Elastic *n*-^{6}He scattering and ^{7}He resonant states in Single State HORSE method

Igor A. Mazur
Pacific National University
Khabarovsk, Russia

Russian Science Foundation

Collaborators:
A. M. Shirokov, Lomonosov Moscow State University, Russia
A. I. Mazur, PNU, Khabarovsk, Russia
I. J. Shin, Y. Kim, Institute for Basic Sciences, Daejeon, Republic of Korea
J. P. Vary, Iowa State University, Ames, USA

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Motivation

* Resonances in neutron rich nuclear systems are very interesting
* Single State HORSE method gives reasonable results for ^{5}He
* We continue studying resonances in He isotopes. Resent: ^{7}He

HORSE (J-matrix)

- Our method based on Harmonic Oscillator Representation of Scattering Equations. HORSE is reliable method for describing twoparticle scattering
- $H^{\ell} u_{\ell}(E,r) = E u_{\ell}(E,r)$ $\qquad$ $u_{\ell}(E,r) = \sum_{n=0}^{\infty} a_{N\ell}(E)\, R_{N\ell}(r)$
  $R_N^{\ell}(r)$ – oscillator functions
  $N = 2n + \ell$ – oscillator quanta
- Approximation of potential (short range) $\tilde{V}^{\ell}_{NN'} = \begin{cases} V^{\ell}_{NN'}, & N, N' \leq \mathbb{N} = N_{\max} + N_{\min} \quad P\text{-space} \\ 0, & N, N' > \mathbb{N} = N_{\max} + N_{\min} \quad Q\text{-space} \end{cases}$
- Kinetic energy matrix stay full
- Phase shifts $\tan \delta_{\ell}(E) = -\dfrac{S_{\mathbb{N}\ell}(E) - G_{\mathbb{N}\mathbb{N}}(E) T^{\ell}_{\mathbb{N},\mathbb{N}+2} S_{\mathbb{N}+2,\ell}(E)}{C_{\mathbb{N}\ell}(E) - G_{\mathbb{N}\mathbb{N}}(E) T^{\ell}_{\mathbb{N},\mathbb{N}+2} C_{\mathbb{N}+2,\ell}(E)}$

$$S_{N\ell}(E) = \sqrt{\frac{\pi n!}{\Gamma(n+\ell+\frac{3}{2})}} \left(\frac{2E}{\hbar\Omega}\right)^{\frac{\ell+1}{2}} \exp\left(-\frac{E}{\hbar\Omega}\right) L_n^{\ell+\frac{1}{2}} \left(\frac{2E}{\hbar\Omega}\right)$$

$$C_{N\ell}(E) = \sqrt{\frac{\pi n!}{\Gamma(n+\ell+\frac{3}{2})}} \frac{(-1)^n}{\Gamma(-\ell+\frac{1}{2})} \left(\frac{2E}{\hbar\Omega}\right)^{-\frac{\ell}{2}} \exp\left(-\frac{E}{\hbar\Omega}\right) {}_1F_1\left(-n-\ell-\frac{1}{2}, -\ell+\frac{1}{2}; \frac{2E}{\hbar\Omega}\right)$$

$$G_{NN'}(E) = -\sum_{\nu=0}^{\mathcal{N}-1} \frac{\langle \nu | N\ell \rangle \langle N'\ell | \nu \rangle}{E_{\nu} - E} \qquad \sum_{n'=0}^{(\mathbb{N}-\ell)/2} H^{\ell}_{NN'} \langle N'\ell | \nu \rangle = E_{\nu} \langle N\ell | \nu \rangle$$

Single State HORSE

- $$\tan\delta_\ell(E) = -\frac{S_{\mathbb{N}\ell}(E) - G_{\mathbb{NN}}(E)T^{\ell}_{\mathbb{N},\mathbb{N}+2}S_{\mathbb{N}+2,\ell}(E)}{C_{\mathbb{N}\ell}(E) - G_{\mathbb{NN}}(E)T^{\ell}_{\mathbb{N},\mathbb{N}+2}C_{\mathbb{N}+2,\ell}(E)}$$
  $$G_{NN'}(E) = -\sum_{\nu=0}^{\mathcal{N}-1}\frac{\langle\nu|N\ell\rangle\,\langle N'\ell|\nu\rangle}{E_\nu - E}$$
  $$\sum_{n'=0}^{(\mathbb{N}-\ell)/2} H^{\ell}_{NN'}\langle N'\ell|\nu\rangle = E_\nu\langle N\ell|\nu\rangle, \quad n = 0, ..., (\mathbb{N}-\ell)/2 \qquad N' = 2n' + \ell$$
- In *P*-space we may use various calculations with oscillator basis, for example, *ab initio* No-Core Shell Model calculations
- Phase shift requires ALL eigenstates, number of them increase rapidly
- $E = E_\nu$: $\tan\delta_\ell(E_\nu) = -\dfrac{S_{\mathbb{N}+2,\ell}(E_\nu)}{C_{\mathbb{N}+2,\ell}(E_\nu)}$
- By varying $N_{\max}$ and $\hbar\Omega$ we obtain E_ν and δ_l in some interval
- Parametrization of phase shifts
- Search of *S*-matrix poles, which associated with bound, resonant states

Parametrization & *S*-matrix poles search

- Effective-range function $K(E) = k^{2\ell+1}\cot\delta$ $\qquad k = \frac{\sqrt{2\mu E}}{\hbar}$
- Padé-approximation $K(E) = \frac{w_0^{(n)} + w_1^{(n)}E + w_2^{(n)}E^2 + \cdots}{1 + w_1^{(d)}E + w_2^{(d)}E^2 + \cdots}$
- Scattering amplitude $f(E) = \frac{k^{2\ell}}{K(E) - ik^{2\ell+1}}$
- *S*-matrix $S = e^{2i\delta}$ has the same poles as *f(E)*
- *f(E)* have poles where $F(E) \equiv K(E) - ik^{2\ell+1} = 0$
- From theory of functions of complex variables:

$$\Upsilon = \frac{1}{2\pi i}\oint_C \frac{\mathcal{F}'(E)}{\mathcal{F}(E)}\,dE$$

number of zeroes

$$E_p = \frac{1}{2\pi i}\oint_C E\,\frac{\mathcal{F}'(E)}{\mathcal{F}(E)}\,dE$$

zero's position (*S*-matrix pole)

- Bound, false pole $E_p = -E_b\,;$ virtual pole $E_p = |E_v|$

  Resonance pole $E_p = E_r + i\frac{\Gamma}{2}$

Scattering with two clusters on infinity

- Let's consider scattering of two nuclei with A^{proj} and A^{targ} nucleons

- Approximation: projectile and target described by only lowest oscillator states

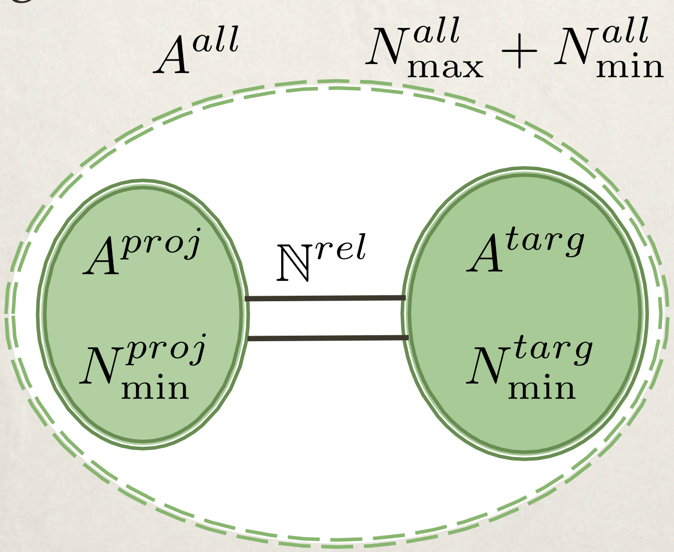
- No-Core Shell Model calculations:
  - Eigenenergies of system with $A^{\mathrm{all}} = A^{\mathrm{proj}} + A^{\mathrm{targ}}$ nucleons
  - Ground state energies of projectile and target

- Energy is calculated regarding the channel threshold

  $$E^{rel} \equiv E = E^{A^{all}} - E^{A^{proj},gs} - E^{A^{targ},gs}$$

- Relative motion oscillator quanta

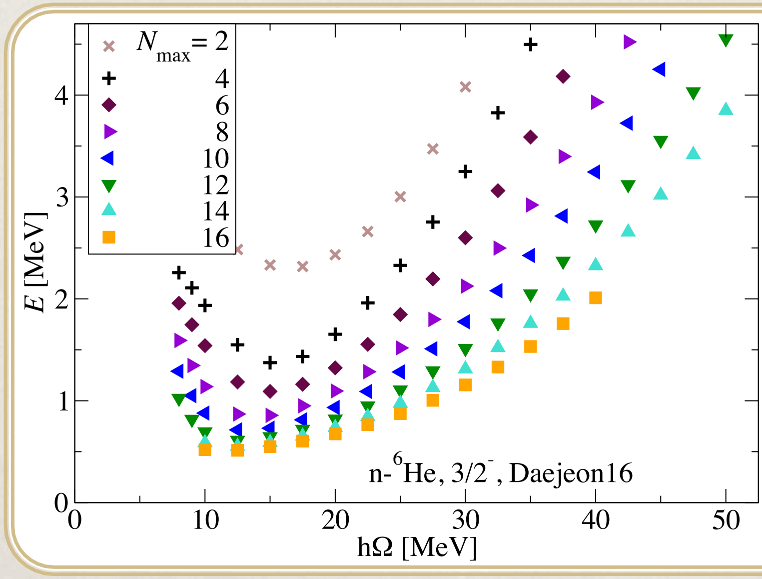
  $$\mathbb{N}^{rel} \equiv \mathbb{N} = N^{all}_{\max} + N^{all}_{\min} - N^{proj}_{\min} - N^{targ}_{\min}$$

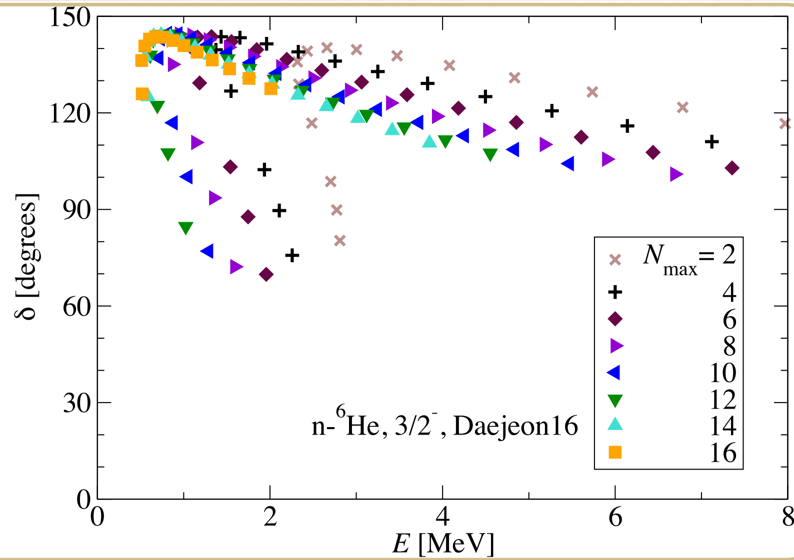


n–^{6}He scattering with Daejeon16

* No-Core Shell Model calculations:
  * NN-interaction: Daejeon16
  * lowest eigenenergies E_0 of $3/2^-$, $1/2^-$, $5/2^-$, $1/2^+$, $3/2^+$ and $5/2^+$ states of ^{7}He nuclei in bases with $N_{max} \leq 17$, $10 \leq \hbar\Omega \leq 50$ MeV
  * ground state energy of ^{6}He nuclei in the same bases
* Approximation: ^{6}He described by only lowest oscillator states
* Energy is calculated regarding the channel threshold
  * for waves with negative parity
  $$E_0(N_{\max}, \hbar\Omega) = E_0^{^7\mathrm{He}}(N_{\max}, \hbar\Omega) - E_0^{^6\mathrm{He,\ gs}}(N_{\max}, \hbar\Omega)$$
  * for waves with positive parity
  $$E_0(N_{\max}, \hbar\Omega) = E_0^{^7\mathrm{He}}(N_{\max}, \hbar\Omega) - E_0^{^6\mathrm{He,\ gs}}(N_{\max} - 1, \hbar\Omega)$$
* Link oscillator quanta of relative motion and NCSM excitation quanta
$$\mathbb{N} = N_{\max}^{^7\mathrm{He}} + N_{\min}^{^7\mathrm{He}} - N_{\min}^{^6\mathrm{He}} = N_{\max} + 1$$

3/2⁻ state of n–^{6}He scattering: Daejeon16



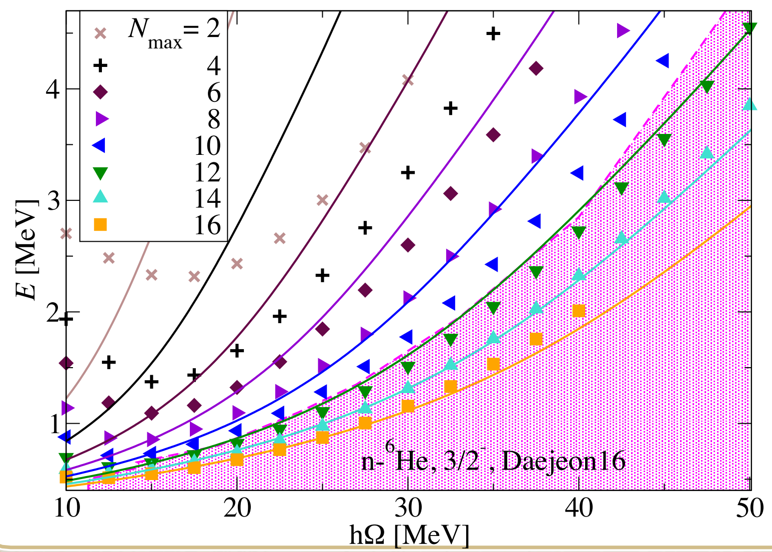


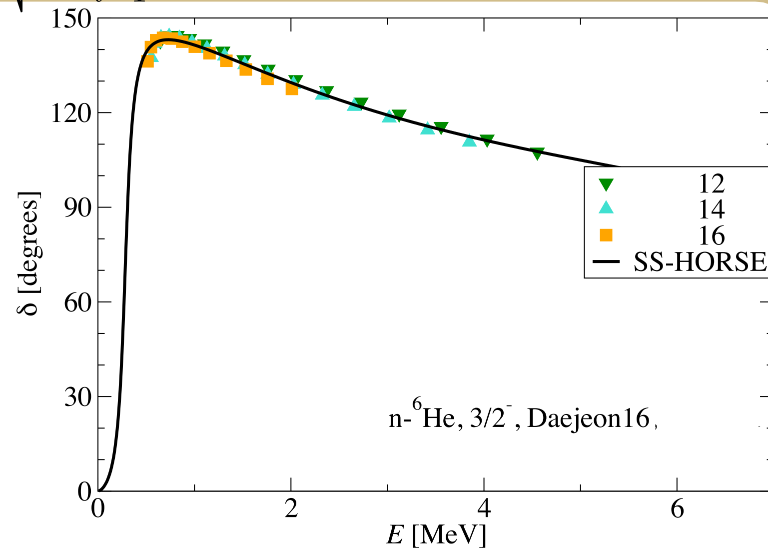
Energies E_0 from NCSM

$$\tan \delta_\ell(E_0) = -\frac{S_{\mathbb{N}+2,\ell}(E_0)}{C_{\mathbb{N}+2,\ell}(E_0)}$$

$3/2^-$ state of n–^{6}He scattering: Daejeon16

- Convergence: phase shifts form smooth curve
- Parameterization based on equation $$\frac{w_0^{(n)} + w_1^{(n)} E + w_2^{(n)} E^2}{1 + w_1^{(d)} E + w_2^{(d)} E^2} = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$
- Parameters regarding rule: $$\Xi = \min_{w} \sqrt{\frac{1}{D} \sum_{i=1}^{D} \left(E(N_{\max}^{(i)}, \hbar\Omega^{(i)}) - E_0^{(i),NCSM} \right)^2}$$





	E_r, MeV	Γ, MeV	Ξ, keV	D
SS-HORSE $N_{\max} \leq 16$	0.28	0.13	320	43
From experiment*	0.44	0.16		

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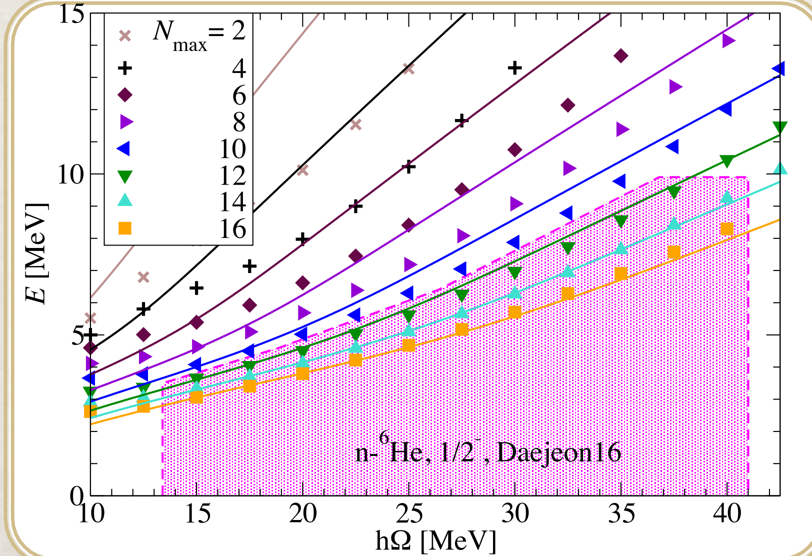
$1/2^-$ state of n–^{6}He scattering: Daejeon16



	E_r, MeV	Γ, MeV	Ξ, keV	D
SS-HORSE $N_{max} \le 16$	2.8	4.3	571	32
From experiment*	1.2	1.0		

$$w_0 + w_1 E + w_2 E^2 = -k^{2l+1}\,\frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

Pure convergence

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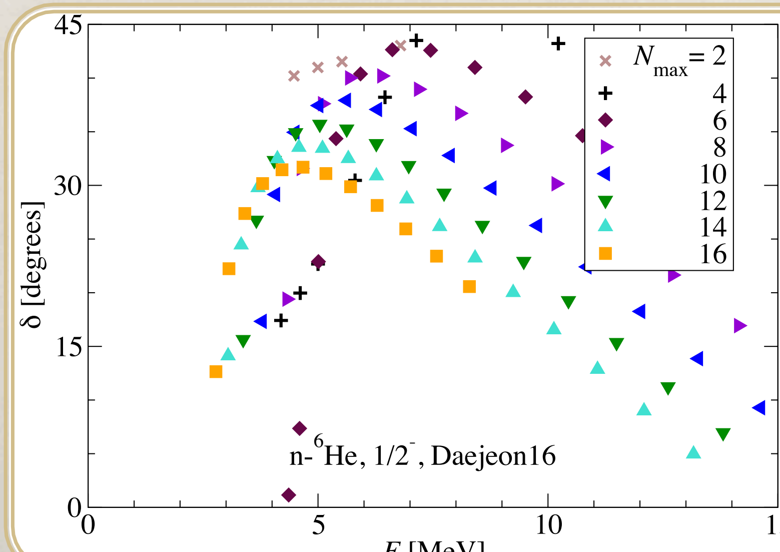
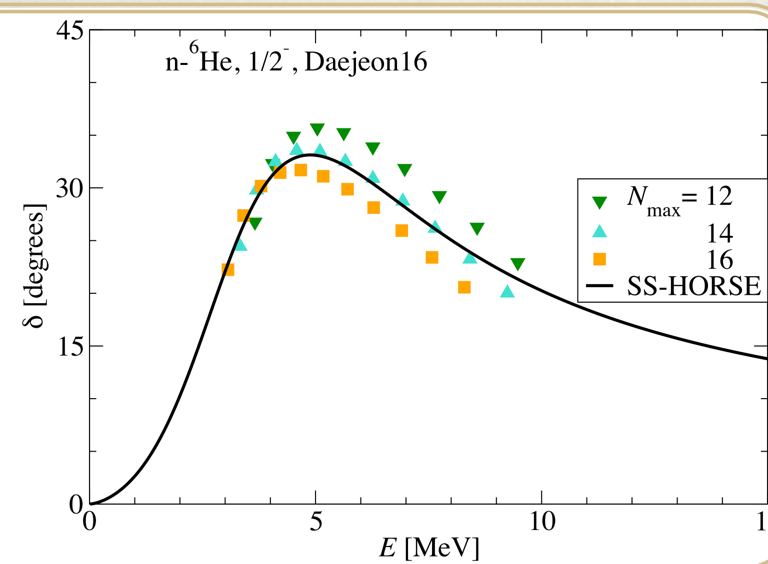
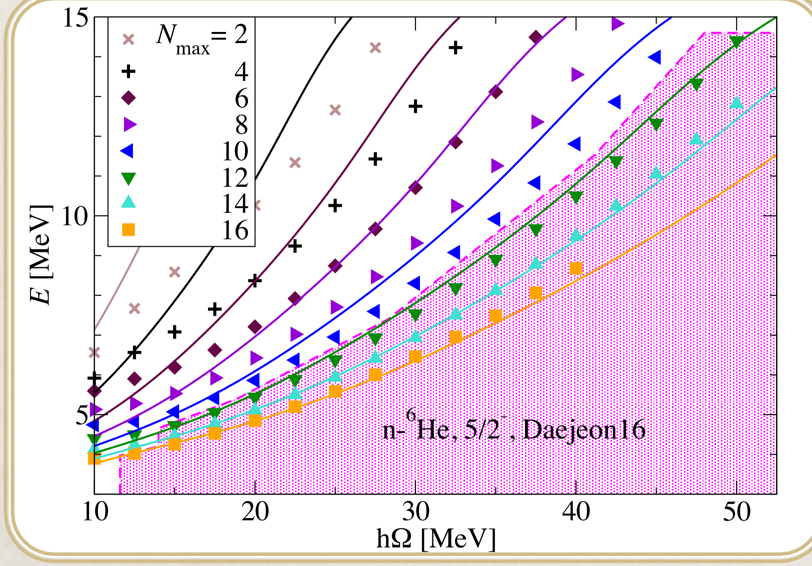




5/2⁻ state of n–^{6}He scattering: Daejeon16



	E_r, MeV	Γ, MeV	Ξ, keV	D
SS-HORSE $N_{max} \leq 16$	3.7	1.4	651	42
From experiment*	3.3	2.2		

$$\frac{w_0^{(n)} + w_1^{(n)} E}{1 + w_1^{(d)} E + w_2^{(d)} E^2 + w_3^{(d)} E^3} = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

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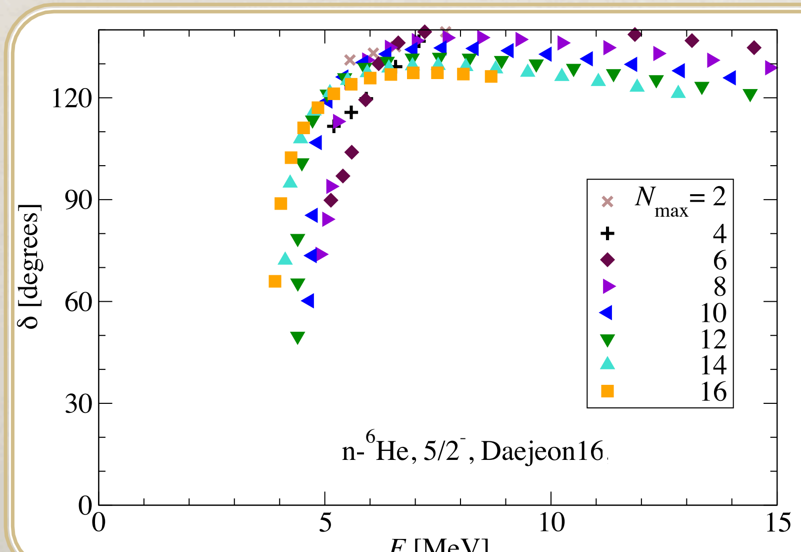
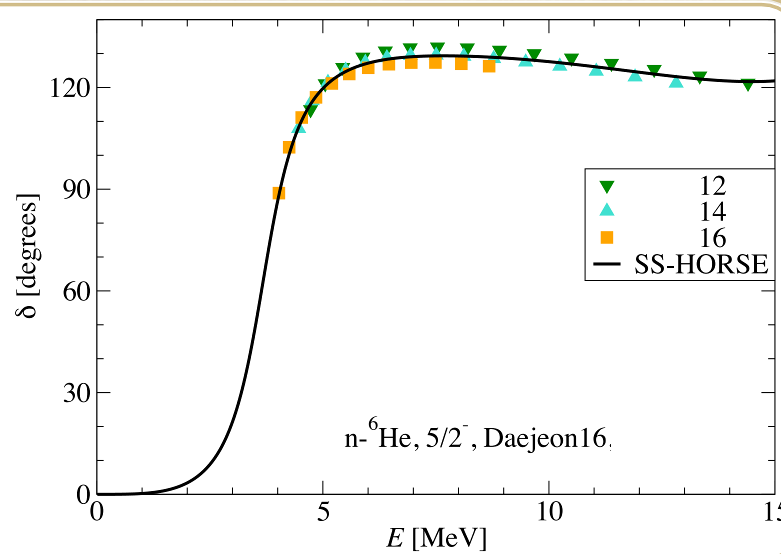
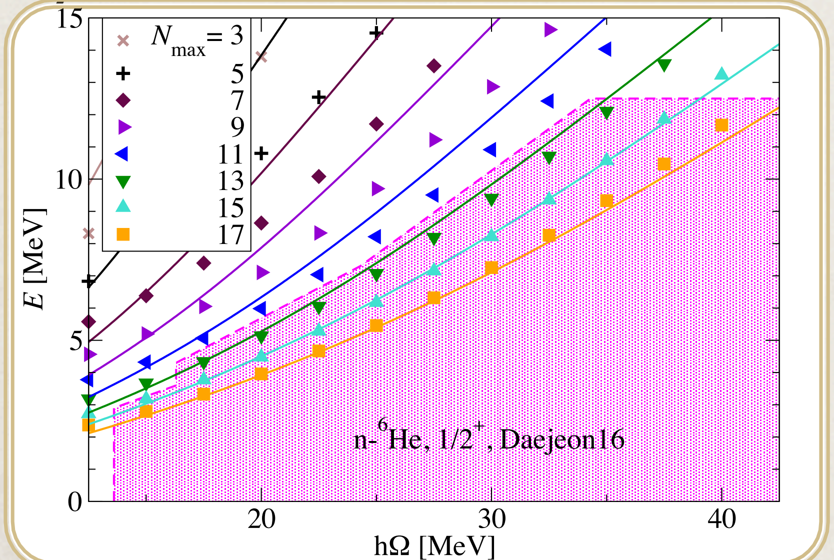
Convergence: like a 3/2⁻ state





$1/2^+$ state of n–^{6}He scattering: Daejeon16



	E_r, MeV	Γ, MeV	Ξ, keV	D
SS-HORSE $N_{max} \leq 17$	*non-resonant*		885	29
From experiment*	*non-resonant*			

$$w_0 + w_1 E + w_2 E^2 = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

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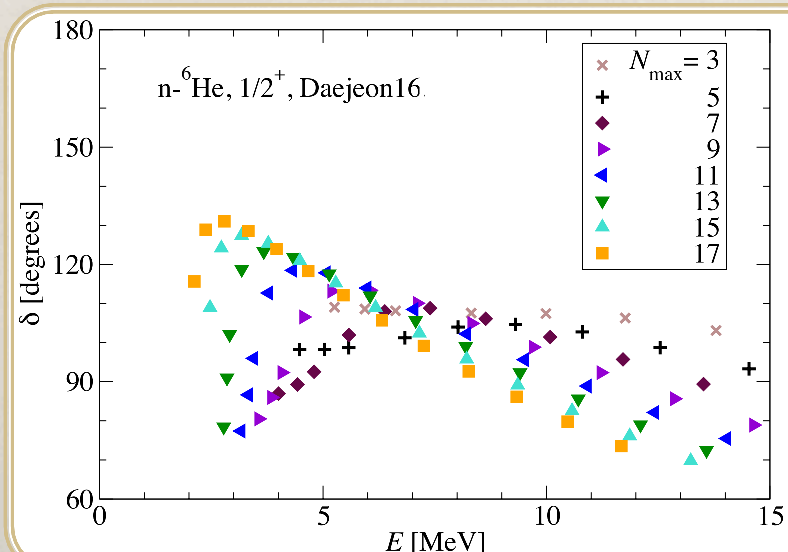
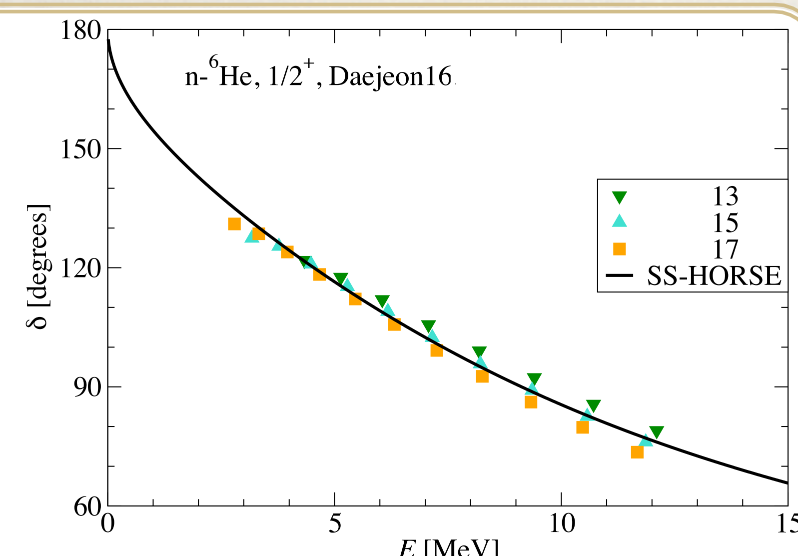
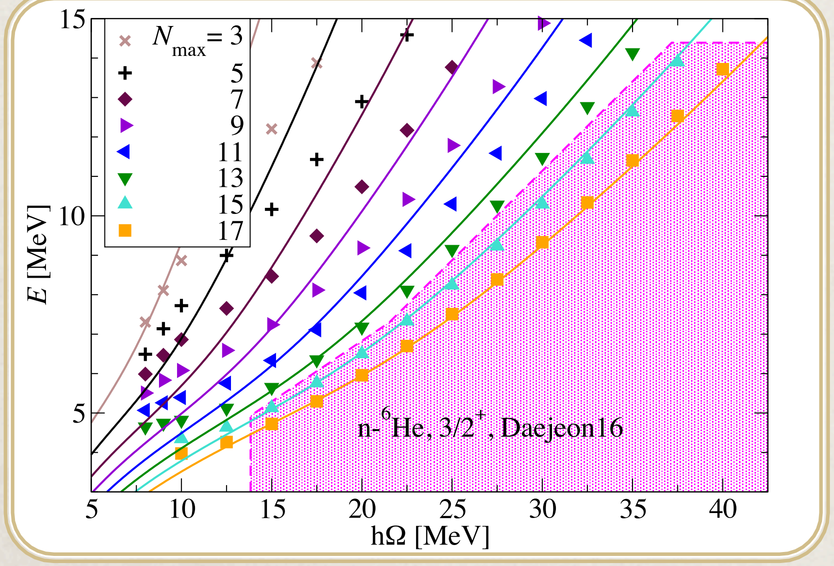
Reasonable convergence





$3/2^+$ state of n–^{6}He scattering: Daejeon16



	E_r, MeV	Γ, MeV	Ξ, keV	D
SS-HORSE $N_{max} \leq 17$	4.0	4.4	131	22
From experiment*	*non-resonant*			

$$\frac{w_0^{(n)} + w_1^{(n)} E + w_2^{(n)} E^2}{1 + w_1^{(d)} E} = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

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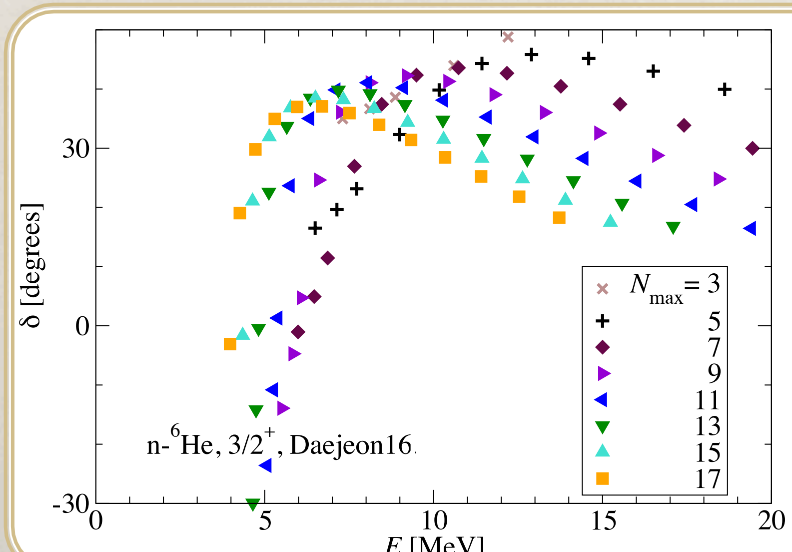
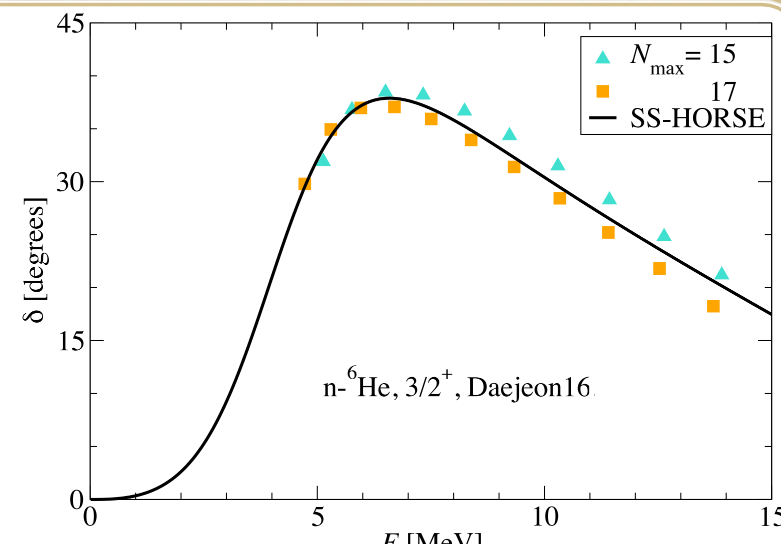
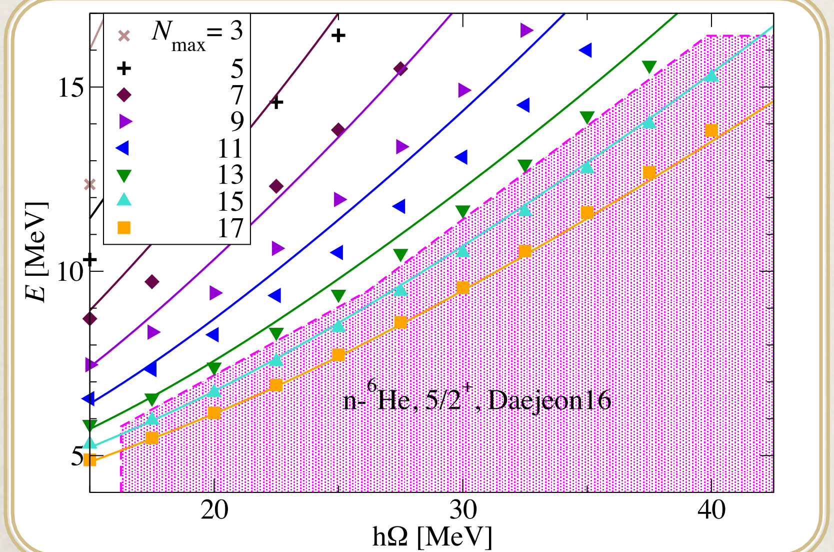
Pure convergence





$5/2^+$ state of n–^{6}He scattering: Daejeon16



	E_r, MeV	Γ, MeV	Ξ, keV	D
SS-HORSE $N_{max} \leq 17$	3.9	4.7	135	20
From experiment*	*non-resonant*			

$$\frac{w_0^{(n)} + w_1^{(n)} E + w_2^{(n)} E^2}{1 + w_1^{(d)} E} = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

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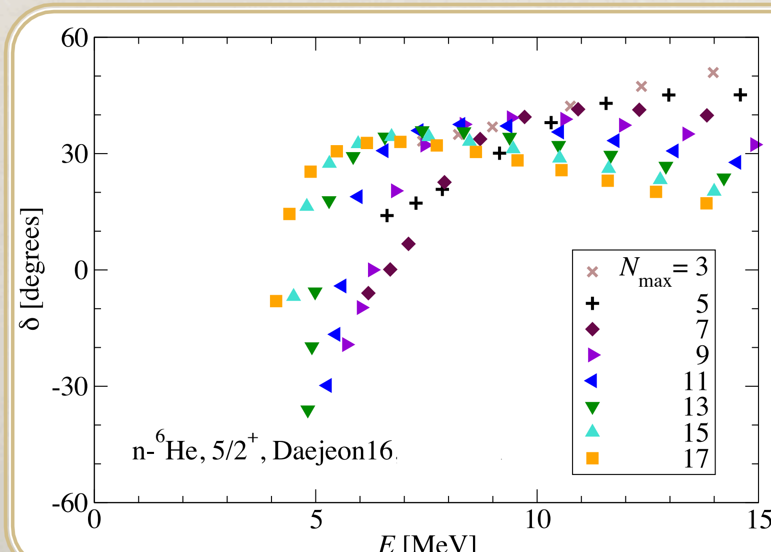
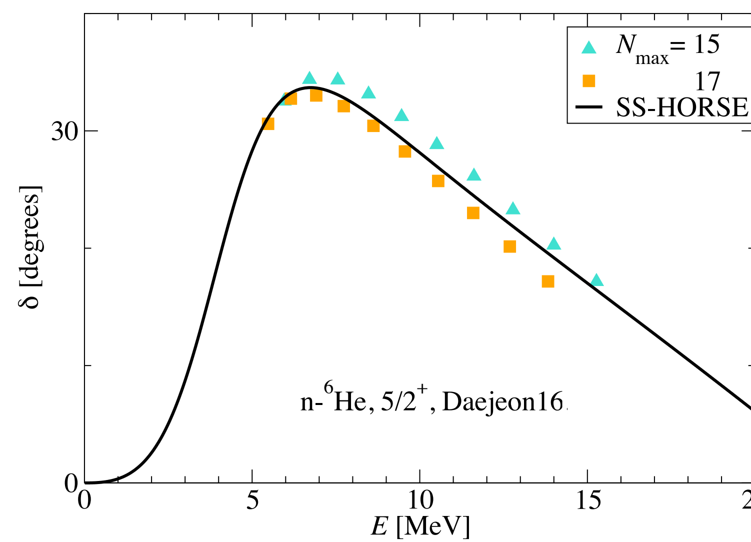
Pure convergence

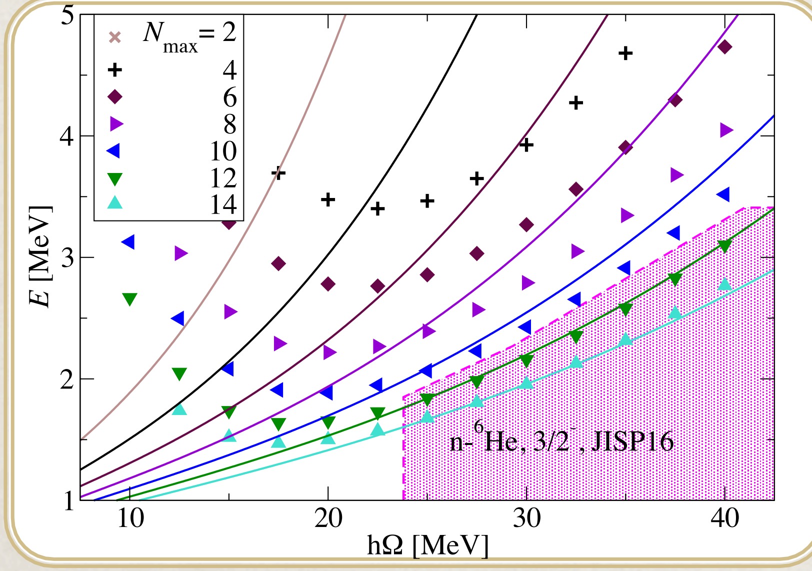




n–^{6}He scattering with JISP16

* No-Core Shell Model calculations:
    * NN-interaction: JISP16
    * lowest eigenenergies E_0 of $3/2^-$, $1/2^-$, $5/2^-$, $1/2^+$, $3/2^+$ and $5/2^+$ states of ^{7}He nuclei in bases with $N_{\max} \leq 15$, $10 \leq \hbar\Omega \leq 40$ MeV
    * ground state energy of ^{6}He nuclei in the same bases
* Approximation: ^{6}He described by only lowest oscillator states
* Energy is calculated regarding the channel threshold
    * for waves with negative parity $$E_0(N_{\max},\hbar\Omega) = E_0^{^7\mathrm{He}}(N_{\max},\hbar\Omega) - E_0^{^6\mathrm{He,\ gs}}(N_{\max},\hbar\Omega)$$
    * for waves with positive parity $$E_0(N_{\max},\hbar\Omega) = E_0^{^7\mathrm{He}}(N_{\max},\hbar\Omega) - E_0^{^6\mathrm{He,\ gs}}(N_{\max}-1,\hbar\Omega)$$
* Link oscillator quanta of relative motion and NCSM excitation quanta $$\mathbb{N} = N_{\max}^{^7\mathrm{He}} + N_{\min}^{^7\mathrm{He}} - N_{\min}^{^6\mathrm{He}} = N_{\max} + 1$$

3/2⁻ state of n–^{6}He scattering: JISP16



	E_r, MeV	Γ, MeV	Ξ, keV	D
JISP16 $N_{max} \leq 14$	0.71	0.60	36	14
Daejeon16	0.28	0.13	320	43
From experiment*	0.44	0.16		

$$w_0 + w_1 E + w_2 E^2 = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

Reasonable convergence

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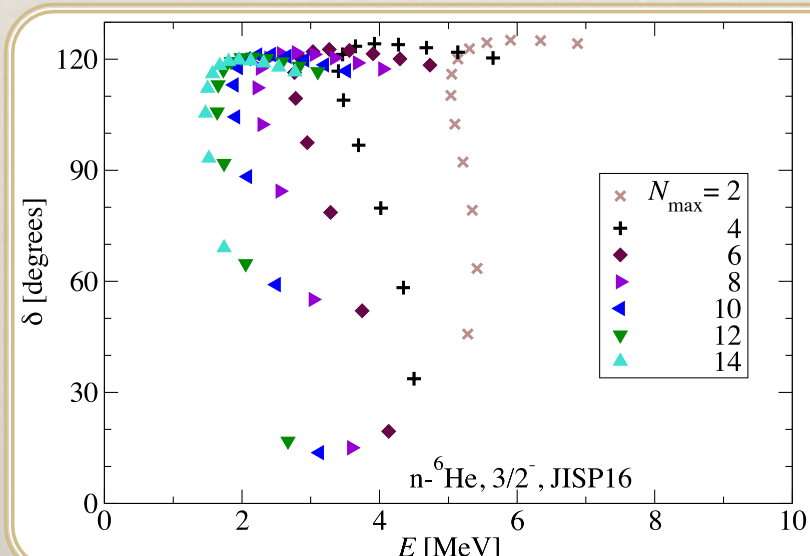
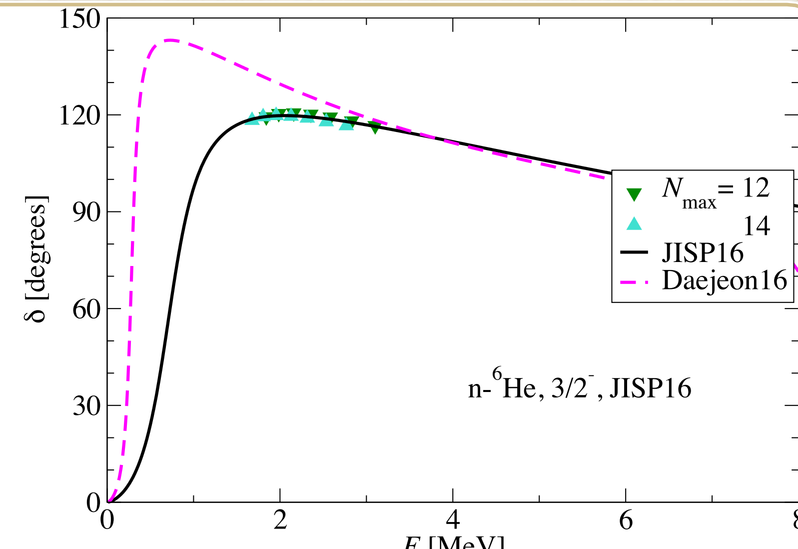
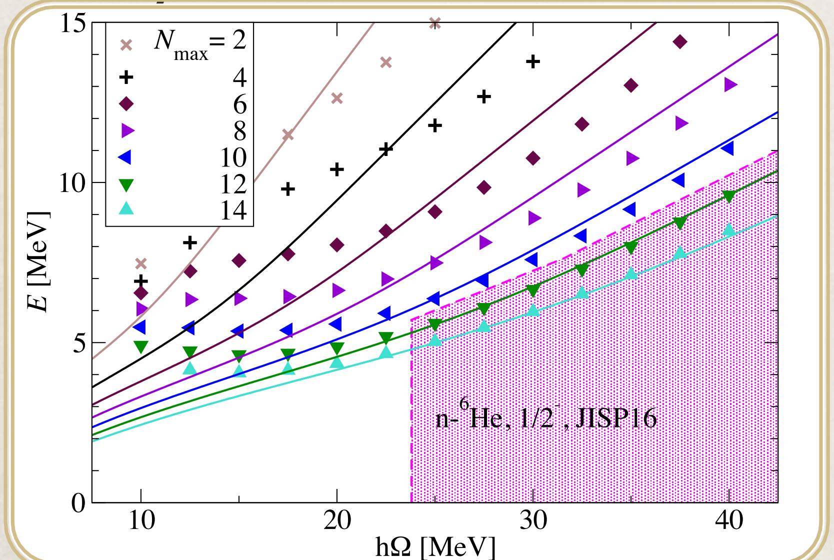




1/2⁻ state of n–^{6}He scattering: JISP16



	E_r, MeV	Γ, MeV	Ξ, keV	D
JISP16 $N_{max} \le 14$	3.0	5.0	89	12
Daejeon16	2.8	4.3	571	32
From experiment*	1.2	1.0		

$$w_0 + w_1 E + w_2 E^2 = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

Pure convergence

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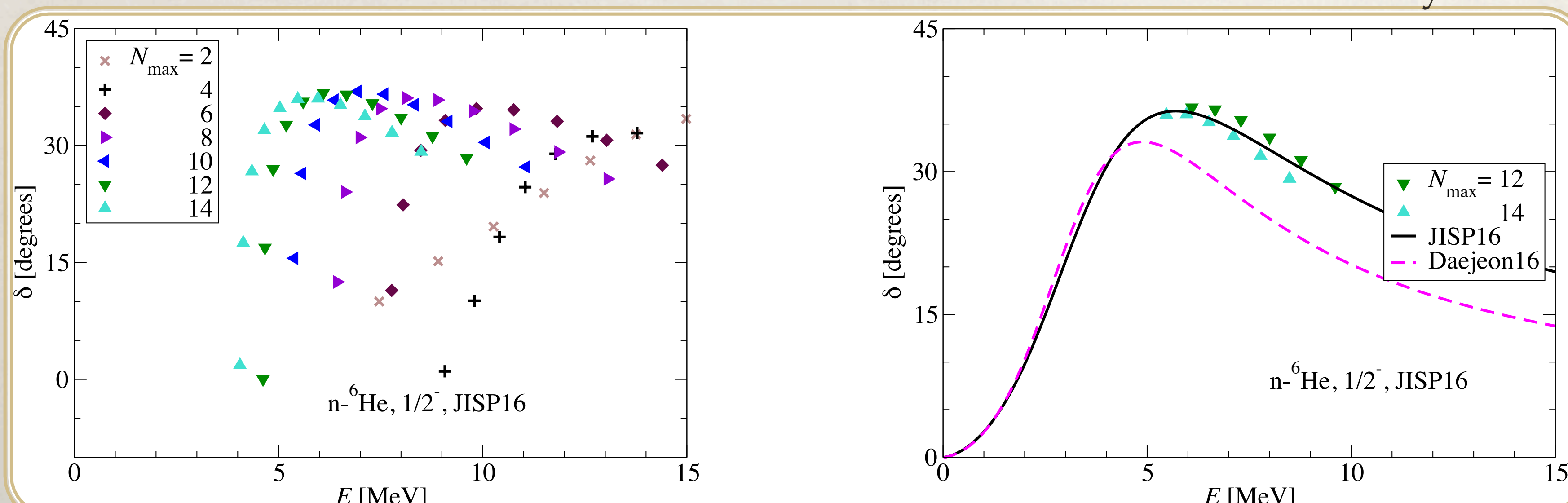
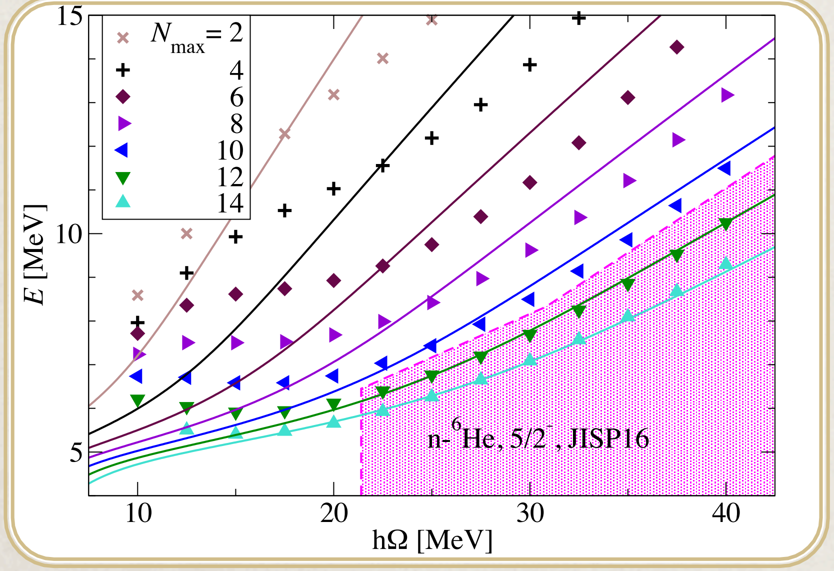


5/2⁻ state of n–^{6}He scattering: JISP16



	E_r, MeV	Γ, MeV	Ξ, keV	D
JISP16 $N_{max} \leq 14$	4.2	1.3	76	16
Daejeon16	3.7	1.4	651	42
From experiment*	3.3	2.2		

$$w_0 + w_1 E + w_2 E^2 + w_3 E^3 = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

Reasonable convergence

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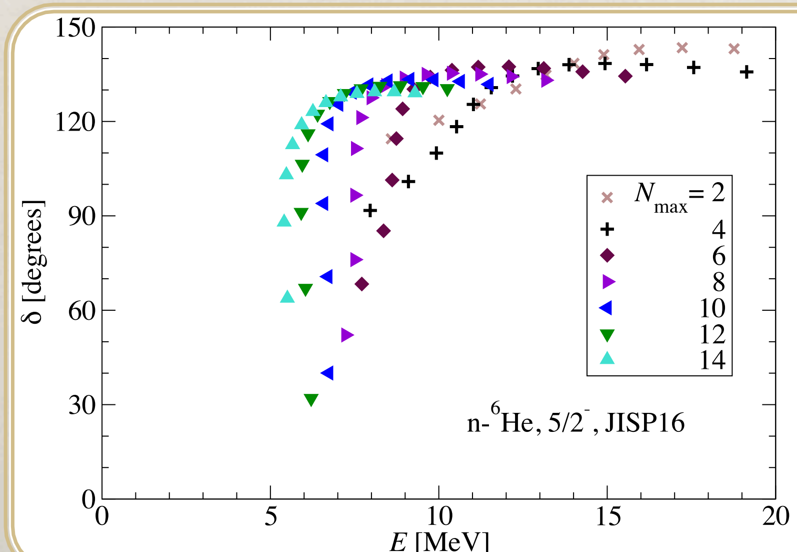
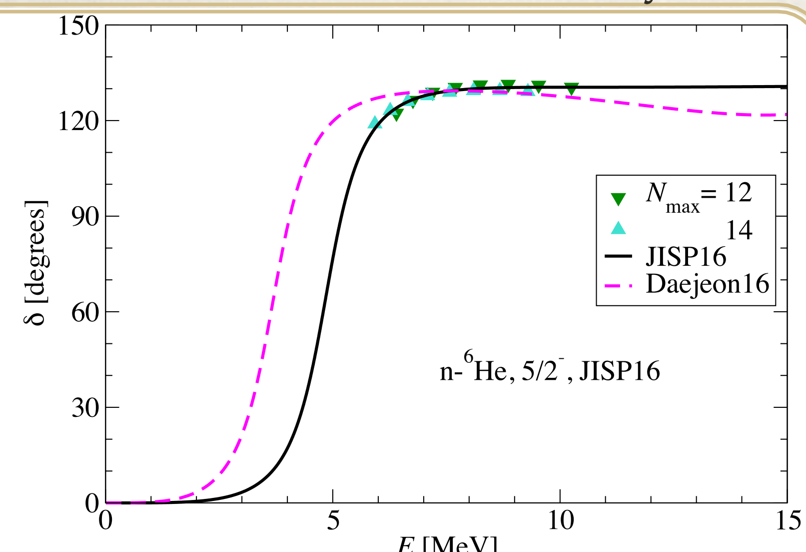
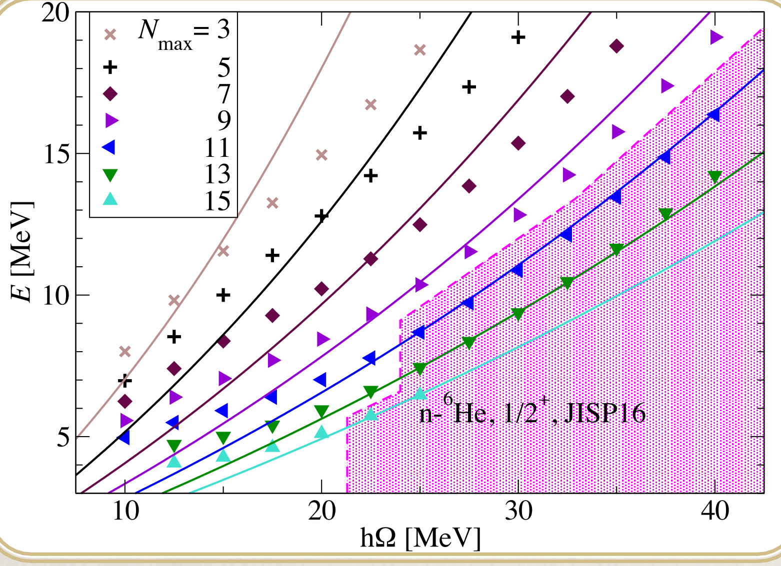




$1/2^+$ state of n–^{6}He scattering: JISP16



	E_r, MeV	Γ, MeV	Ξ, keV	D
JISP16 $N_{max} \le 15$	*non-resonant*		155	16
Daejeon16	*non-resonant*		885	29
From experiment*	*non-resonant*			

$$w_0 + w_1 E + w_2 E^2 = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

Pure convergence

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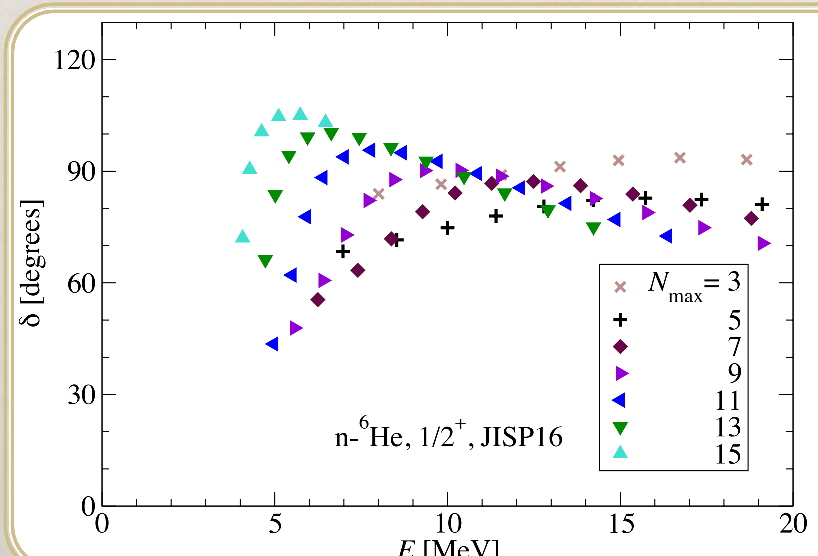
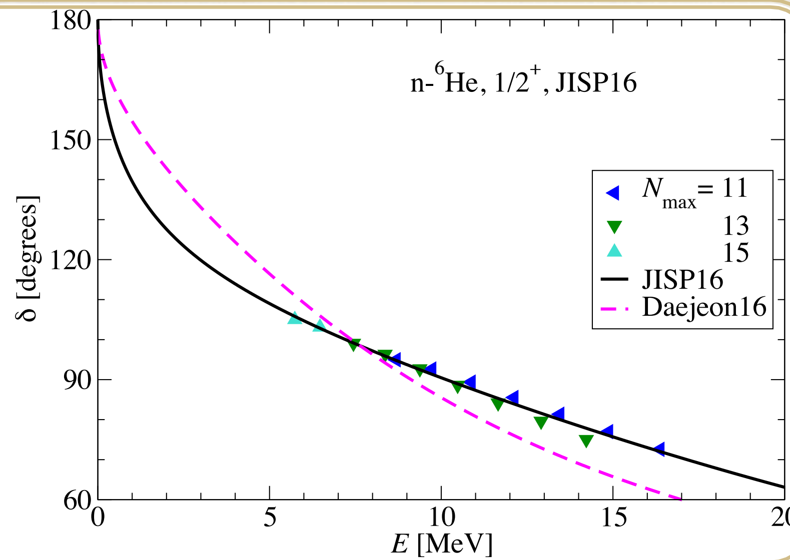
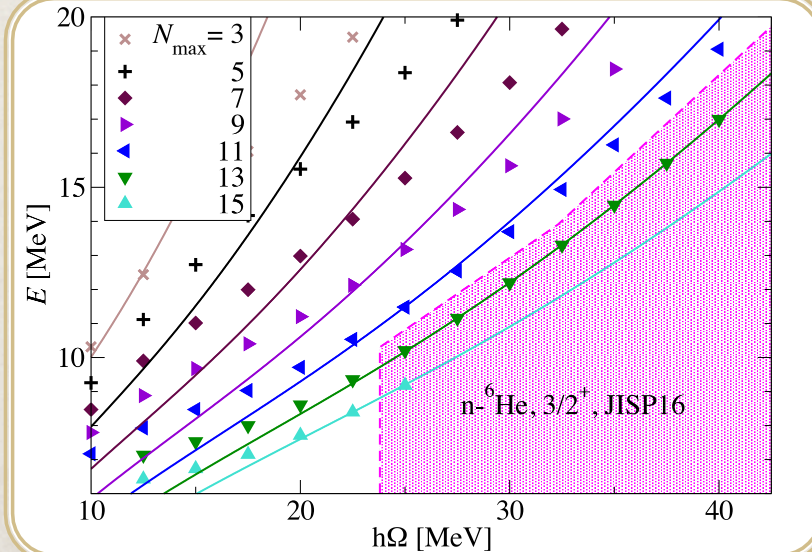




$3/2^+$ state of n–^{6}He scattering: JISP16



	E_r, MeV	Γ, MeV	Ξ, keV	D
JIPS16 $N_{\max} \leq 15$	6.3	9.8	12	8
Daejeon16	4.0	4.4	131	22
From experiment*	*non-resonant*			

$$\frac{w_0^{(n)} + w_1^{(n)} E + w_2^{(n)} E^2}{1 + w_1^{(d)} E} = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

Pure convergence

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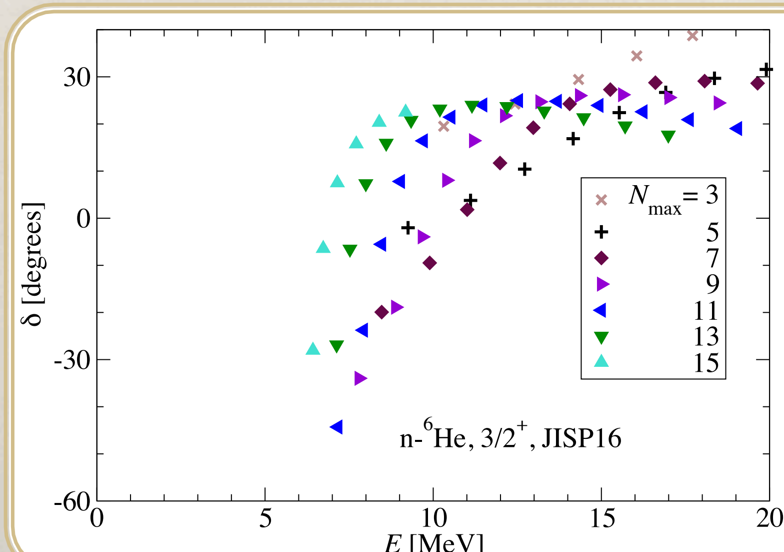
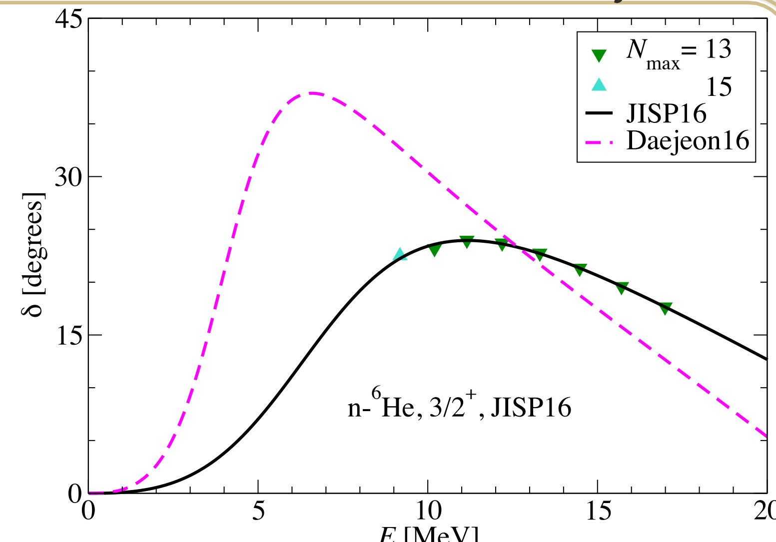
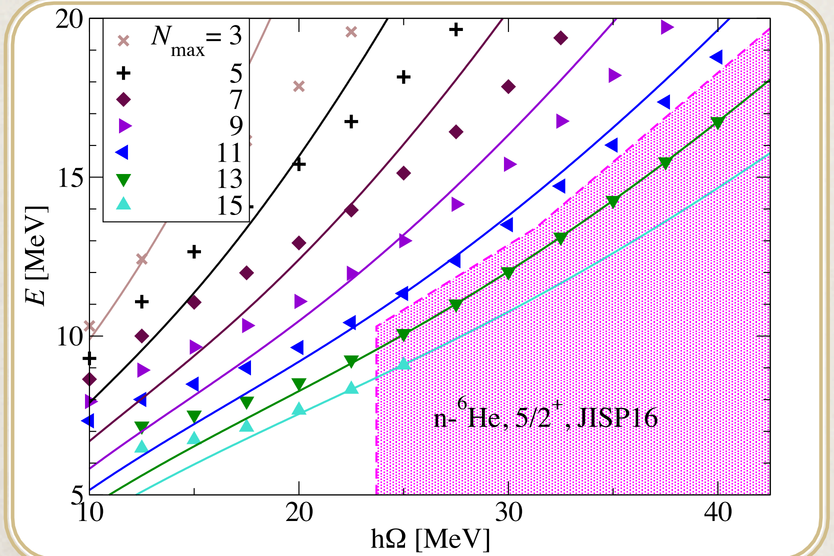




5/2$^+$ state of n–^{6}He scattering: JISP16



	E_r, MeV	Γ, MeV	Ξ, keV	*D*
ISP16 $N_{max} \le 15$	6.3	9.1	12	8
Daejeon16	3.9	4.7	135	20
From experiment*	*non-resonant*			

$$\frac{w_0^{(n)} + w_1^{(n)} E + w_2^{(n)} E^2}{1 + w_1^{(d)} E} = -k^{2l+1} \frac{C_{\mathbb{N}+2,l}(E)}{S_{\mathbb{N}+2,l}(E)}$$

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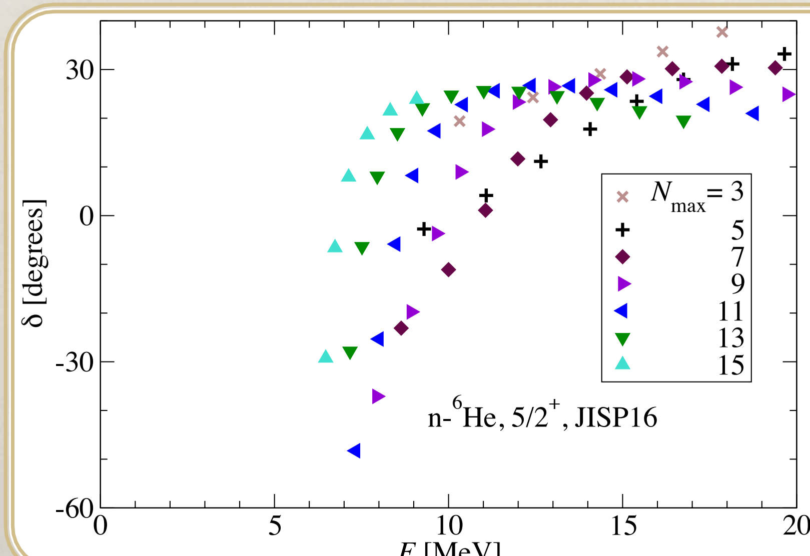

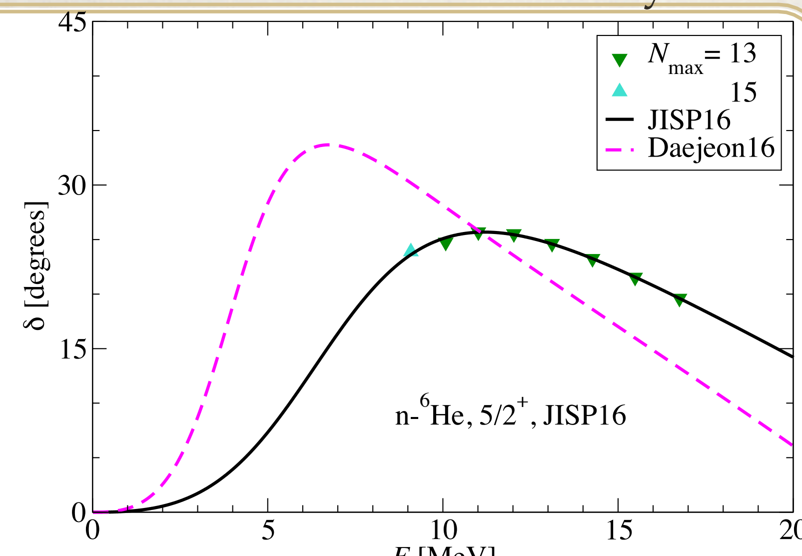

Results

State		Daejeon16	JISP16	Experiment*
$3/2^-$	E_r	**0.28**	0.71	0.44
	Γ	**0.13**	0.60	0.16
$1/2^-$	E_r	*2.8*	*3.0*	1.2
	Γ	*4.3*	*5.0*	1.0
$5/2^-$	E_r	**3.3**	4.2	3.3
	Γ	**1.4**	1.3	2.2
$1/2^+$	E_r	non-resonant	non-resonant	non-resonant
	Γ			
$3/2^+$	E_r	*4.4*	*6.3*	non-resonant
	Γ	*4.0*	*9.8*	
$5/2^+$	E_r	*3.9*	*6.3*	non-resonant
	Γ	*4.7*	*9.1*	

Good Reasonable *Poor* convergence (all numbers in MeV)

Convergence with Daejeon16 in general looks better then with JISP16

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Thank you for attantion!