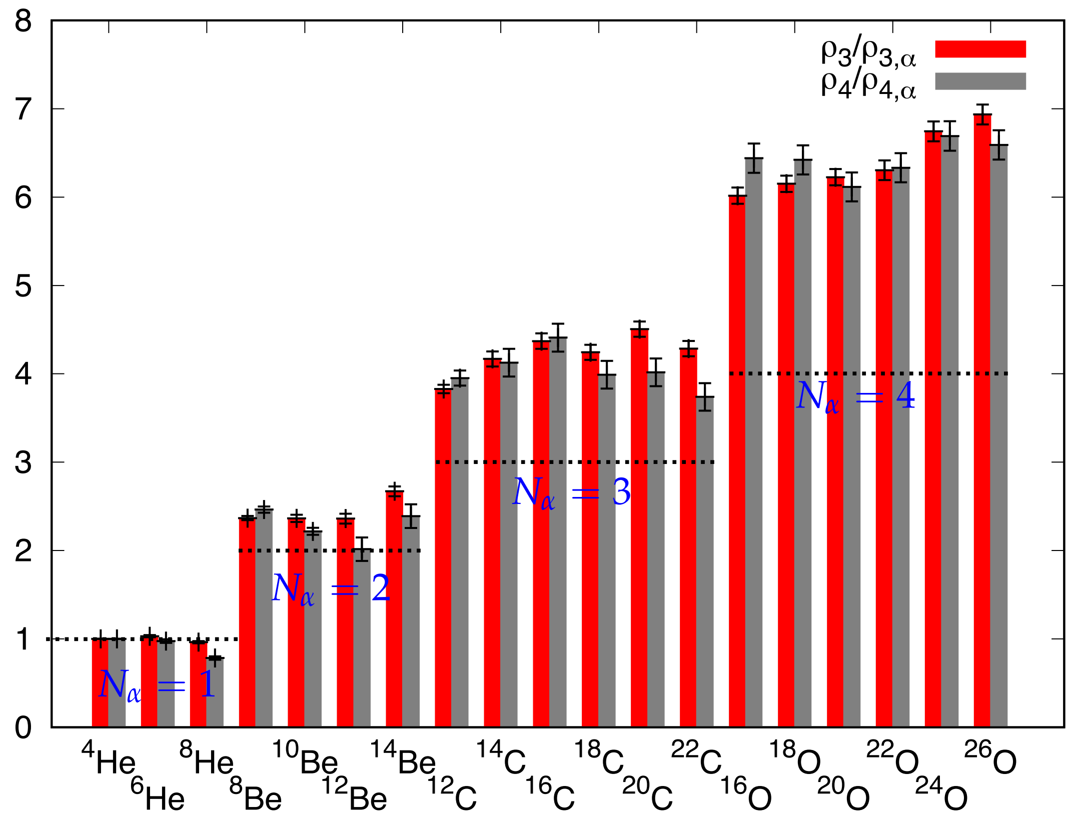
- The ratios $\rho_3/\rho_{3,\alpha}$ and $\rho_4/\rho_{4,\alpha}$ free of short-distance ambiguities and model-independent

- $\rho_3/\rho_{3,\alpha}$ [or $\rho_4/\rho_{4,\alpha}$] measures the effective number of alpha-cluster N_α

$\Rightarrow$ Any deviation from $N_\alpha =$ integer measures the entanglement of the α-clusters in a given nucleus

PROBING NUCLEAR CLUSTERING

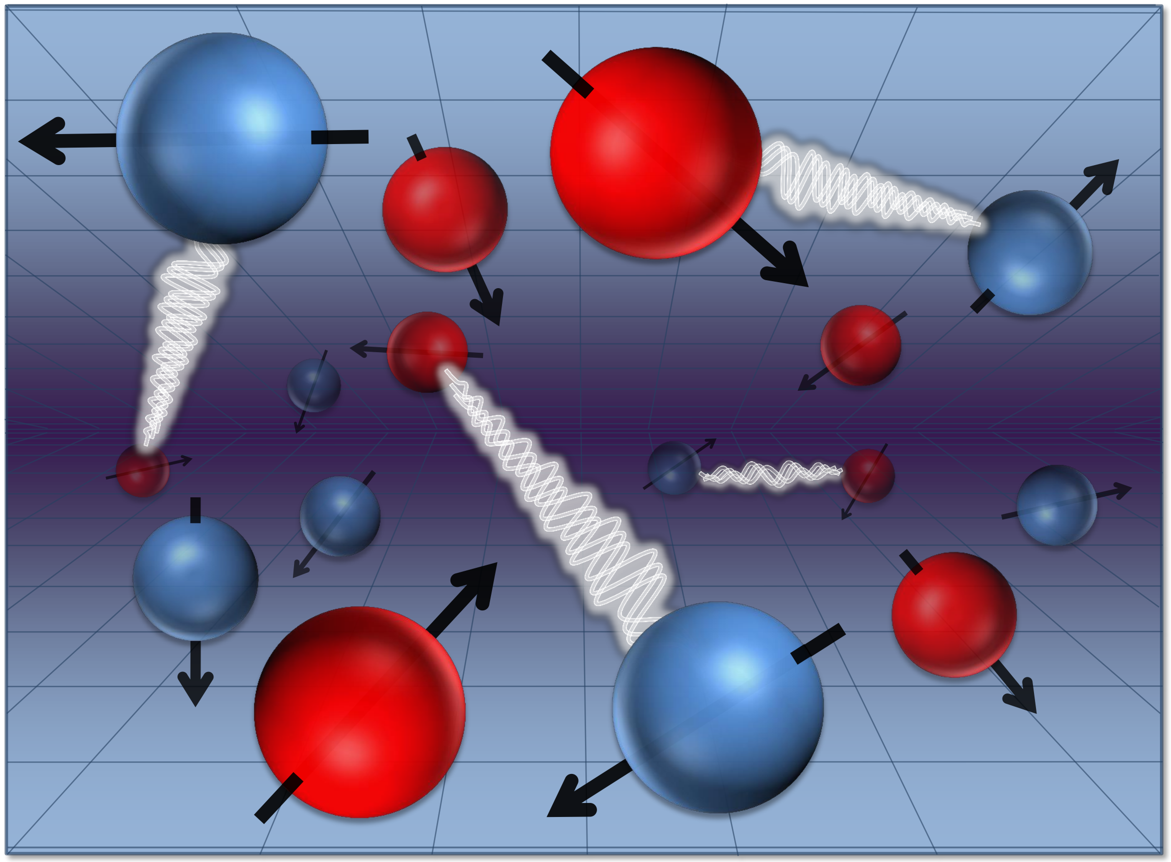
- ρ_3-entanglement of the α-clusters: $$\frac{\Delta_\alpha^{\rho_3}}{N_\alpha} = \frac{\rho_3/\rho_{3,\alpha}}{N_\alpha} - 1$$



Nucleus	^{4,6,8}He	^{8,10,12,14}Be	^{12,14,16,18,20,22}C	^{16,18,20,22,24,26}O
$\Delta_\alpha^{\rho_3}/N_\alpha$	0.00 - 0.03	0.20 - 0.35	0.25 - 0.50	0.50 - 0.75

PROBING NUCLEAR CLUSTERING

- The transition from cluster-like states in light systems to nuclear liquid-like states in heavier systems should not be viewed as a simple suppression of multi-nucleon short-distance correlations, but rather as an increasing *entanglement* of the nucleons involved in the multi-nucleon correlations.



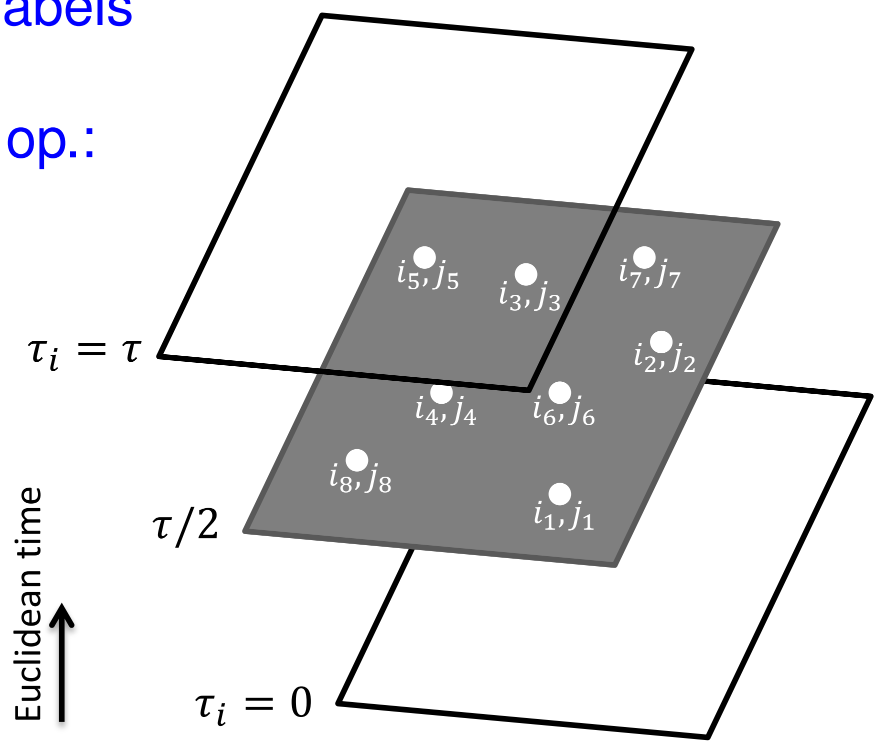
PINHOLE ALGORITHM

- AFQMC calculations involve states that are superpositions of many different center-of-mass positions

$\longrightarrow$ density distributions of nucleons can not be computed directly

- Insert a screen with pinholes with spin & isospin labels that allows nucleons with corresponding spin & isospin to pass = insertion of the A-body density op.:

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A)$$
$$=:\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$



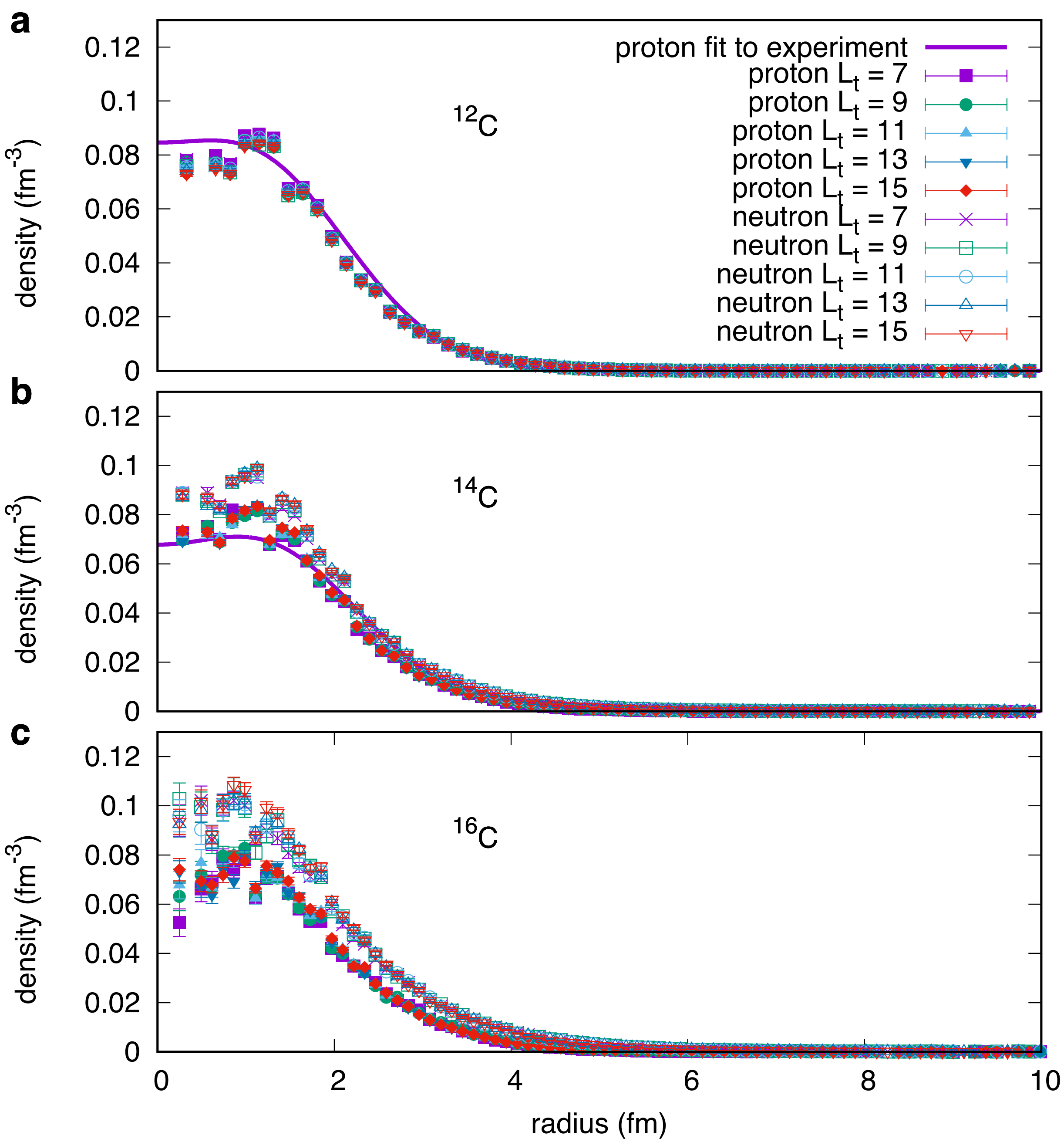
- MC sampling of the amplitude:

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A,L_t)$$
$$=\langle\psi(\tau/2)|\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A)|\psi(\tau/2)\rangle$$

- Allows to measure proton and neutron distributions

- Resolution scale $\sim a/A$ as cm position $\mathbf{r}_{\rm cm}$ is an integer $\mathbf{n}_{\rm cm}$ times a/A

PROTON and NEUTRON DENSITIES in CARBON



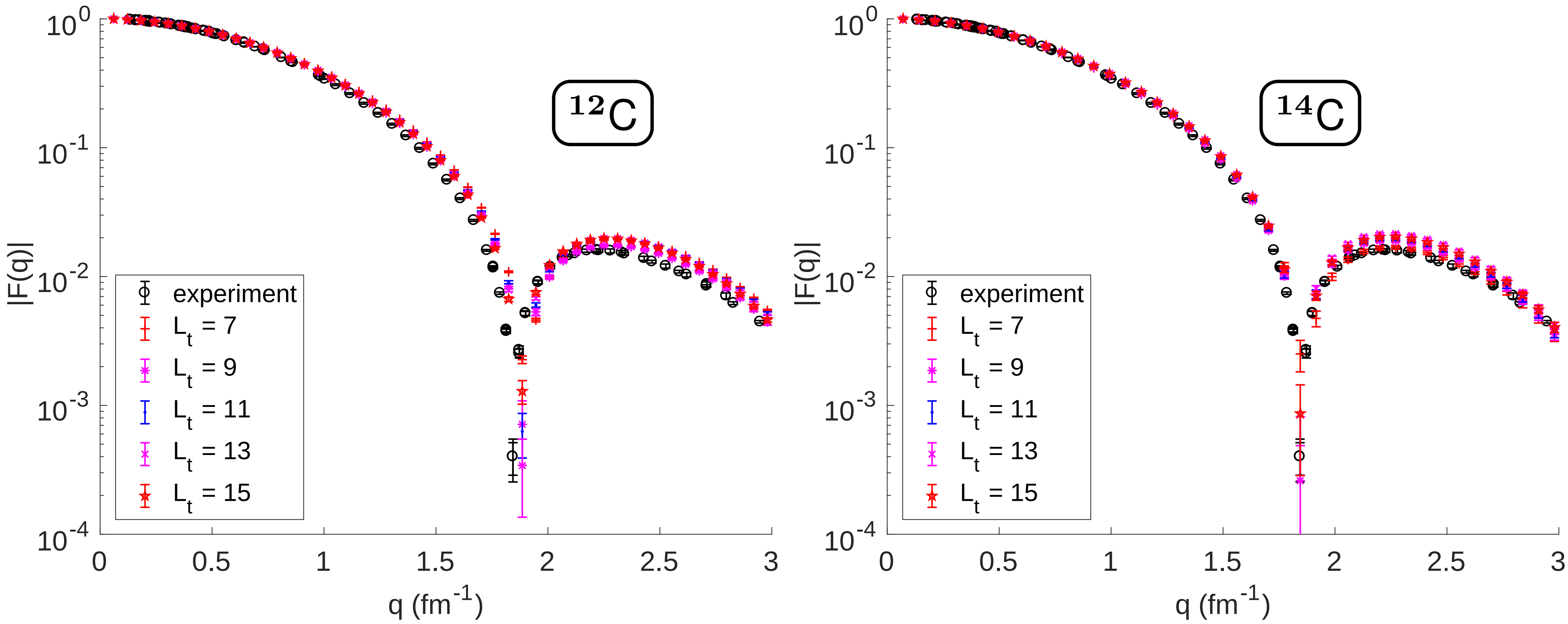
- open symbols: neutron
- closed symbols: proton
- proton size accounted for
- asymptotic properties of the distributions from the volume dependence of N-body bound states

  König, Lee, Phys. Lett. B779 (2018) 9
- consistent with data
- fit to data from

  Kline et al., Nucl. Phys. A209 (1973) 381

FORM FACTORS

- Fit charge distributions by a Wood-Saxon shape
  $\hookrightarrow$ get the form factor from the Fourier-transform (FT)
  $\hookrightarrow$ uncertainties from a direct FT of the lattice data

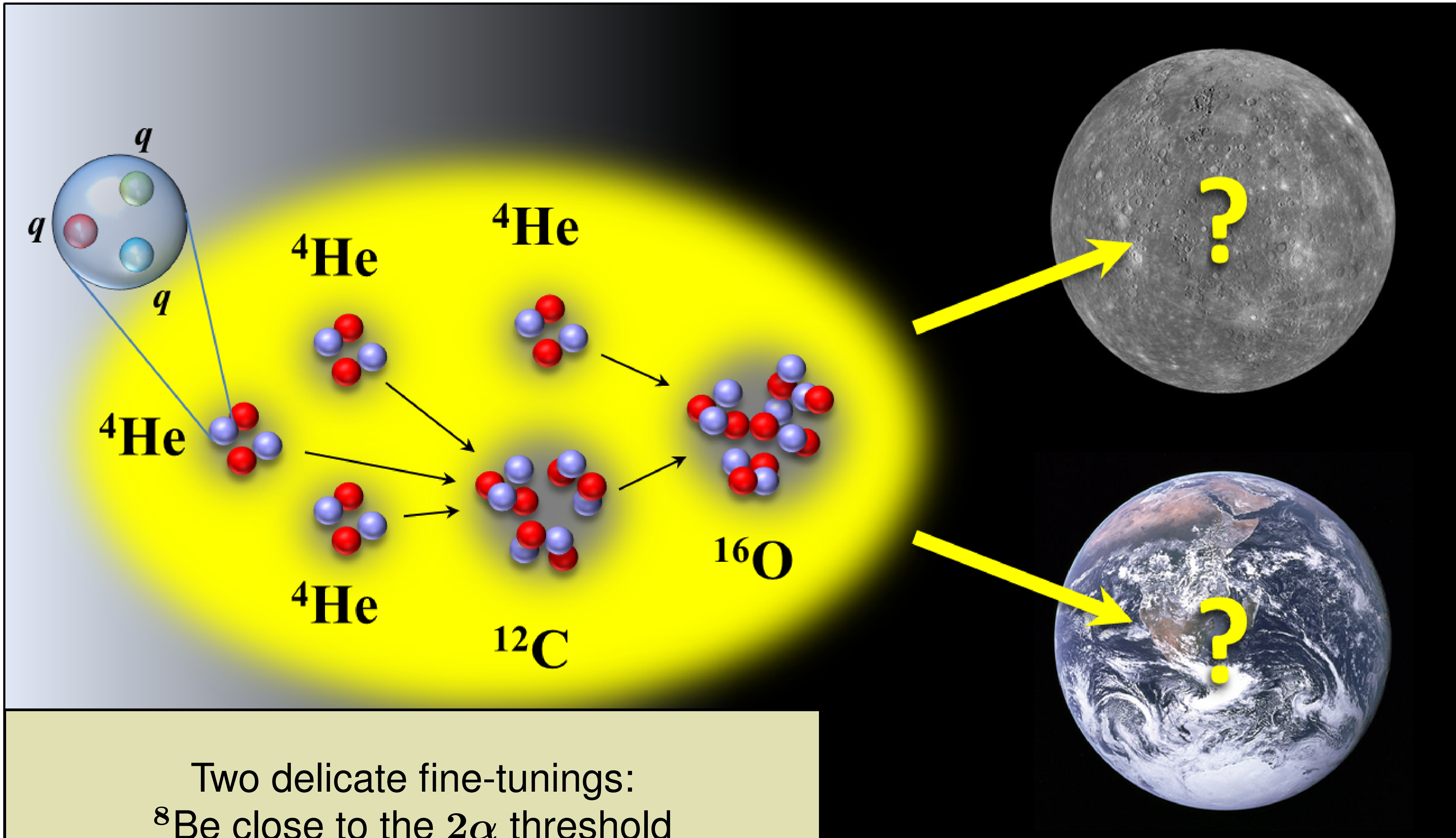


$\Rightarrow$ detailed structure studies become possible

Fine-tunings and the multiverse

UGM, Sci. Bull. **60** (2015) no.1, 43-54
Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **110** (2013) 112502
Epelbaum, Krebs, Lähde, Lee, UGM, Eur. Phys. J. **A 49** (2013) 82

FINE-TUNING of FUNDAMENTAL PARAMETERS

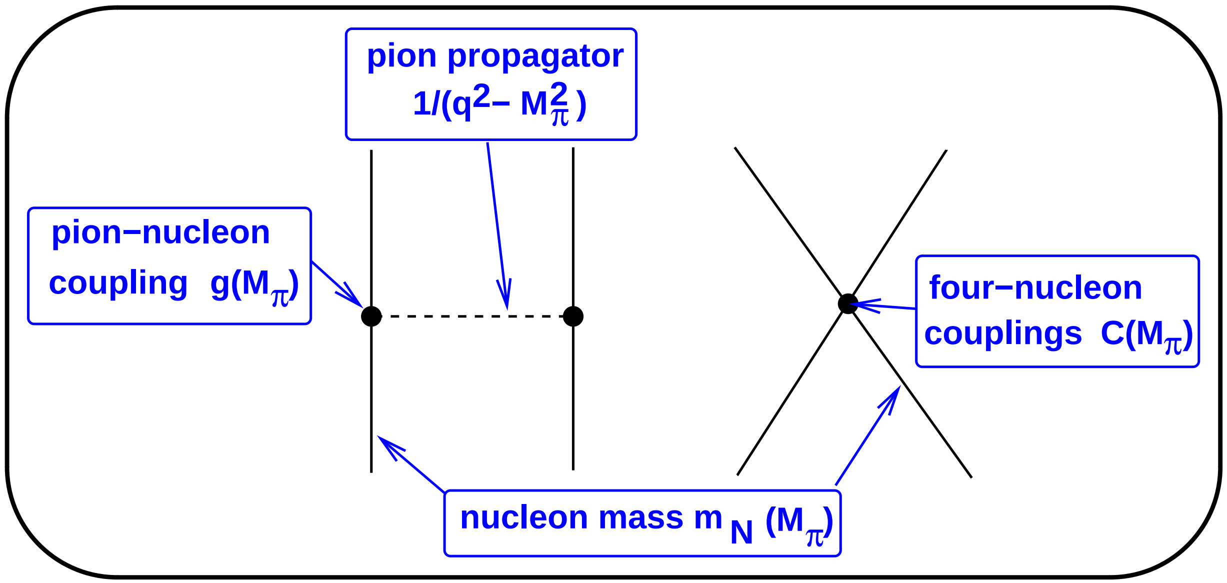


Two delicate fine-tunings:
^{8}Be close to the 2α threshold
Hoyle state close to the 3α threshold
are these related or unrelated?

Fig. courtesy Dean Lee

NUCLEAR FORCES for VARYING QUARK MASSES

- Nuclear forces: Pion-exchange contributions & short-distance multi-N operators
- graphical representation of the quark mass dependence of the LO potential



- always use the Gell-Mann–Oakes–Renner relation: $M^2_{\pi^\pm} \sim (m_u + m_d)$
- fulfilled in QCD to better than 94% Colangelo, Gasser, Leutwyler 2001

⇒ Quark mass dependence of hadron properties from lattice QCD,
contact interaction require modeling → challenge to lattice QCD

FINE-TUNING: MONTE-CARLO ANALYSIS

Epelbaum, Krebs, Lähde, Lee, UGM, PRL **110** (2013) 112502

- consider first QCD only $\rightarrow$ calculate $\partial\Delta E/\partial M_\pi$

- relevant quantities (energy *differences*)

^{4}He + ^{4}He $\leftrightarrow$ ^{8}Be $\rightsquigarrow$ $\boxed{\Delta E_b \equiv E_8 - 2E_4}$

^{4}He + ^{8}Be $\rightarrow$ ^{12}C* $\rightsquigarrow$ $\boxed{\Delta E_h \equiv E_{12}^* - E_8 - E_4}$

- energy differences depend on parameters of QCD (LO analysis)

$$\boxed{E_i = E_i\Big(M_\pi^{\rm OPE}, m_N(M_\pi), g_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\Big)}$$

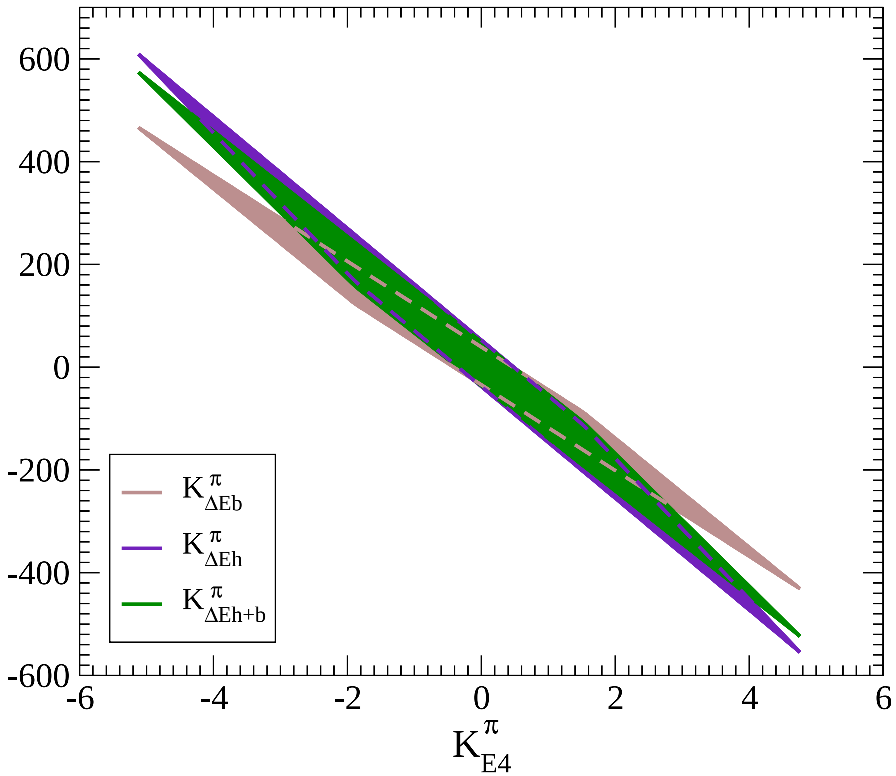
$$g_{\pi N} \equiv g_A/(2F_\pi)$$

- QED in the same manner $\rightarrow$ calculate $\partial\Delta E/\partial\alpha_{\rm EM}$

CORRELATIONS

- map $C_{0,I}(M_\pi)$ onto $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ [singlet/triplet scatt. length]

- vary the derivatives $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ within $-1, \ldots, +1$:



$$\Delta E_b = E(^8\text{Be}) - 2E(^4\text{He})$$

$$\Delta E_h = E(^{12}\text{C}^*) - E(^8\text{Be}) - E(^4\text{He})$$

$$\Delta E_{h+b} = E(^{12}\text{C}^*) - 3E(^4\text{He})$$

$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \frac{O_H}{M_\pi}}$$

$\Rightarrow$ the fine-tunings in the triple-alpha process are *correlated*, as speculated

Weinberg (2000)

SUMMARY & OUTLOOK

- Nuclear lattice simulations: a new quantum many-body approach

  $\rightarrow$ based on the successful continuum nuclear chiral EFT

  $\hookrightarrow$ Evgeny Epelbaum's talk

  $\rightarrow$ a number of intriguing results already obtained

  $\rightarrow$ clustering emerges naturally, α-cluster nuclei well described

  $\rightarrow$ fine-tuning in nuclear reactions can be studied

  $\rightarrow$ N3LO precision w/ highly improved LO action upcoming

  $\hookrightarrow$ Dean Lee's talk

- Various bridges to lattice QCD studies need to be explored

- Many open issues can now be addressed in a truly quantitative manner

  $\rightarrow$ the "holy grail" of nuclear astrophysics ^{4}He+$^{12}\mathbf{C} \rightarrow ^{16}\mathbf{O} + \gamma$

  Fowler (1983)

SPARES

EARLIER RESULTS on NUCLEAR CLUSTERING

- Already a number of intriguing results on clustering:

  Ab initio calculation of the spectrum and structure of ^{12}C (esp. the Hoyle state)

  Ab initio calculation of the spectrum and structure of ^{16}O

  Ground state energies of α-type nuclei up to ^{28}Si within 1%

  Ab initio calculation of α-α scattering

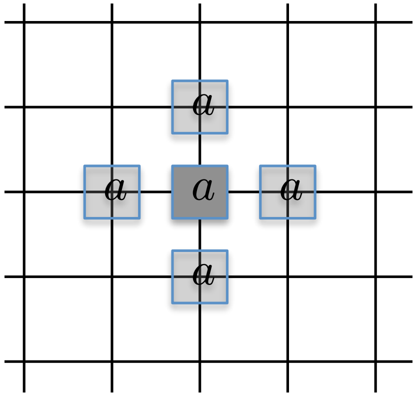
  Quantum phase transition from Bose gas of α's to nuclear liquid for α-type nuclei

- However: when adding extra neutrons/protons, the precision quickly deteriorates due to sign oscillations

- New LO action with smeared SU(4) local+non-local symmetric contact interactions & smeared one-pion exchange

$$a_{\rm NL}(\mathbf{n}) = a(\mathbf{n}) + s_{\rm NL} \sum_{\langle \mathbf{n}'\,\mathbf{n}\rangle} a(\mathbf{n}')$$

$$a^\dagger_{\rm NL}(\mathbf{n}) = a^\dagger(\mathbf{n}) + s_{\rm NL} \sum_{\langle \mathbf{n}'\,\mathbf{n}\rangle} a^\dagger(\mathbf{n}')$$

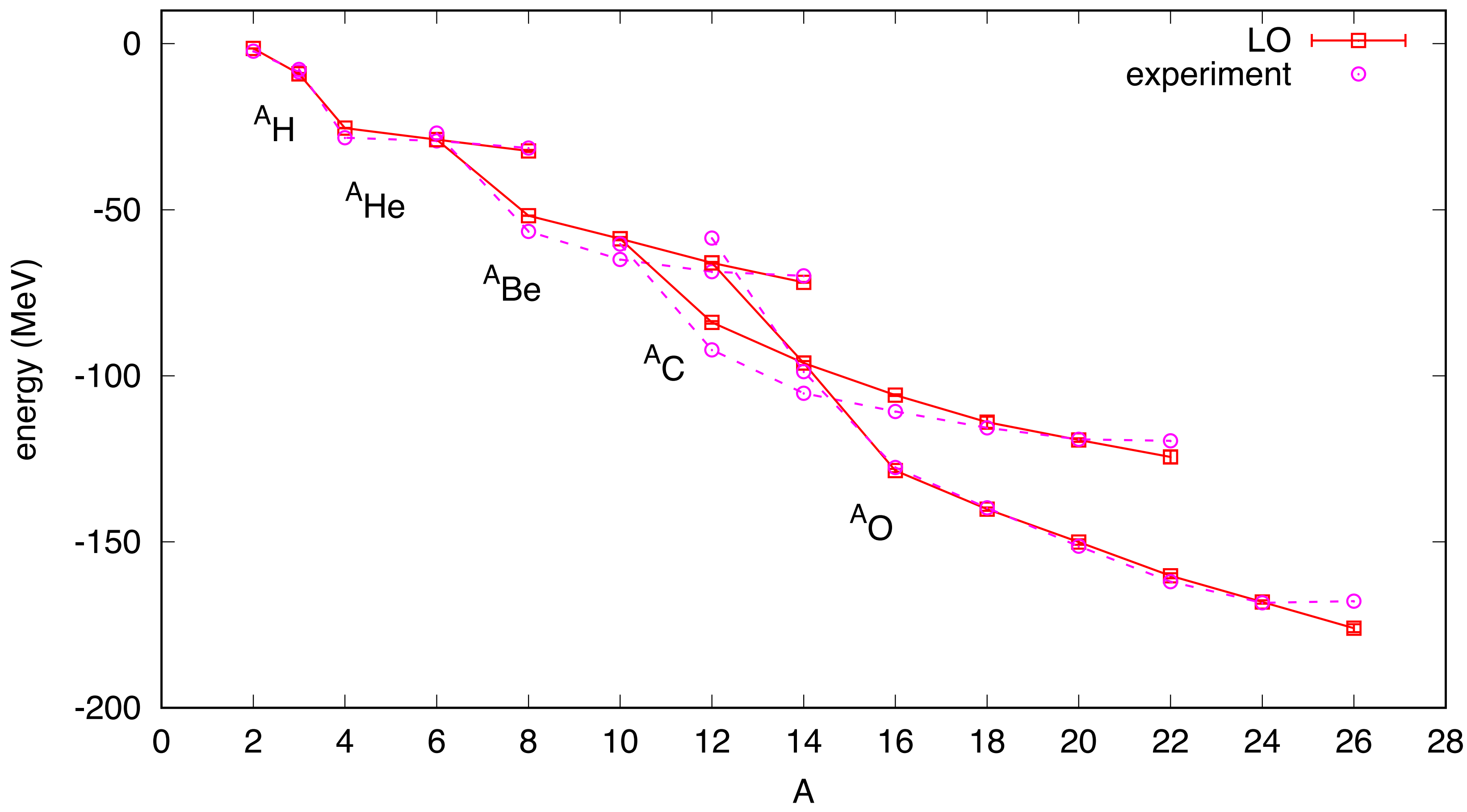


$\longrightarrow$ Dean Lee's talk

GROUND STATE ENERGIES

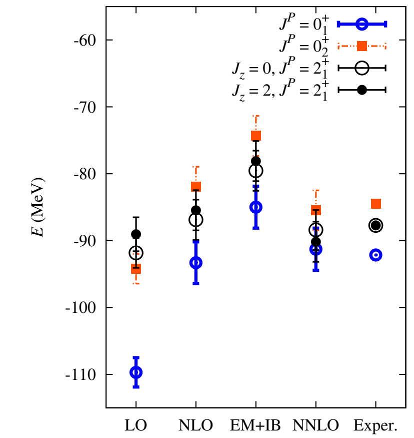
- Fit parameters to average NN S-wave scattering length and effective range and α-α S-wave scattering length

$\rightarrow$ predict g.s. energies of H, He, Be, C and O isotopes $\rightarrow$ quite accurate (LO)



The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600
From: Steven Weinberg ⟨weinberg@zippy.ph.utexas.edu⟩
To: Ulf-G. Meissner ⟨meissner@hiskp.uni-bonn.de⟩
Subject: Re: Hoyle state in 12C



Dear Professor Meissner,

Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,
Steve Weinberg

- How does the Hoyle state move relative to the ^{4}He+^{8}Be threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

EARLIER STUDIES of the ANTHROPIC PRINCIPLE

- rate of the 3α-process: $r_{3\alpha} \sim \Gamma_\gamma \exp\left(-\frac{\Delta E_{h+b}}{kT}\right)$

$$\Delta E_{h+b} = E^\star_{12} - 3E_\alpha = 379.47(18)\ \text{keV}$$

- how much can ΔE_{h+b} be changed so that there is still enough ^{12}C and ^{16}O?

$\Rightarrow$ $\delta|\Delta E_{h+b}| \lesssim 100$ keV

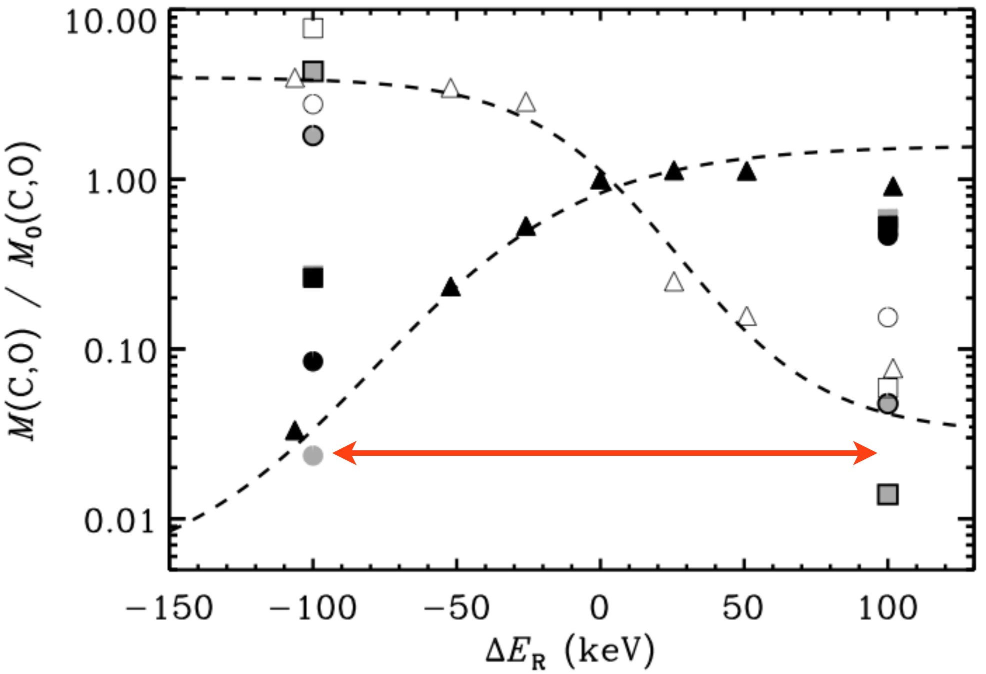
Oberhummer et al., Science **289** (2000) 88

Csoto et al., Nucl. Phys. A **688** (2001) 560

Schlattl et al., Astrophys. Space Sci. **291** (2004) 27

[Livio et al., Nature **340** (1989) 281]

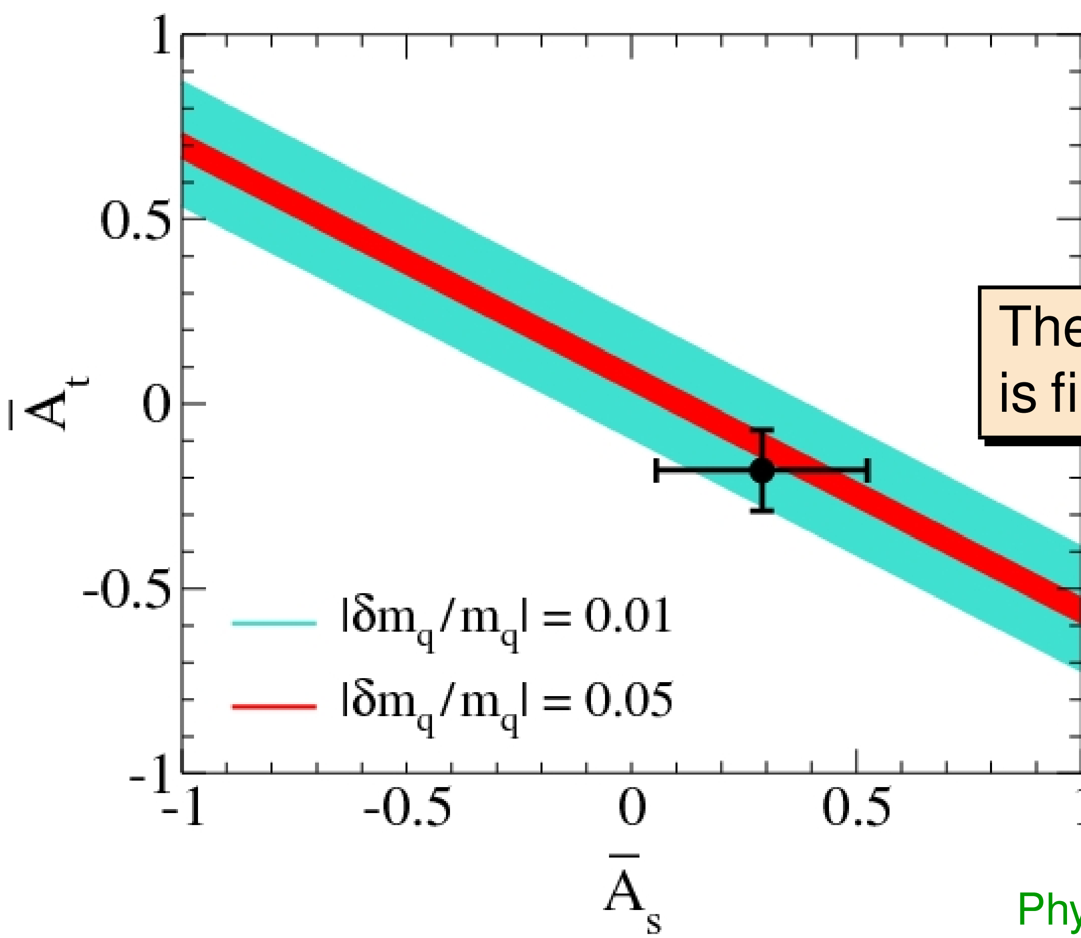
too few ^{16}O too few ^{12}C



THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV [exp: 387 keV]   Oberhummer et al., Science (2000)

$$\rightarrow \boxed{\left|\left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$



$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018