THE RELATIVISTIC DYNAMICS IN MINKOWSKI SPACE: EXPLORING HADRON STRUCTURE

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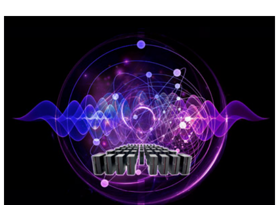


Int. Conf. Nuclear Theory in Supercomputing Era

NTSE2018, IBS, Daejeon, Oct 29- Nov 2

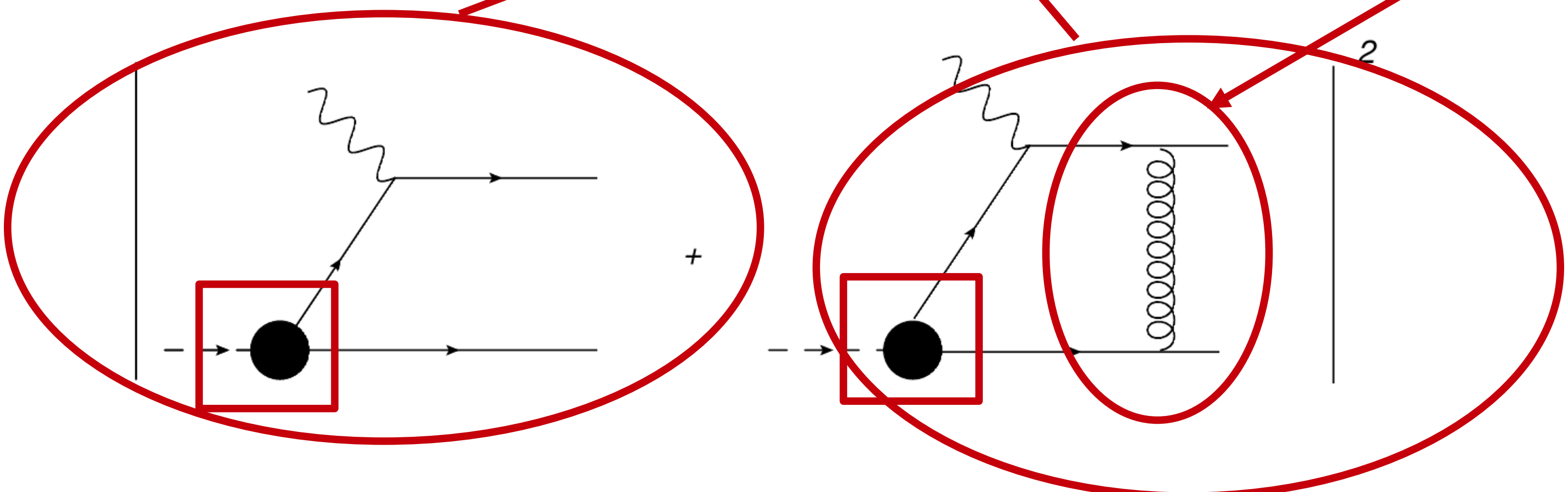
Motivation

- Develop methods in continuous nonperturbative QCD in *Minkowski space-time*
- Solve the Bethe-Salpeter bound state equation 2 & 3 bodies
- Observables: spectrum, SL/TL momentum region
- Relation BSA to LF Fock-space expansion of the hadron wf
- Inversion Problem: Euclidean→Minkowski

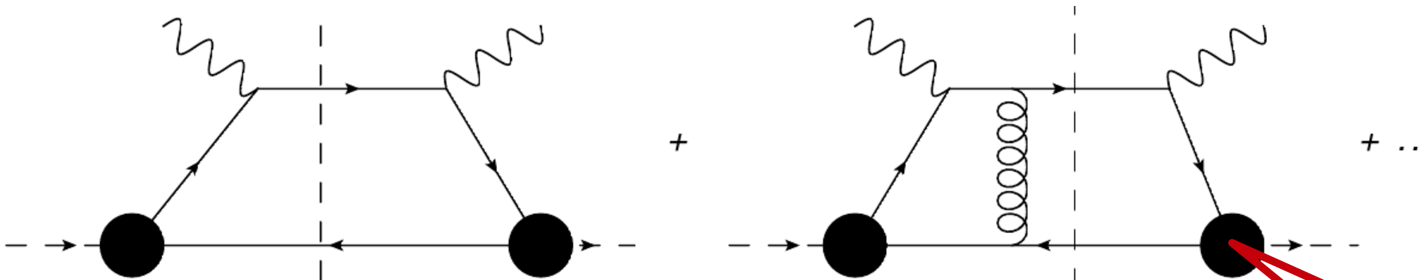
Applications:
parton distributions (pdfs); generalized parton distributions;
transverse momentum distributions (TMDs);
Fragmentation functions; TL form factors ….

TMDs & PDFs

FSI gluon exchange: T-odd



TF & Miller PRD 50 (1994)210



$$q^2 = q^+q^- - q^2_T$$

$$q^+ = q^0+q^3 \quad q^- = q^0-q^3$$

$q^- \longrightarrow$ *infty*

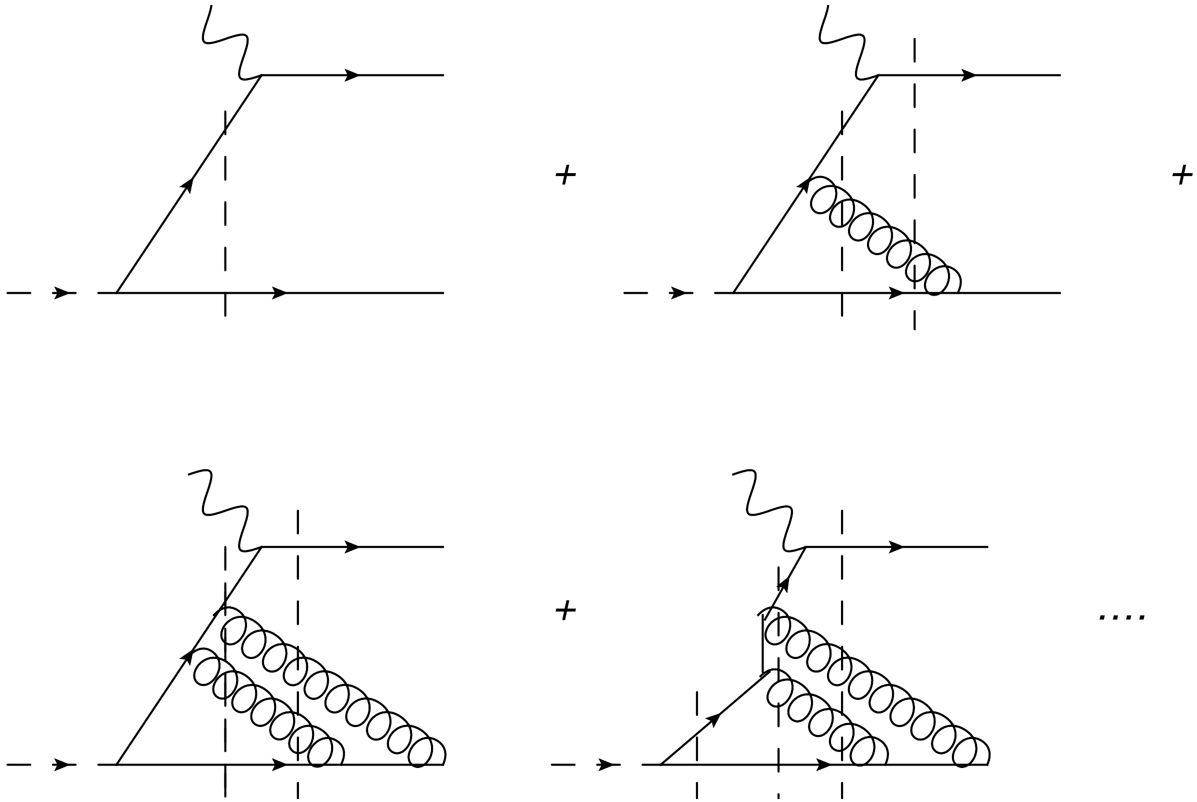
DIS

Bethe-Salpeter Amplitude @ $x^+=0$

Beyond the valence ....

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001)

Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)



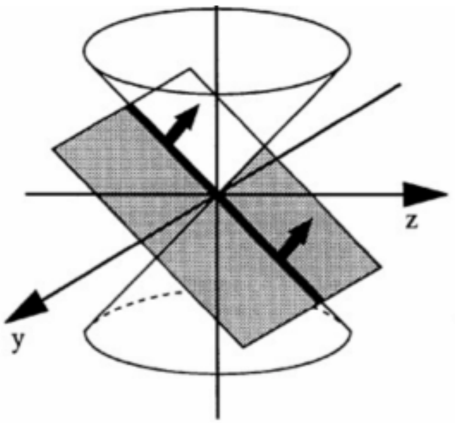
Population of lower x, due to the gluon radiation!
(LF initial state interaction)

Bethe-Salpeter Amplitude →Light-Front WF (LFWF)

- **basic ingredient in PDFs, GPDs and TMDs**

$$\tilde{\Phi}(x, p) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Phi(k, p)$$

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$



$$\begin{aligned}\tilde{\Phi}(x, p) &= \langle 0|T\{\varphi_H(x^\mu/2)\varphi_H(-x^\mu/2)\}|p\rangle \\ &= \theta(x^+)\langle 0|\varphi(\tilde{x}/2)e^{-iP^- x^+/2}\varphi(-\tilde{x}/2)|p\rangle e^{ip^- x^+/4} + \bullet\ \bullet\ \bullet \\ &= \theta(x^+)\sum_{n,n'} e^{ip^- x^+/4}\langle 0|\varphi(\tilde{x}/2)|n'\rangle\langle n'|e^{-iP^- x^+/2}|n\rangle\langle n|\varphi(-\tilde{x}/2)|p\rangle + \bullet\ \bullet\ \bullet\end{aligned}$$

$x^+ = 0$ only valence state remains! How to rebuilt the full BS amplitude?

Iterated Resolvents: Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998)

Reminder…

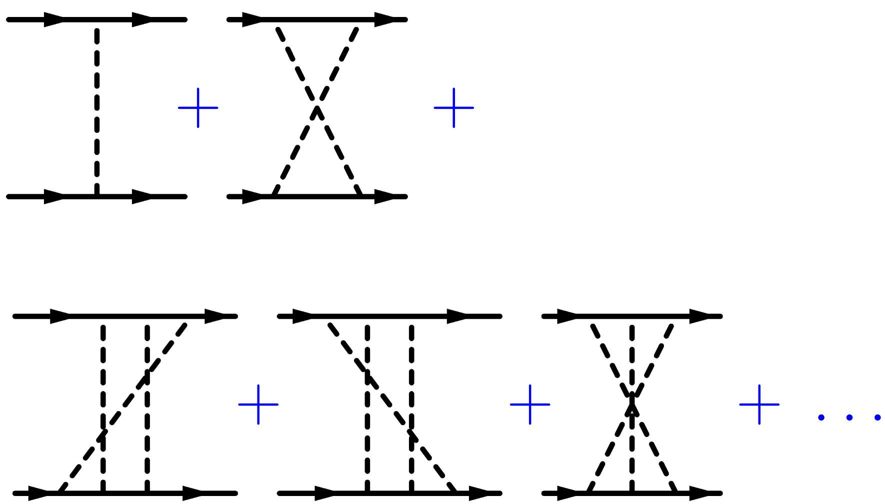
Bethe-Salpeter Bound-State Equation

(2 bosons)

$$\Phi(k,p) = G_0^{(12)}(k,p) \int \frac{d^4k'}{(2\pi)^4} \mathrm{i}K(k,k';p)\Phi(k',p)$$

$$G_0^{(12)}(k,p) = \frac{\mathrm{i}}{\left[(p/2+k)^2 - m^2 + \mathrm{i}\epsilon\right]} \frac{\mathrm{i}}{\left[(p/2-k)^2 - m^2 + \mathrm{i}\epsilon\right]}$$

Kernel: sum 2PI diagrams



- **Valence LF wave function → BSA ?**
- **Valence→ full Fock Space w-f ?**

Sales, et al. PRC61, 044003 (2000)

Quasi-potential approach to LF projection: Expressing the BSE in the LF

Reverse operation: valence wave function ⇒ BS amplitude

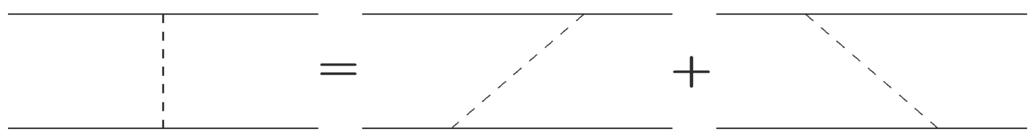
$$|\Psi\rangle = \Pi(p)\, |\phi_{LF}\rangle$$

Sales, et al. PRC61, 044003 (2000); PRC63, 064003 (2001); Frederico et al. NPA737, 260c (2004); Marinho et al., PRD 76, 096001 (2007); Marinho et al. PRD77, 116010 (2008); Frederico and Salmè, FBS49, 163 (2011).

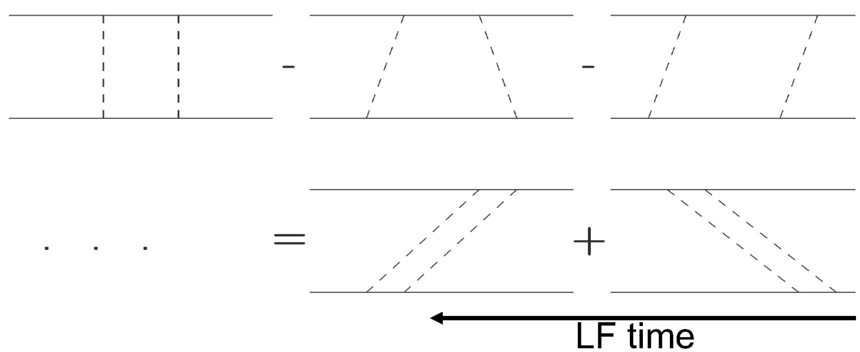
Example: Bosonic Yukawa model   Ladder approx.

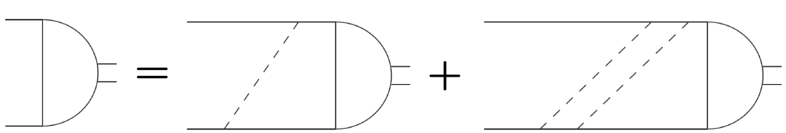
$$\mathcal{L}_I = g_S \phi_1^\dagger \phi_1 \sigma + g_S \phi_2^\dagger \phi_2 \sigma$$

$w^{(1)} =$



$w^{(2)} =$





$$\left[g_0^{-1} - w \right]$$

Mass² eigenvalue eq. & valence wf:

$$g(K_\lambda)^{-1} \, |\phi_\lambda\rangle = 0$$

Main Tool: Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" [Nakanishi PR130(1963)1230]

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

$$\kappa^2 = m^2 - \frac{M^2}{4}$$

BSE in Minkowski space with NIR for bosons

Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^-

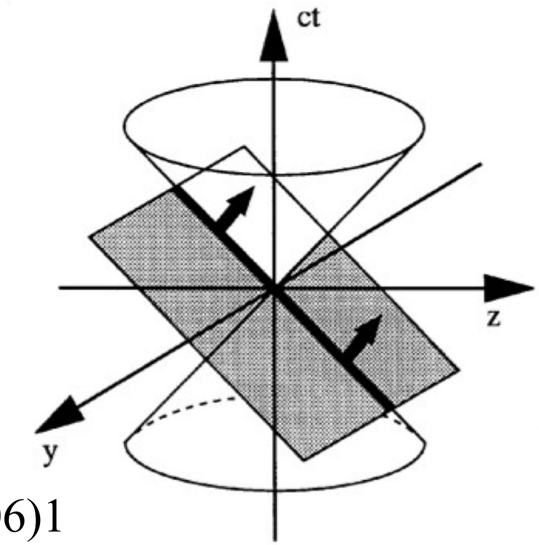
Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD85(2012)036009;PRD89(2014) 016010,EPJC75(2015)398 (application to scattering)

LF wave function

& NAKANISHI INTEGRAL REPRESENTATION

Carbonell&Karmanov EPJA27(2006)1



$$\psi_{LF}(\gamma,z)=\frac{1}{4}(1-z^2)\int_0^\infty \frac{g(\gamma',z)d\gamma'}{\Big[\gamma'+\gamma+z^2m^2+\kappa^2(1-z^2)\Big]^2}$$

$$\gamma=k_\perp^2 \qquad\qquad z=2x-1$$

Generalized Stietjes transform and the LF valence wave function

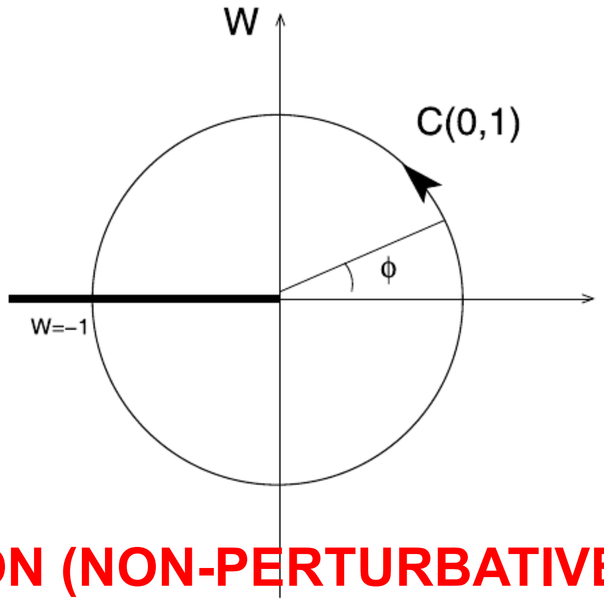
Jaume Carbonell, TF, Vladimir Karmanov PLB769 (2017) 418

$$\psi_{LF}(\gamma,z) = \frac{1-z^2}{4}\int_0^{\infty}\frac{g(\gamma',z)d\gamma'}{\left[\gamma'+\gamma+z^2m^2+(1-z^2)\kappa^2\right]^2}.$$

$$f(\gamma) \equiv \int_0^{\infty} d\gamma'\, L(\gamma,\gamma')g(\gamma') = \int_0^{\infty} d\gamma' \,\frac{g(\gamma')}{(\gamma'+\gamma+b)^2}$$

denoted symbolically as $f = \hat{L}\, g$.

$$g(\gamma) = \hat{L}^{-1} f = \frac{\gamma}{2\pi}\int_{-\pi}^{\pi} d\phi\; e^{i\phi}\, f(\gamma\, e^{i\phi} - b).$$



J.H. Schwarz, J. Math. Phys. 46 (2005) 014501,

- **UNIQUENESS OF THE NAKANISHI REPRESENTATION (NON-PERTURBATIVE)**
- **PHENOMENOLOGICAL APPLICATIONS from the valence wf → BSA!**

Solution Method of the Bethe-Salpeter eq.:

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11

$$\Phi(k,p) = G_0(k,p) \int d^4k' \, \mathcal{K}_{BS}(k,k',p) \, \Phi(k',p)$$

$\Rightarrow$

$$\int_0^\infty d\gamma' \frac{g_b(\gamma', z; \kappa^2)}{[\gamma' + \gamma + z^2 m^2 + (1-z^2)\kappa^2 - i\epsilon]^2} =$$

$$= \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; V_b^{LF}(\gamma, z; \gamma', z') g_b(\gamma', z'; \kappa^2).$$

with $V_b^{LF}(\gamma, z; \gamma', z')$ determined by the irreducible kernel $\mathcal{I}(k,k',p)$!

UNIQUENESS OF THE NAKANISHI WEIGHT FUNCTION?

PERTURBATIVE PROOF BY NAKANISHI.

NON-PERTURBATIVE PROOF? Inverse Stieltjes: Int. Eq. in the normal form.

Two-Boson System: ground-state

Building a solvable model…

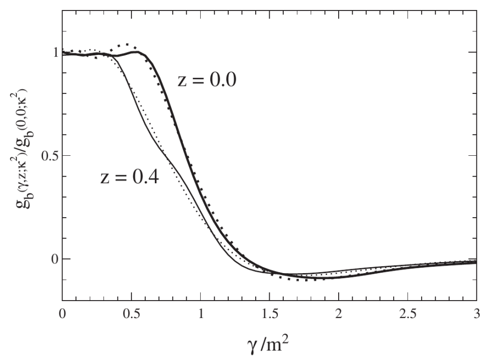
Nakanishi weight function

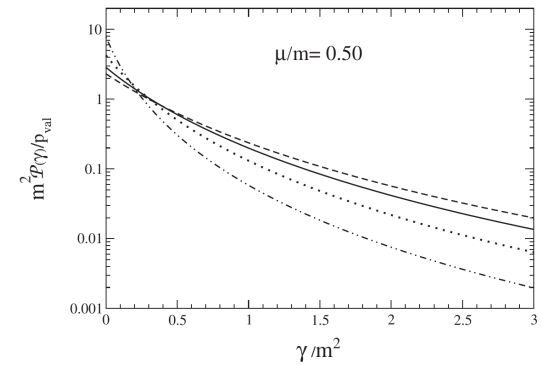
Valence wave function

3+1 n=1

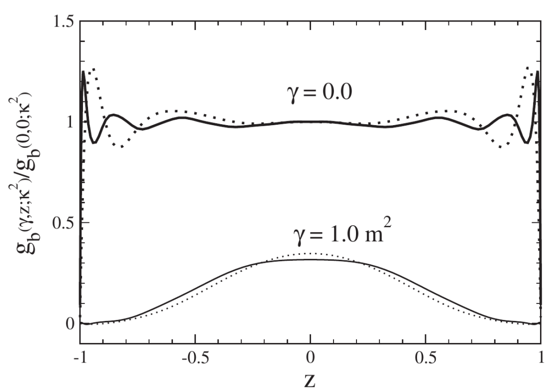
LADDER KERNEL

3+1 n=1





$\mu = 0.5 \;\; B/M = 1$



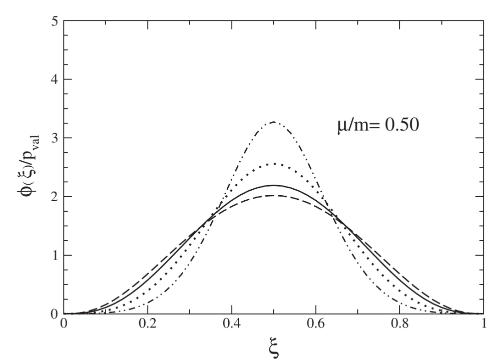


FIG. 3. The longitudinal LF distribution $\phi(\xi)$ for the valence component Eq. (34) vs the longitudinal-momentum fraction ξ for $\mu/m = 0.05$, 0.15, 0.50. Dash-double-dotted line: $B/m = 0.20$. Dotted line: $B/m = 0.50$. Solid line: $B/m = 1.0$. Dashed line: $B/m = 2.0$. Recall that $\int_0^1 d\xi \phi(\xi) = P_{\rm val}$ (cf. Table III).

Karmanov, Carbonell, EPJA 27, 1 (2006)
Frederico, Salmè, Viviani PRD89, 016010 (2014)

Light-front valence wave function L+XL

Large momentum behavior

$$\psi_{LF}(\gamma,\xi) \to \alpha\, \gamma^{-2}\, C(\xi)$$

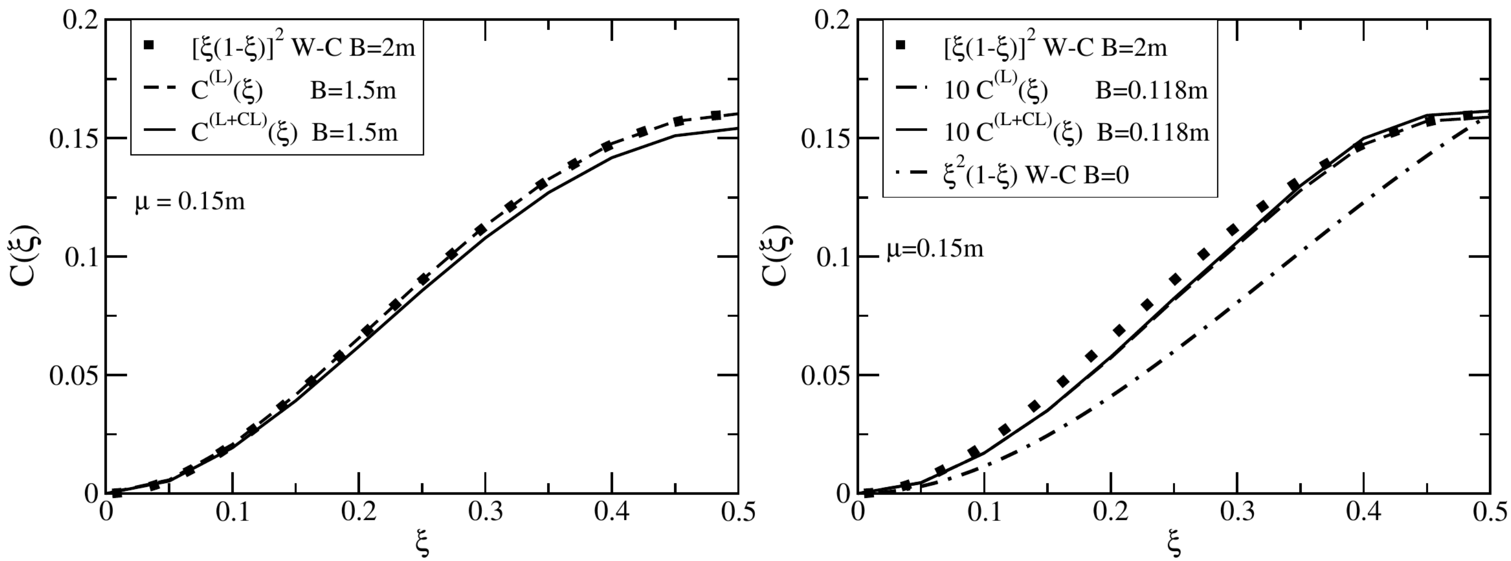


Fig. 2. Asymptotic function $C(\xi)$ defined from the LF wave function for $\gamma \to \infty$ (6) computed for the ladder kernel, $C^{(L)}(\xi)$ (dashed line), and ladder plus cross-ladder kernel, $C^{(L+CL)}(\xi)$ (solid line), with exchanged boson mass of $\mu = 0.15\,m$. Calculations are performed for $B = 1.5\,m$ (left frame) and $B = 0.118\,m$ (right frame). A comparison with the analytical forms of $C(\xi)$ valid for the Wick-Cutkosky model for $B = 2m$ (full box) and $B \to 0$ (dash-dotted line) both arbitrarily normalized.

Gigante, Nogueira, Ydrefors, Gutierrez, Karmanov, TF, PRD95(2017)056012.

Transverse distribution: Euclidean and Minkowski

$$\phi_M^T(\mathbf{k}_\perp) \equiv \int dk^0 dk^3 \Phi(k,p) = \frac{1}{2}\int dk^+ dk^- \Phi(k,p) \text{ and}$$

$$\phi_E^T(\mathbf{k}_\perp) \equiv \mathrm{i}\int dk_E^0 dk^3 \Phi_E(k_E,p),$$

136 C. Gutierrez et al. / Physics Letters B 759 (2016) 131–137

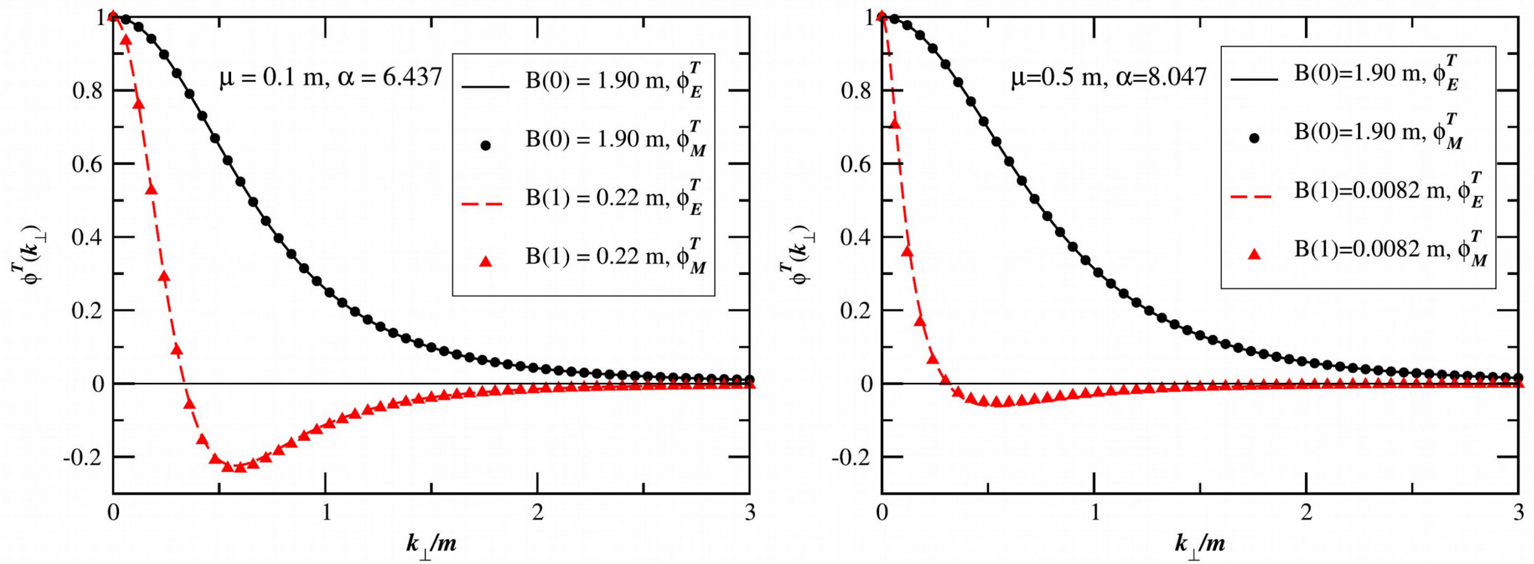


Fig. 6. Transverse momentum amplitudes *s*-wave states, in Euclidean and Minkowski spaces, vs $k_\perp$, for both ground- and first-excited states, and two values of μ/m and α_{gr} (as indicated in the insets). The amplitudes ϕ_E^T and ϕ_M^T, arbitrarily normalized to 1 at the origin, are not easily distinguishable.

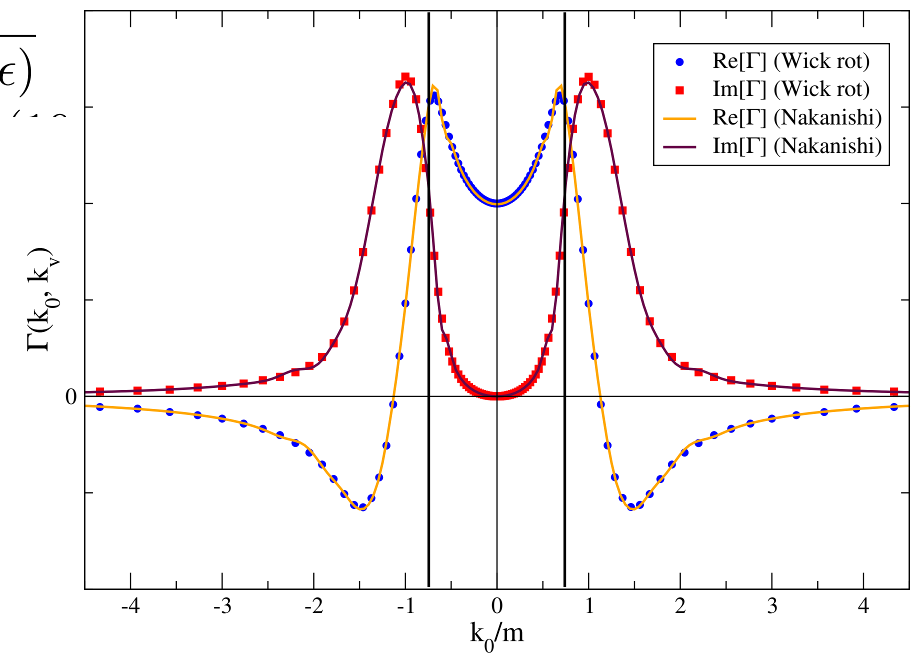
Gutierrez, Gigante, TF, Salmè, Viviani, Tomio PLB759 (2016) 131

Rotation in Complex Plane

Comparison between solution for the vertex function in the complex plane and NIR

$$\Gamma(k;P) = \imath g^2 \int \frac{d^4k'}{(2\pi)^4} \frac{\Gamma(k';P)}{((k-k')^2 - \mu^2 + i\epsilon)}$$
$$\times \frac{1}{((\frac{1}{2}P + k')^2 - m^2 + i\epsilon)((\frac{1}{2}P - k')^2 - m^2 + i\epsilon)}$$

α = 5.48, /m = 0.2, B/m = 1.0, θ = π/16, k_v/m = 0.067



$$k_0 \to k_0 \exp \imath\, \theta$$

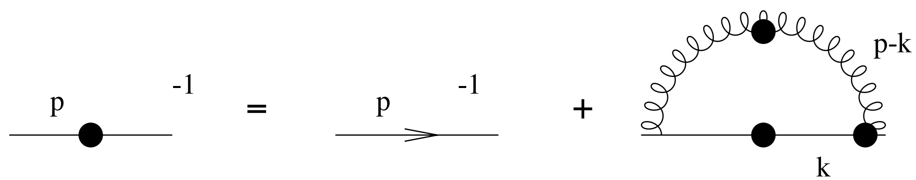
Peaks Branching points: $k_0^{\pm} = \pm\sqrt{(m+\mu)^2 + (\vec{k})^2} \mp \frac{\sqrt{p^2}}{2}$

Castro, de Paula, TF, Maris, Nogueira, Ydrefors; in preparation

Dyson-Schwinger equation in Rainbow ladder truncation

from Euclidean to Minkowski: Un-Wick rotating

In collaboration with Duarte, de Paula, Maris, Nogueira, Ydrefors



$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

QED-like, Landau Gauge, bare vertices, massive vector boson, Pauli-Villars regulator

Wick-rotated SD equation

$$A(p_0, \vec{p}^2) = 1 + g^2 \int_{-\infty}^{\infty} dk_0 \int \frac{d^3k}{(2\pi)^4} \frac{A(k_0, \vec{k}^2)\, K^A(p_0, k_0, \vec{p}, \vec{k}, \vec{p}\cdot\vec{k})}{(k_0^2 + \vec{k}^2)\, A^2(k_0, \vec{k}^2) + B^2(k_0, \vec{k}^2)}$$

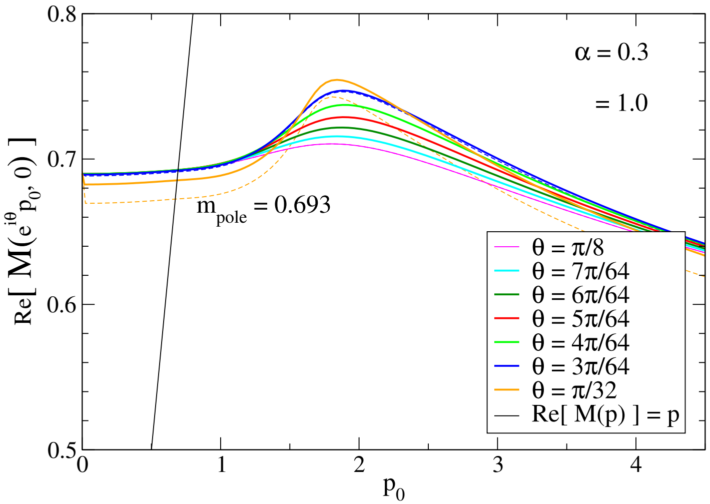
$$B(p_0, \vec{p}^2) = m_0 + g^2 \int_{-\infty}^{\infty} dk_0 \int \frac{d^3k}{(2\pi)^4} \frac{B(k_0, \vec{k}^2)\, K^B(p_0, k_0, \vec{p}, \vec{k}, \vec{p}\cdot\vec{k})}{(k_0^2 + \vec{k}^2)\, A^2(k_0, \vec{k}^2) + B^2(k_0, \vec{k}^2)}$$

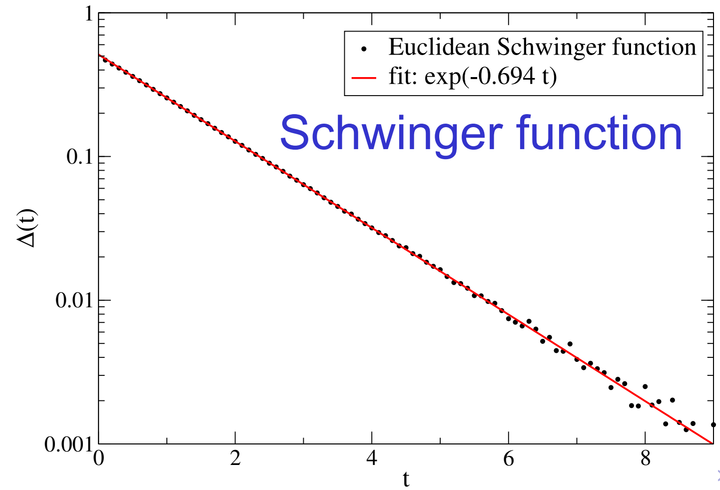
Un-Wick rotation: $k_0 \to k_0 \exp\imath(\theta - \frac{\pi}{2}) \quad p_0 \to p_0 \exp\imath(\theta - \frac{\pi}{2})$

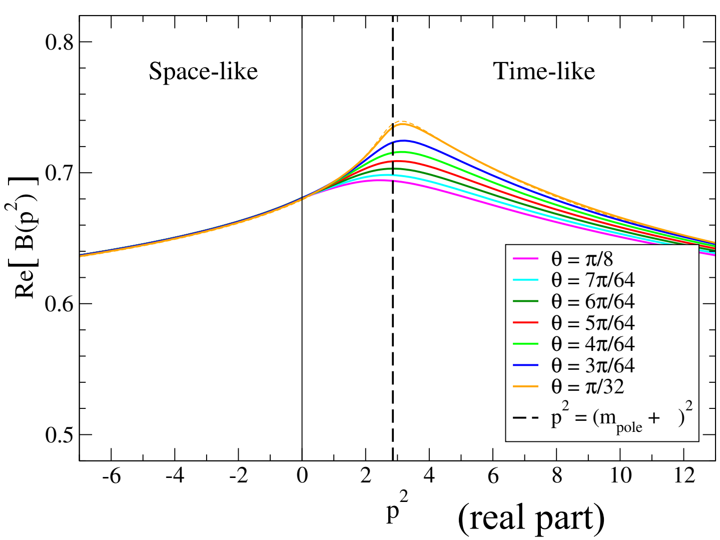
Euclidean $\theta = \dfrac{\pi}{2}$ **Minkowski** $\theta = 0$

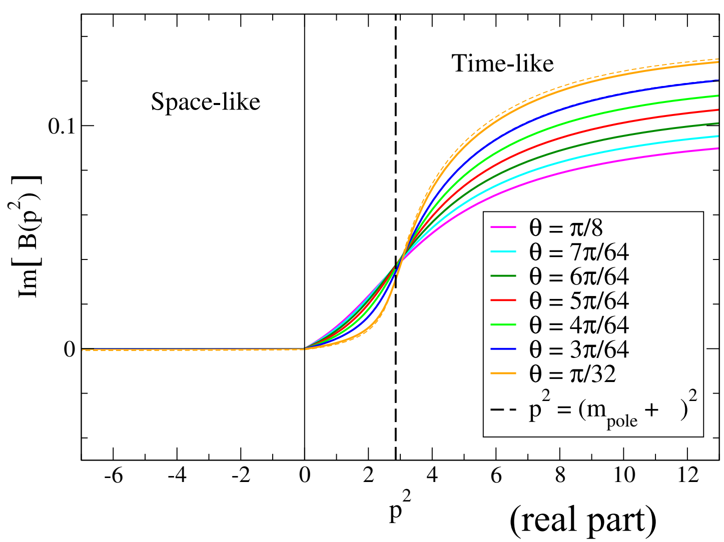
Parameters: $m_0 = 0.5 \quad \mu = 1 \quad \alpha = 0.3 \quad \Lambda = 10$

$$M = B/A \qquad \Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_0 + \vec{x}\cdot\vec{p})} S_{s,v}(p^2) \propto e^{-m\,t}$$





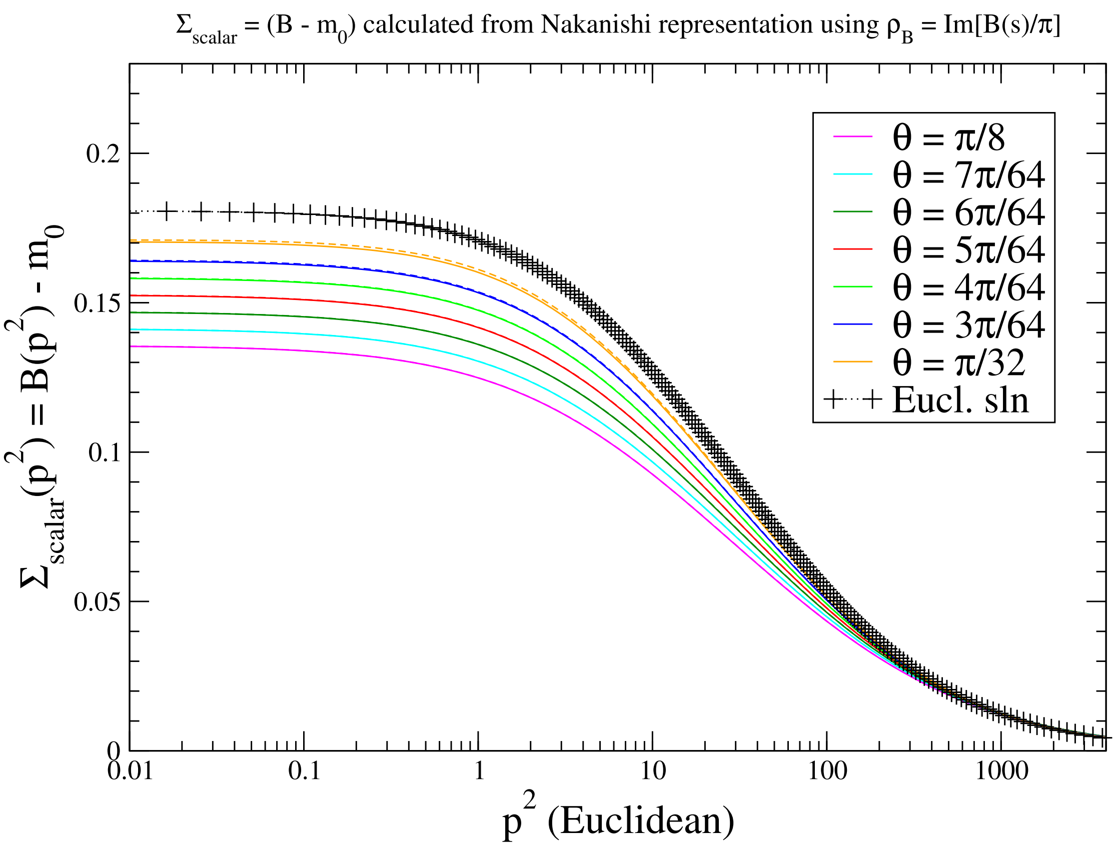




Spectral Representation (Nakanishi Integral representation)

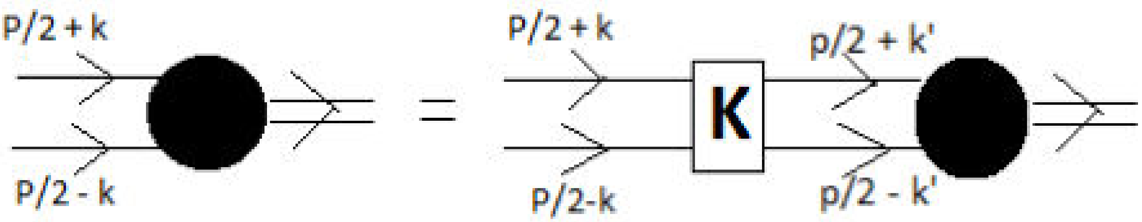
$$\Sigma_{\text{scalar}}(p^2) = B(p^2) - m_0 = \int_0^\infty \frac{\rho_B(s)}{p^2 - s + \imath\epsilon}$$

$$\rho_B(s) = -\text{Im}[B(s)/\pi]$$



BSE for two-fermions

Carbonell and Karmanov EPJA 46 (2010) 387; de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



Vector Exchange in Ladder

$$i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$$

Vertex Form-Factor

$$F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$$

fermion-antifermion 0$^-$: $\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

Nakanishi Integral Representation :

$$\phi_i(k,p) = \int_{-1}^{+1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{(k^2 + p\cdot k\ z' + M^2/4 - m^2 - \gamma' + i\epsilon)^3}$$

Light-front projection: integration over k^- (LF singularities)

For the two-fermion BSE, singularities have generic form:

$$\mathcal{C}_j = \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} (k^-)^j \, \mathcal{S}(k^-, v, z, z', \gamma, \gamma') \qquad j = 1, 2, 3$$

with $\mathcal{S}(k^-, v, z, z', \gamma, \gamma')$ explicitly calculable

N.B., in the worst case

$$\mathcal{S}(k^-, v, z, z', \gamma, \gamma') \sim \frac{1}{[k^-]^2} \qquad \text{for} \quad k^- \to \infty$$

End-point singularities: T.M. Yan , Phys. Rev. **D 7**, 1780 (1973)

$$\mathcal{I}(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^2} = \pm \frac{2\pi i \; \delta(\beta)}{\left[-y \mp i\epsilon\right]}$$

→ Kernel with delta's and its derivatives!

End-point singularities– more intuitive: can be treated by the pole-dislocation method de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87

Example: Scalar boson exchange

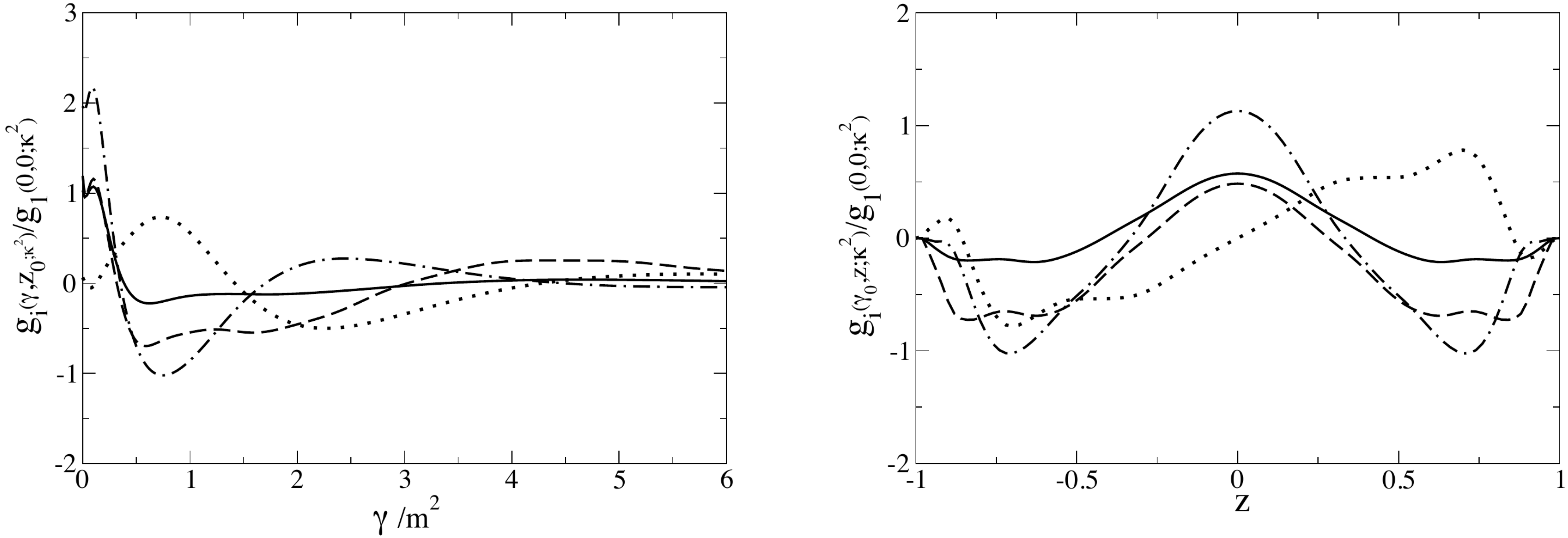


Figure 2. Nakanishi weight-functions $g_i(\gamma,\, z; \kappa^2)$, Eqs. 3.1 and 3.2 evaluated for the 0^+ two-fermion system with a scalar boson exchange such that $\mu/m = 0.5$ and $B/m = 0.1$ (the corresponding coupling is $g^2 = 52.817$ [17]). The vertex form-factor cutoff is $\Lambda/m = 2$. Left panel: $g_i(\gamma,\, z_0; \kappa^2)$ with $z_0 = 0.6$ and running γ/m^2. Right panel: $g_i(\gamma_0,\, z; \kappa^2)$ with $\gamma_0/m^2 = 0.54$ and running z, The Nakanishi weight-functions are normalized with respect to $g_1(0, 0; \kappa^2)$. Solid line: g_1. Dashed line: g_2. Dotted line: g_3. Dot-dashed line: g_4.

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;

PION MODEL

W. de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

- **Gluon effective mass ~ 500 MeV – Landau Gauge LQCD**

[Oliveira, Bicudo, JPG 38 (2011) 045003;
Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240]

- **Mquark = 250 MeV**

[Parappilly, et al, PR D73 (2006) 054504]

- **Λ/m =1, 2, 3**

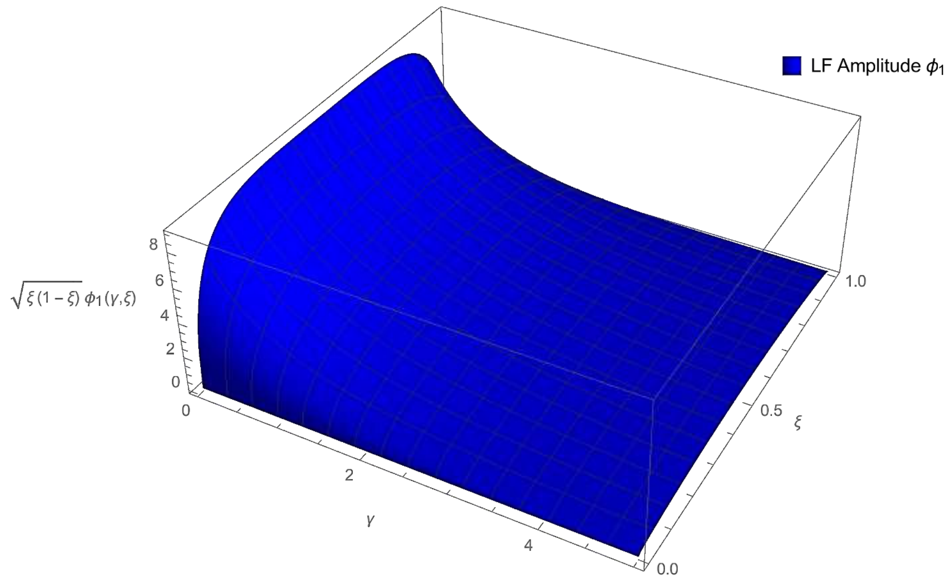
Ladder approximation (L): suppression of XL (non-planar diagram) for N_c=3

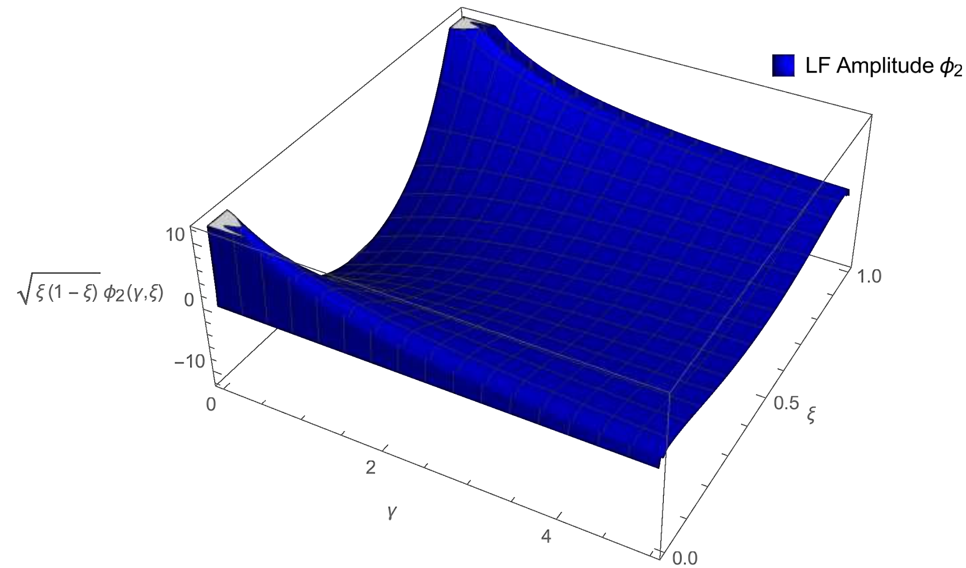
[A. Nogueira, CR Ji, Ydrefors, TF, PLB 777 (2018) 207]

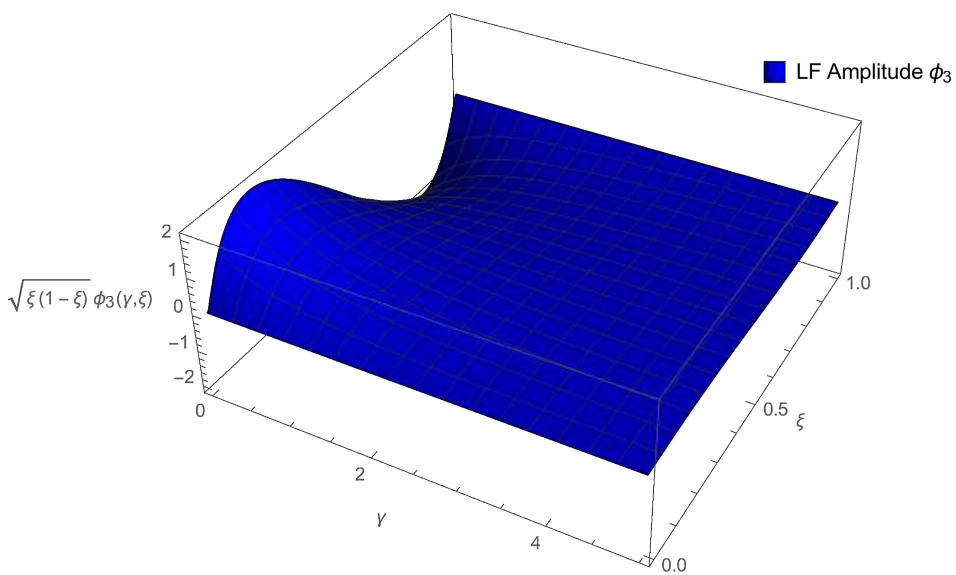
Light-front amplitudes

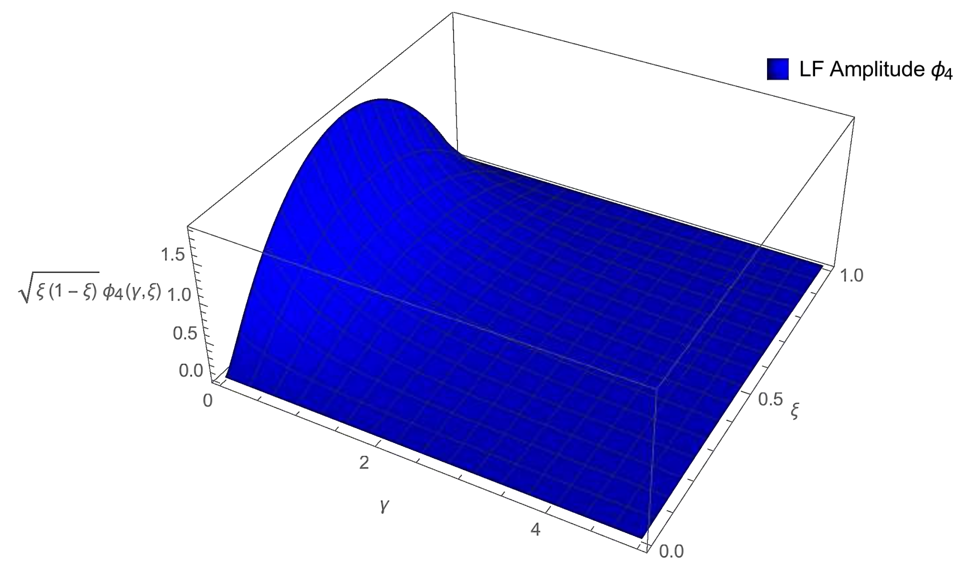
$(B/m = 1.35, \mu/m = 2.0, \Lambda/m = 1.0, \tilde{m}_q$=215 MeV): $f_\pi = 96$ MeV

$P_{val} = 0.68$









Valence distribution functions

W. de Paula, et. al, in preparation

Valence probability:

$$N_2 = \frac{1}{32\,\pi^2}\int_{-1}^{1} dz \int_0^\infty d\gamma \left\{ \tilde{\psi}_{val}(\gamma,\xi)\ \tilde{\psi}_{val}(\gamma,\xi) + \frac{\gamma}{M^2}\ \psi_{val;4}(\gamma,\xi)\ \psi_{val;4}(\gamma,\xi) \right\}$$

$$\tilde{\psi}_{val}(\gamma,z) = -\frac{i}{M}\int_0^\infty d\gamma' \frac{g_2(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2-i\epsilon]^2}$$
$$-\frac{i}{M}\frac{z}{2}\int_0^\infty d\gamma' \frac{g_3(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2-i\epsilon]^2}$$
$$+\frac{i}{M^3}\int_0^\infty d\gamma' \frac{\partial g_3(\gamma',z)/\partial z}{[\gamma+\gamma'+z^2m^2+(1-z^2)\kappa^2-i\epsilon]}$$

$$\psi_{val;4}(\gamma,z) = -\frac{i}{M}\int_0^\infty d\gamma' \frac{g_4(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2-i\epsilon]^2}\,.$$

Valence distribution functions: longitudinal and transverse

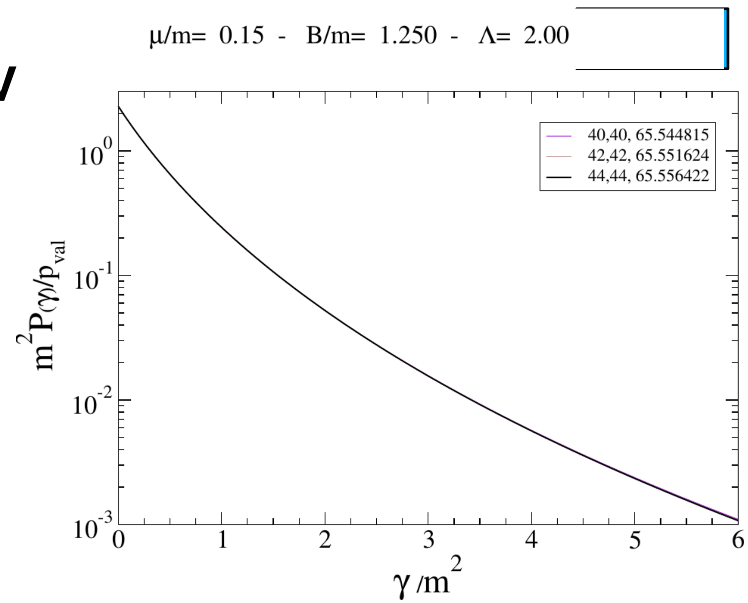
Mquark 187 MeV

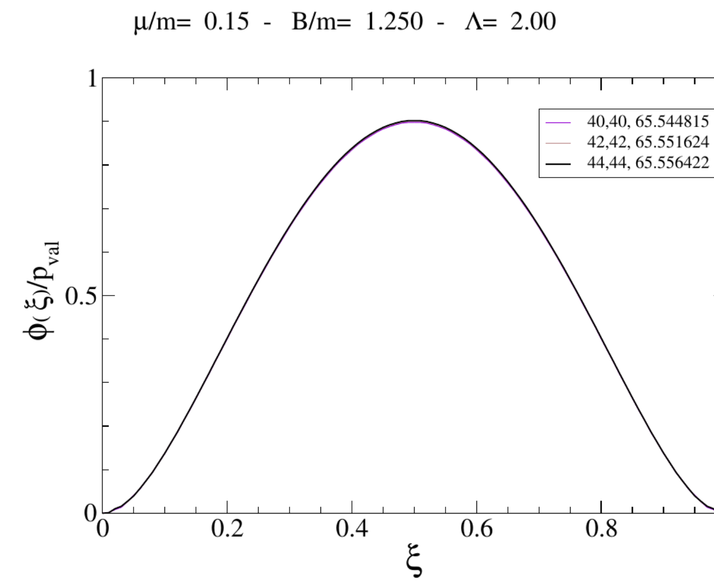
Mgluon 28 MeV

Λ/m =2

Pval=0.64

$f_\pi \; = \; 77\,\mathrm{MeV}$





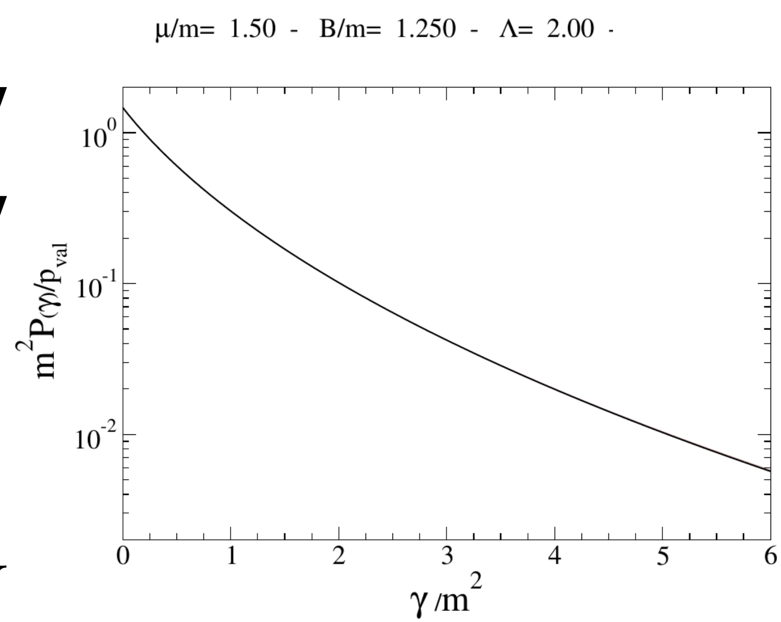
Mquark 187 MeV

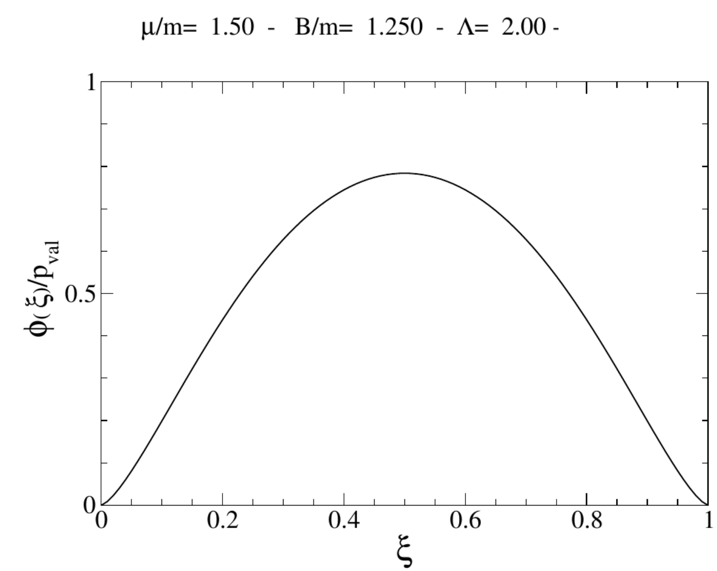
Mgluon 280 MeV

Λ/m =2

Pval=0.78

$f_\pi \; = \; 99\,\mathrm{MeV}$





Preliminary result for a fermion-scalar bound system

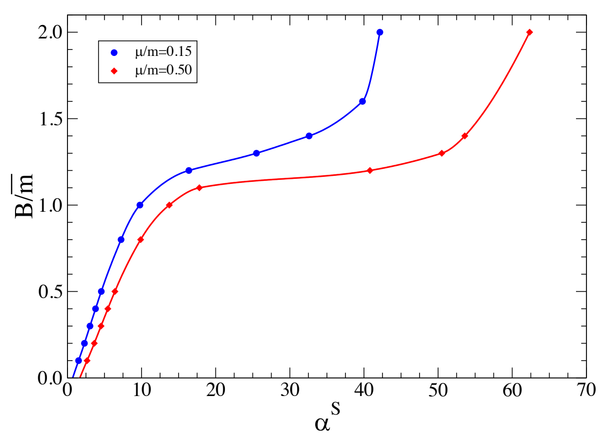
The covariant decomposition of the BS amplitude for a $(1/2)^+$ bound system, composed by a fermion and a scalar, reads

with A. Nogueira, Salmè and Pace

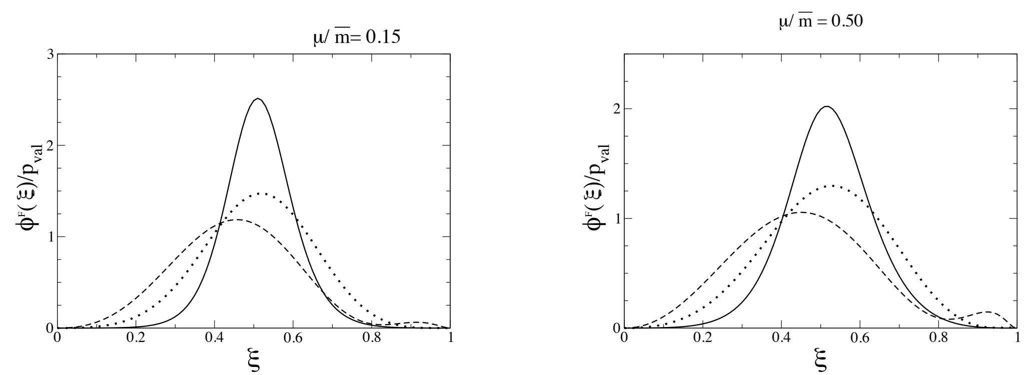
$$\Phi(k,p) = \left[S_1 \ \phi_1(k,p) + S_2 \ \phi_2(k,p) \right] \ U(p,s)$$

with $U(p,s)$ a Dirac spinor, $S_1(k) = 1$, $S_2(k) = \not{k}/M$,and $M^2 = p^2$

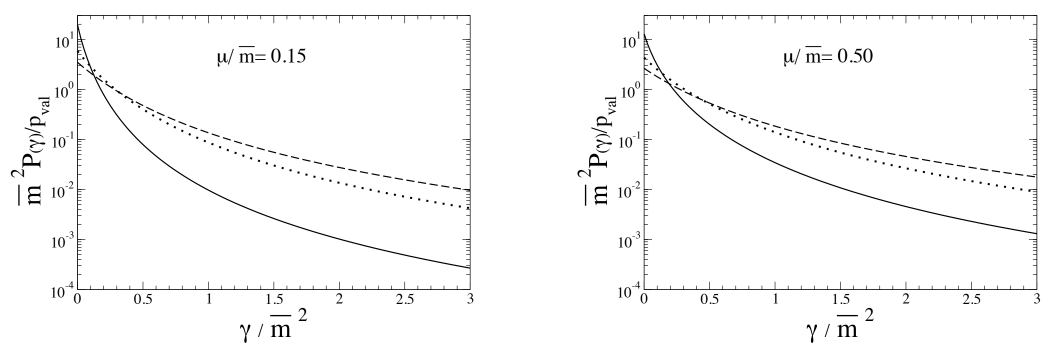
A first check: scalar coupling $\alpha^s = \lambda^s_F \lambda^s_S/(8\pi m_S)$, for $m_F = m_S$ and $\mu/\bar{m} = 0.15, \ 0.50$



Fermion-scalar system interacting through a massive scalar exchange



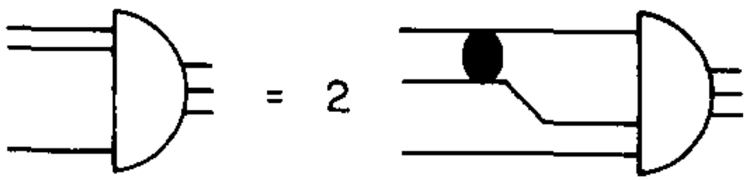
Longitudinal light-cone distribution for a fermion in the valence component. Solid line : $B/\bar{m} = 0.1$. Dotted line: $B/\bar{m} = 0.5$. Dotted line: $B/\bar{m} = 1.0$



Transverse light-cone distribution for a fermion in the valence component.

Relativistic Three-body Bound states with contact interaction

TF, PLB 282 (1992) 409



$$v(q,p) = 2iF(M_{12})\int \frac{d^4k}{(2\pi)^4}\frac{i}{[k^2-m^2+i\epsilon]}\frac{i}{[(p-q-k)^2-m^2+i\epsilon]}v(k,p).$$

$$F(M_{12}) = \begin{cases} \dfrac{8\pi^2}{\dfrac{1}{2y'_{M_{12}}}\log\dfrac{1+y'_{M_{12}}}{1-y'_{M_{12}}} - \dfrac{\pi}{2am}}, & \text{if } M_{12}^2 < 0 \\[2ex] \dfrac{8\pi^2}{\dfrac{\arctan y_{M_{12}}}{y_{M_{12}}} - \dfrac{\pi}{2am}}, & \text{if } 0 \le M_{12}^2 < 4m^2 \end{cases}$$

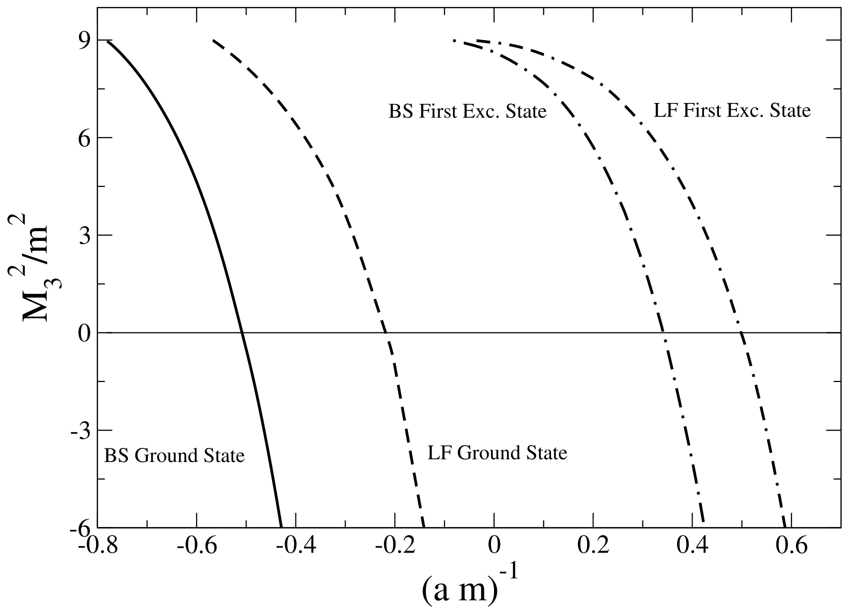
The LF projection in the NREL limit reduces to the SKT equation!

Euclidean space solution

Ydrefors, Alvarenga Nogueira, TF, Karmanov PLB 770 (2017)131

Wick rotation after the transformation $k = k' + \frac{1}{3}p, \quad q = q' + \frac{1}{3}p.$

Faddeev-BSE in Eucl. Space vs. Truncation in the LF valence sector



E. Ydrefors et al. / Physics Letters B 770 (2017) 131–137

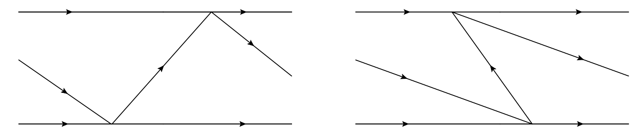


Fig. 2. The three-body LF graphs obtained by time-ordering of the Feynman graph shown in right panel of Fig. 1.

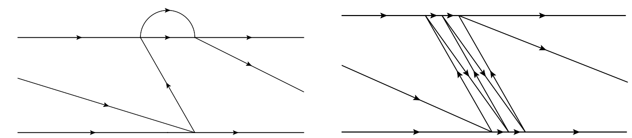


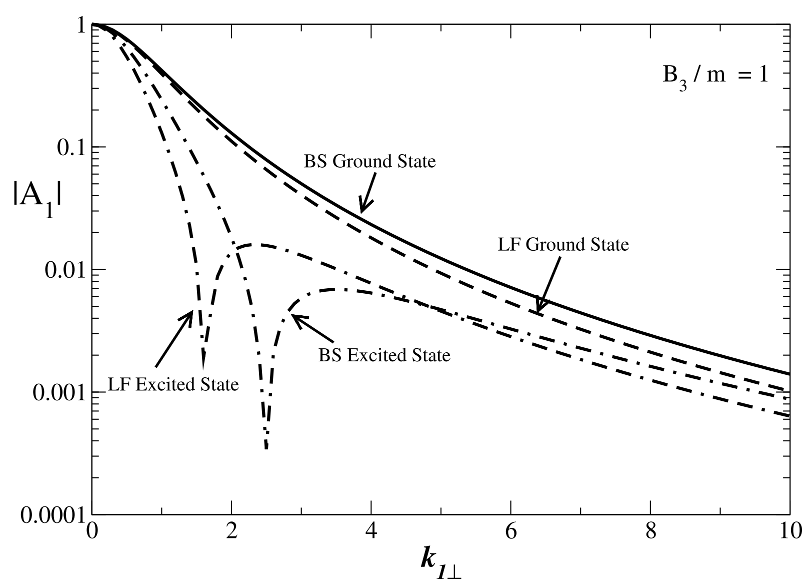
Fig. 3. Examples of many-body intermediate state contributions to the LF three-body forces.

LF Missing induced three-body forces

V.A. Karmanov, P. Maris, Few-Body Syst. 46 (2009) 95.

Transverse amplitude: FULL BSA vs LF valence truncation

$$\int_{-\infty}^{\infty} dk_{10} \int_{-\infty}^{\infty} dk_{1z} \int_{-\infty}^{\infty} dk_{20} \int_{-\infty}^{\infty} dk_{2z}\ i\Phi_M(k_{10}, k_{1z}, k_{20}, k_{2z}; \vec{k}_{1\perp}, \vec{k}_{2\perp}).$$



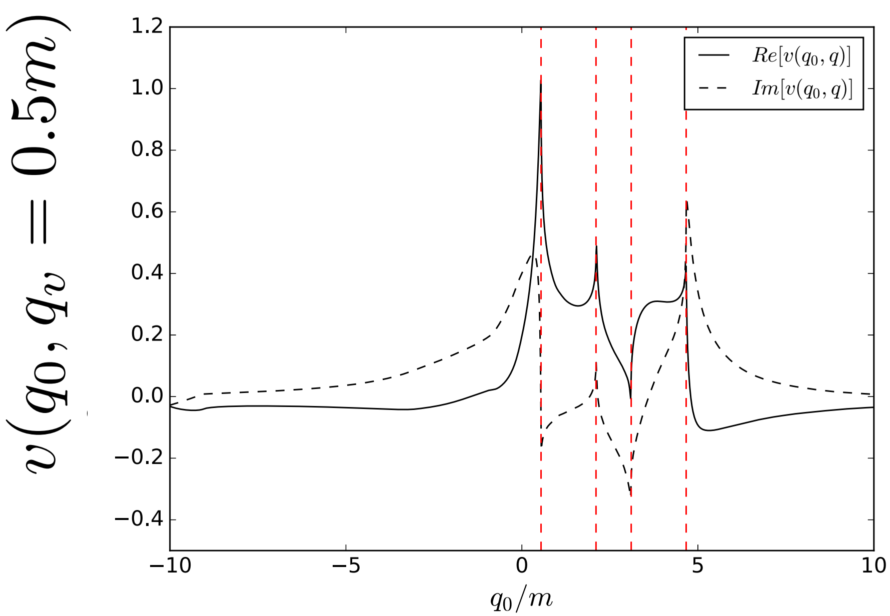
Direct solution in Minkowski space

Ydrefors, Alvarenga Nogueira, Karmanonv and TF; in preparation

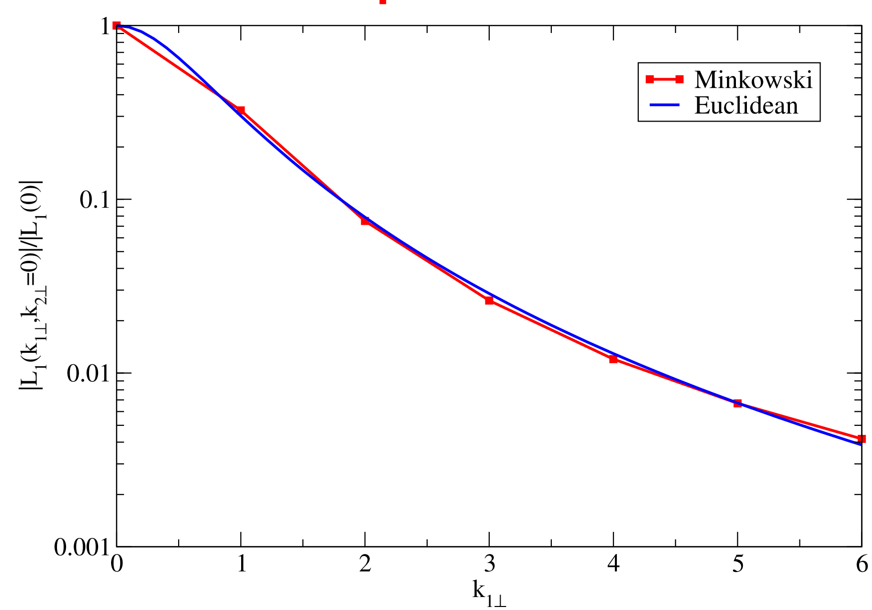
Generalization of the technique used in the two-boson problem in ladder approximation

Carbonell and Karmanov PRD90(2014) 056002

$$\begin{aligned} v(q_0,q_v) &= \frac{\mathcal{F}(M_{12})}{(2\pi)^4}\int_0^\infty k_v^2 dk_v \left\{ \frac{2\pi i}{2\varepsilon_k}\left[\Pi(q_0,q_v;\varepsilon_k,k_v)v(\varepsilon_k,k_v)+\Pi(q_0,q_v;-\varepsilon_k,k_v)v(-\varepsilon_k,k_v)\right] \right. \\ &- 2\int_{-\infty}^0 dk_0 \left[\frac{\Pi(q_0,q_v;k_0,k_v)v(k_0,k_v)-\Pi(q_0,q_v;-\varepsilon_k,k_v)v(-\varepsilon_k,k_v)}{k_0^2-\varepsilon_k^2}\right] \\ &- \left. 2\int_0^\infty dk_0 \left[\frac{\Pi(q_0,q_v;k_0,k_v)v(k_0,k_v)-\Pi(q_0,q_v;\varepsilon_k,k_v)v(\varepsilon_k,k_v)}{k_0^2-\varepsilon_k^2}\right]\right\}, \end{aligned} \quad (9)$$



transverse amplitudes: Mink. vs. Eucl.



$a\,m = -1.5 \quad B_3 = 0.395m$

Conclusions and Perspectives

- A method for solving bosonic and fermionic BSE: NIR (LF singularities-fermions);
- Euclidean and Minkowski BSE for 3-bósons;
- Un-Wick rotation: BSE and SD - promissing tool allied to Integral Representations;
- Self-energies, vertex corrections, Landau gauge, ingredients from LQCD….
- Confinement – Generalized Stieljes transform and LF wave function (hint)?
- Beyond the pion, kaon, D, B, rho…, and the nucleon
- Form-Factors, PDFs, TMDs, Fragmentation Functions...

THANK YOU!

LIA/CNRS - SUBATOMIC PHYSICS: FROM THEORY TO APPLICATIONS

IPNO (Jaume Carbonell)…. + Brazilian Institutions …

Numerical method

$$g_b^{(Ld)}(\gamma, z; \kappa^2) = \sum_{\ell=0}^{N_z} \sum_{j=0}^{N_g} A_{\ell j} G_\ell(z) \mathcal{L}_j(\gamma)$$

$$G_\ell(z) = 4(1-z^2)\Gamma(5/2)\sqrt{\frac{(2\ell+5/2)(2\ell)!}{\pi\Gamma(2\ell+5)}} C_{2\ell}^{(5/2)}(z)$$

even Gegenbauer polynomials

$$\mathcal{L}_j(\gamma) = \sqrt{a}\, L_j(a\gamma) e^{-a\gamma/2}$$

Laguerre polynomials

Solution of the eigenvalue problem for g^2 for each given B

$B=2m-M$ binding energy