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The proton radius puzzle and nuclear structure corrections in light muonic atoms

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NTSE 2018, Deajeon, Korea
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Collaboration

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Vancouver, Canada

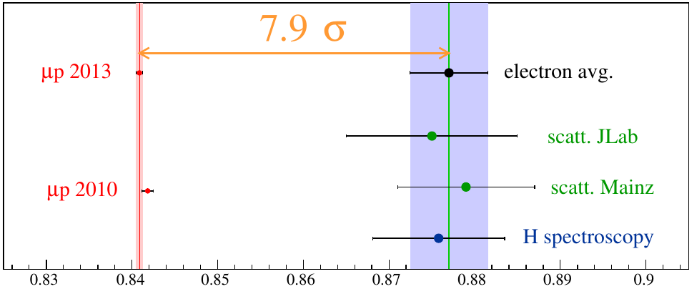
TRIUMF

Introduction

Proton charge measurements

- electron-proton interactions: 0.8770 ± 0.0045 fm
  - eH spectroscopy
  - $e - p$ scattering
- muon-proton interactions: 0.8409 ± 0.0004 fm
  - μH Lamb shift (2S-2P energy splittings) measurements at PSI (Switzerland)

    Pohl *et al.*, Nature (2010); Antognini *et al.*, Science (2013)



Worldwide interests in the proton radius puzzle

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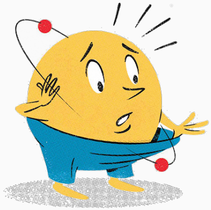
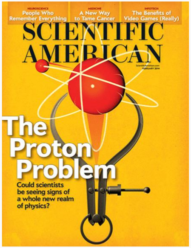
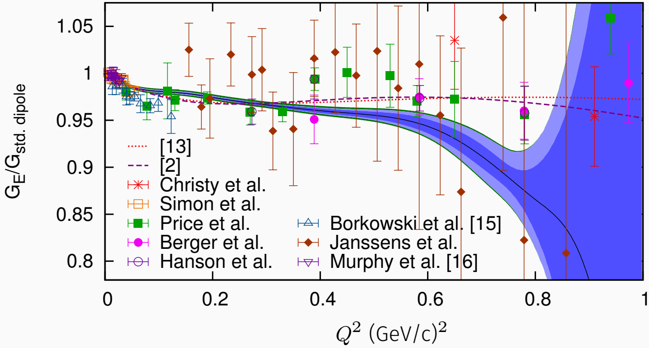


The New York Times Chris Gash



Errors in ep scattering experiment?

- $G_E^p(Q^2) = 1 - \frac{1}{6}\, \boldsymbol{r_p^2}\, Q^2 + \ldots$



- Q^2 not small enough / floating normalization

Errors in ep scattering experiment?

- dispersion analysis + chiral EFT constraints
  global fit to n & p EM form factors $r_p = 0.84(1)$ fm ($\chi^2 \approx 1.4$)

  Lorenz, Hammer, Meißner, EPJA '12

  Lorenz, Hammer, Meißner, Dong, PRD '15

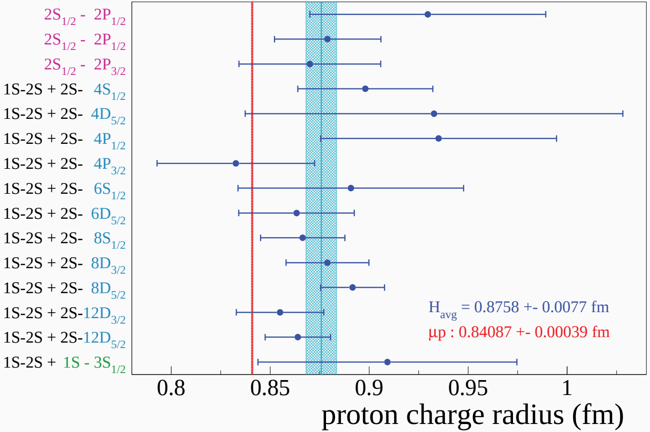
- deficiency in standard radiative correction model?
  requires further analysis

  Lee, Arrington, Hill, PRD '15

- Recent reanalysis suggests $r_p = 0.886(12)$ fm

  Sick, Arxiv 18

Underestimated uncertainties in eH spectroscopies?



Phol *et al.*, ARNPS '13

Origin of the discrepancy?

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- Exotic hadron structure?
  - 2γ subtraction term: Birse, McGovern, EPJA (2012) **vs** Miller, PLB (2013)
  - light sea fermions: Jentschura PRA (2013) **vs** Miller, PRC (2015)
  - new contact term from NRQED: Hill & Paz, PRD '10; PRL '11

- Beyond-standard-model physics?
  - new force carrier, e.g., dark photon: couples differently with e and μ
  - explain both the r_p puzzle & $(g-2)_\mu$ puzzle

Tucker-Smith, Yavin, PRD (2011)
Batell, McKeen, Pospelov, PRL (2011)

Carlson, Rislow, PRD (2012,2014)

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New experiments to shed light on the puzzle

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- Jefferson Lab
  - ep scattering for Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2
- A1 collaboration at Mainz Microtron
  - ed scattering for $Q^2 < 0.04$ GeV^2
- MUSE collaboration at PSI (2018)
  - measure $e^{\pm}p$ and $\mu^{\pm}p$ scattering:
    reduce systematic errors / potentially unveil new physics
- CREMA collaboration at PSI
  - Lamb shift (2S-2P) in μD (published)
  - Lamb shift in $\mu\,^4$He$^+$ (finished), $\mu\,^3$He$^+$ (finished), $\mu\,^3$H (planned?)

high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs

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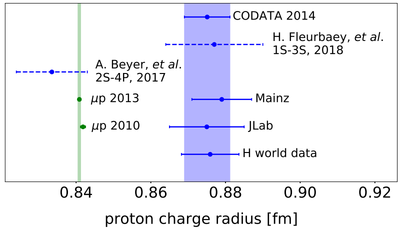
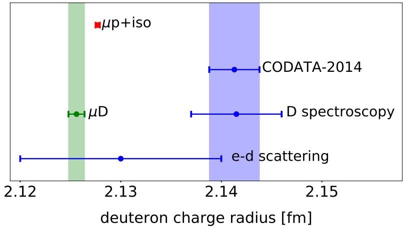
Proton



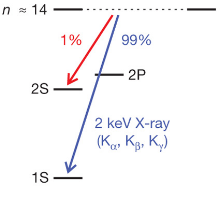
Deuteron



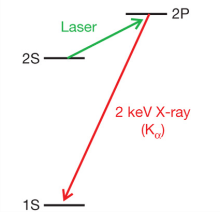
μA Lamb shift experiments

Lamb Shift: 2S-2P splitting in atomic spectrum   Pic: Pohl *et al.* Nature (2010)

- prompt X-ray ($t \sim 0$s): μ^- stopped in H_2 gases



- delayed X-ray ($t \sim 1\mu$s): laser induced 2S→2P



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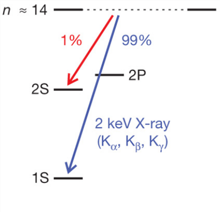

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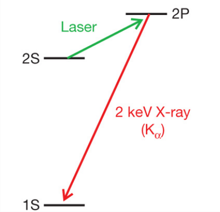

μH Lamb shift experiment

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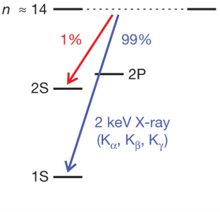

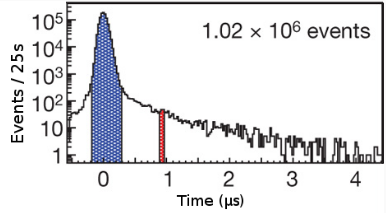

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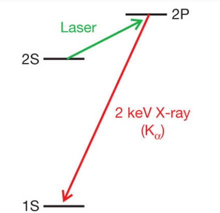

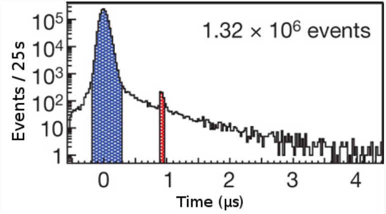

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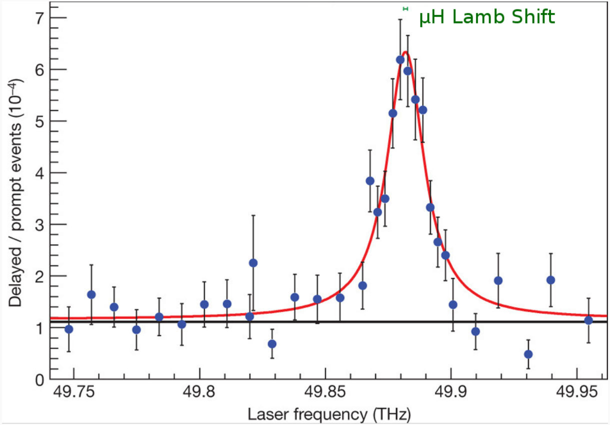
- measure $K_\alpha^{\text{delayed}} / K_\alpha^{\text{prompt}}$
- $\Delta E_{LS} = f_{res}$

Pic: Pohl *et al.* Nature (2010)



Charge radius from the Lambs shift

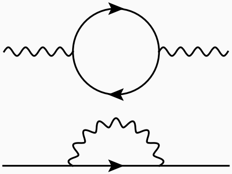
- Extract the nuclear charge radius $\langle r_c^2 \rangle$ from Lamb shift in light muonic atoms

$$\Delta E_{2S-2P} = \delta_{\text{QED}} + \mathcal{A}_{\text{OPE}} \langle r_c^2 \rangle + \delta_{\text{TPE}}$$

- Extract the nuclear charge radius $\langle r_c^2 \rangle$ from Lamb shift in light muonic atoms

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects



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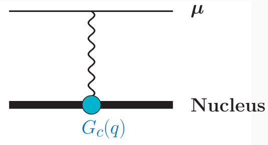
- Nuclear structure effects:
  - linear in $\langle r_c^2 \rangle \Longrightarrow$ structure effects in one photon exchange
  - $\mathcal{A}_{\text{OPE}} \approx m_r^3 (Z\alpha)^4 / 12$



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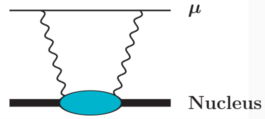
- Nuclear structure effects:
  - $\boldsymbol{\delta_{\rm TPE}} \Longrightarrow$ structure effects in two photon exchange
- elastic $\delta_{\rm TPE}$: Zemach moment $\langle r^3 \rangle_{(2)}$
- inelastic $\delta_{\rm TPE}$: nuclear polarizability δ_{pol}



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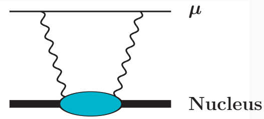

Accuracy in extracting $\langle r_c^2 \rangle$ relies on δ_{TPE} input (especially $\boldsymbol{\delta_{pol}}$)

μD - $\quad \Delta r_c = 0.1\ \% \quad \Longrightarrow \quad \Delta\delta_{TPE} = 3\ \%$

μ^4He - $\quad \Delta r_c = 0.1\ \% \quad \Longrightarrow \quad \Delta\delta_{TPE} = 10\ \%$

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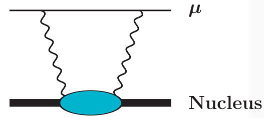

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μD -	$\Delta_{exp} = 0.034$ meV	$\Delta\delta_{TPE} = 0.05$ meV
μ^4He -	$\Delta_{exp} = 0.06$ meV	$\Delta\delta_{TPE} = 0.4$ meV

Extract the nuclear charge radius $\langle r_c^2 \rangle$

$$\boxed{\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE} \langle r_c^2 \rangle + \delta_{\rm TPE}}$$

The deuteron binding energy

$$\boxed{B_D = 2.2245666 \text{ MeV}}$$

The muonic deuterium μD binding energy

$$\boxed{B_{\mu D} = 2.227379825778 \text{ MeV}}$$

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point nucleus $B_\mu(1S)$ → 7379 8

Nuclear radius correction → 25778

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Nuclear radius correction

Two photon exchange

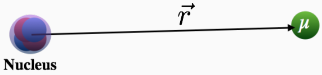
Two-photon exchange

Nuclear polarizability δ_{pol}

- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb

$$\Delta H = \alpha \sum_i^Z \Delta V(\boldsymbol{r}, \boldsymbol{R}_i) \equiv \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- δ_{pol}: ΔH's 2$^{\rm nd}$-order perturbative effects to the atomic spectrum

$$\delta_{\rm pol}^A = \langle N_0 \phi_0 | \Delta H \, G \, \Delta H | N_0 \phi_0 \rangle$$

  - inelastic part of 2γ exchange
  - nucleus excited in intermediate states

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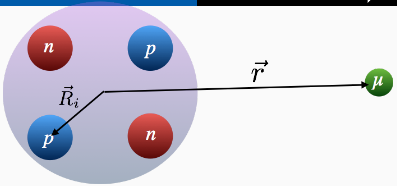

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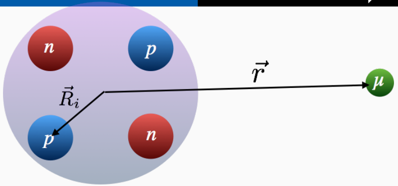

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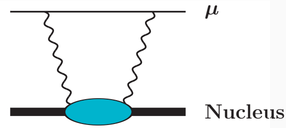

  - inelastic part of 2γ exchange
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- Introducing the transition density

$$\rho_N^p(\boldsymbol{R}) = \langle N| \frac{1}{Z} \sum_a^A \delta(\boldsymbol{R} - \boldsymbol{R}_a) \hat{e}_{p,a} |N_0\rangle$$

- We can write the inelastic TPE contribution as

$$\delta_{\rm pol}^A = \sum\!\!\!\!\!\!\!\int_{N \neq N_0} \int d\boldsymbol{R} d\boldsymbol{R}' \rho_N^{p*}(\boldsymbol{R}) W(\boldsymbol{R}, \boldsymbol{R}', \omega_N) \rho_N^p(\boldsymbol{R}')$$

- where W is the muon matrix element

$$W(\boldsymbol{R}, \boldsymbol{R}', \omega_N) = -Z^2 \langle\mu| \Delta V(\boldsymbol{r}, \boldsymbol{R}) \frac{1}{H_\mu + \omega_N - \epsilon_\mu} \Delta V(\boldsymbol{r}', \boldsymbol{R}') |\mu\rangle$$

Point proton non-relativistic expansion II

- Neglect coulomb effects
- Integrate over the muon
- Some algebra, and W takes the form

$$W \simeq \frac{m_r^3 (Z\alpha)^5}{12} \sqrt{\frac{2m_r}{\omega_N}} \times \left[|\boldsymbol{R} - \boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega_N}}{4} |\boldsymbol{R} - \boldsymbol{R}'|^3 + \frac{m_r\omega_N}{10} |\boldsymbol{R} - \boldsymbol{R}'|^4 \right]$$

- $|\boldsymbol{R} - \boldsymbol{R}'| \Longrightarrow$ "virtual" distance a proton travels in 2γ exchange
- uncertainty principal $|\boldsymbol{R} - \boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
- $\sqrt{2m_r\omega}|\boldsymbol{R} - \boldsymbol{R}'| \approx \sqrt{\frac{m_r}{m_N}} \approx 0.2$ for $^4He^+$
- LO + NLO + N^{2}LO $\Longrightarrow$ $\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$

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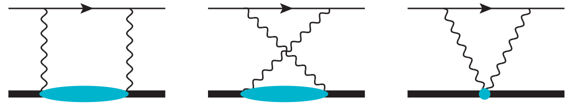
- Integrate over the muon
- Some algebra, and W takes the form

$$W \simeq \frac{m_r^3 (Z\alpha)^5}{12} \sqrt{\frac{2m_r}{\omega_N}} \times \Big[\underbrace{|\boldsymbol{R} - \boldsymbol{R}'|^2}_{\text{LO}} \underbrace{- \frac{\sqrt{2m_r\omega_N}}{4} |\boldsymbol{R} - \boldsymbol{R}'|^3}_{\text{NLO}} + \underbrace{\frac{m_r \omega_N}{10} |\boldsymbol{R} - \boldsymbol{R}'|^4}_{\text{N}^2\text{LO}} \Big]$$

- $|\boldsymbol{R} - \boldsymbol{R}'| \Longrightarrow$ “virtual” distance a proton travels in 2γ exchange
- uncertainty principal $|\boldsymbol{R} - \boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
- $\sqrt{2m_r\omega}|\boldsymbol{R} - \boldsymbol{R}'| \approx \sqrt{\frac{m_r}{m_N}} \approx 0.2$ for $^4\text{He}^+$
- LO + NLO + N^2LO $\Longrightarrow$ $\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$

$$\delta^{\rm R}_{\rm pol} = -8\alpha^2\phi^2(0)\int_0^\infty dq\,[\mathcal{R}_L+\mathcal{R}_T+\mathcal{R}_S]$$



$$\begin{aligned}
\mathcal{R}_L &= \int_0^\infty d\omega S_L(\omega,\boldsymbol{q})g(\omega,q)\\
\mathcal{R}_T &= \int_0^\infty d\omega S_T(\omega,\boldsymbol{q})\left[-\frac{1}{4m_r q}\frac{\omega+2q}{(\omega+q)^2}+\frac{q^2}{4m_r^2}g(\omega,q)\right]\\
\mathcal{R}_S &= \int_0^\infty d\omega S_T(\omega,0)\frac{1}{4m_r\omega}\left[\frac{1}{q}-\frac{1}{E_q}\right]
\end{aligned}$$

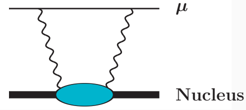
with $E_q = \sqrt{q^2+m_r^2}$ and

$$g(\omega,q) = \frac{1}{2E_q}\left[\frac{1}{(E_q-m_r)(E_q-m_r+\omega)}-\frac{1}{(E_q+m_r)(E_q+m_r+\omega)}\right]$$

Rosenfelder, NPA (1983)

The calculation of δ_{pol}

$$\delta_{pol} = \sum_{g,\, S_{\hat{O}}} \int_{\omega_{th}}^{\infty} d\omega \; \underbrace{g(\omega)}_{\text{weight func}} \; \underbrace{S_{\hat{O}}(\omega)}_{\text{response func}}$$



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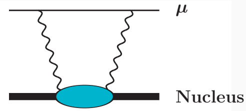
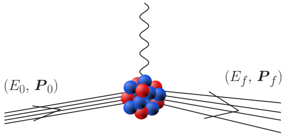


- energy-dependent weight $g\left(\omega\right)$
- nuclear response function $S_{\hat{O}}\left(\omega\right)$

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2 \delta(E_f - E_0 - \omega)$$



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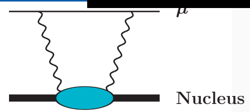

Contributions to δ_{pol} in muonic atoms

- non-relativistic terms (multipole expansion $\Longrightarrow$ equiv. to $\dfrac{m_\mu}{m_{nucl}}$ expansion)
  - leading order: sum rule of dipole response function $S_{D1}(\omega)$
  - higher order: sum rules of other response functions $S_{R2}(\omega)$, $S_{Q2}(\omega)$ & $S_{D1\,D3}(\omega)$
- relativistic corrections
  - treat the muon relativistically in the intermediate state
  - longitudinal & transverse contributions
- Coulomb corrections
  - consider Coulomb interactions in the intermediate state
- finite nucleon size corrections
  - consider charge distributions of p and n

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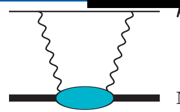

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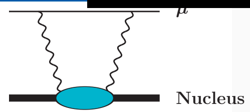

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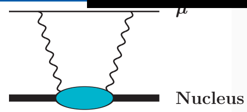

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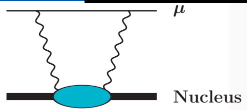

- The calculation of δ_{pol} needs information on the response function $S_{\hat{O}}(\omega)$

Previous/competing works on $S_{\hat{O}}(\omega)$ & δ_{pol}

- Simple potential models
  - $\mu\,^{12}$C (square-well) Rosenfelder '83
  - μD (Yamaguchi) Lu & Rosenfelder '93

- From experimental photoabsorption cross sections
  - $\mu\,^{4}$He: Bernabeu & Jarlskog '74; Rinker '76; Friar '77 20% uncertainty
  - μD: Carlson, Gorchtein, Vanderhagen '14 35% uncertainty
  - $\mu\,^{3}$He: Carlson, Gorchtein, Vanderhagen '17 3.5% uncertainty

- State-of-the-art potentials
  - μD: AV14 Leidemann & Rosenfelder, '95;
    AV18 Pachucki, '11,'15
    zero-range Friar, '13
    uncertainty $< 2\%$
  - The dipole response function $S_{D1}(\omega)$ was calculated for the light nuclei

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ab-initio calculations

Calculations of δ_{pol} in light nuclei

ab-initio calculation of nuclear polarizability effects in μD, μT, $\mu\,^3$He$^+$, $\mu\,^4$He$^+$

C. Ji, N. Nevo-Dinur, S. Bacca, N. Barnea, PRL **111**, 143402 (2013);
Few-Body Syst **55**, 917 (2014)
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PLB **755**, 380 (2016)

O. J. Hernandez, A. Ekstrm, et al. PLB **778**, 377 (2018)

- Few-body methods
  - Hyperspherical Harmonics expansion
  - Lorentz Integral Transform (response function)
  - Lanczos Algorithm (integral of response)
- Nuclear Hamiltonian
  - AV18+UIX
  - χEFT
  - At first, the difference used to estimate the NP uncertainty.
  - For the duetron we have done a rather comprehensive error analysis.
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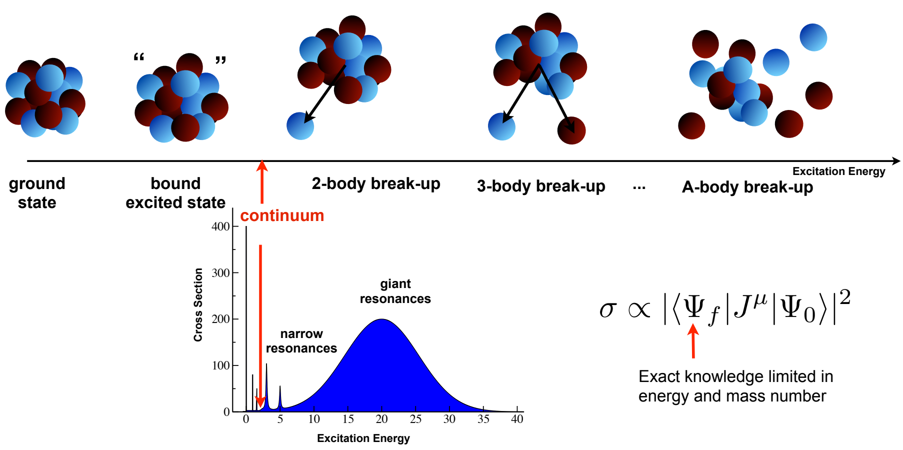
methods

- Response in continuum

$$S_J(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|J^\mu|\psi_0\rangle|^2 \delta(E_f - E_0 - \omega)$$



$$\sigma \propto \left|\langle \Psi_f | J^\mu | \Psi_0 \rangle\right|^2$$

Exact knowledge limited in energy and mass number

Lorentz integral transform method   continuum → bound-state

Ab-initio response functions

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- Response in continuum

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$

- Lorentz integral transform (LIT) method

Efros et al., '07

$$\mathcal{L}(\sigma, \Gamma) = \int d\omega \frac{S_O(\omega)}{(\sigma - \omega)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

$$(H - E_0 - \sigma + i\Gamma)|\tilde{\psi}\rangle = \hat{O}|\psi_0\rangle$$

  - Since r.h.s. is finite, $|\tilde{\psi}\rangle$ has bound-state asymptotic behavior

- Few-body methods for bound-state problems $\rightarrow$ **Hyperspherical Harmonics**
  - applicable for $3 \leqslant A \leqslant 7$
  - can accommodate local and non-local two-/three-nucleon forces

Ab-initio response functions

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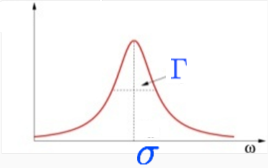

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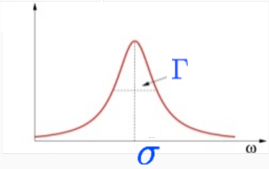

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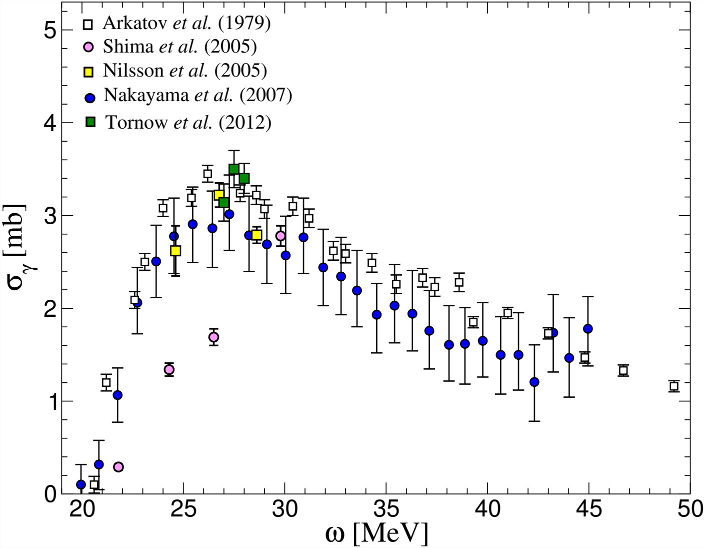
    AV18 + UIX & NN(N^{3}LO) + NNN(N^{2}LO)

^{4}He photoabsorption cross sections $\sigma_\gamma(\omega)$

$$\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega S_{D1}(\omega)$$



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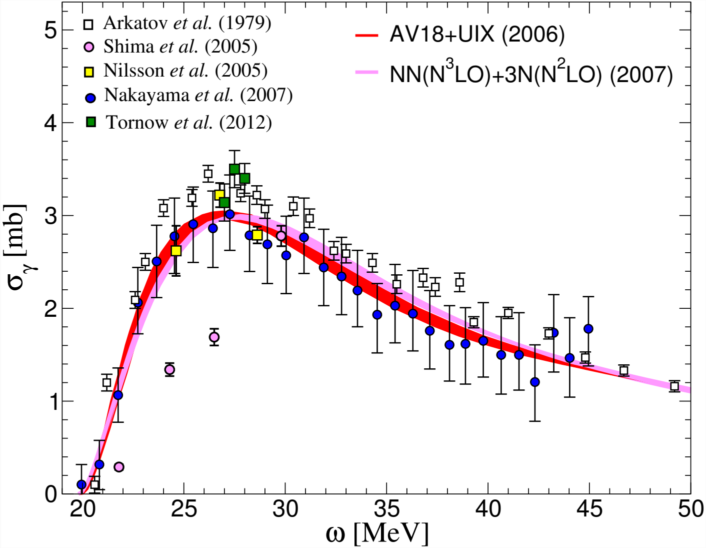

- Nuclear polarizability $\Longrightarrow$ energy-dependent sum rules of the response functions

$$I = \int_0^\infty d\omega \, S_O(\omega) \, g(\omega)$$

- We use Lanczos method to directly calculate I without explicitly solving $S_O(\omega)$.

$$I_M = \sum_{n \neq 0}^{M} |\langle N_m | \hat{O} | N_0 \rangle|^2 g(\omega_m) = \langle N_0 | \hat{O}^\dagger \hat{O} | N_0 \rangle \sum_{m \neq 0}^{M} |Q_{m0}|^2 g(\omega_m)$$

- I_M converges fast in the Lanczos method, if $g(\omega)$ is smooth.
- If $|\mathcal{L}(\sigma,\Gamma) - \mathcal{L}_M(\sigma,\Gamma)| \leq \varepsilon_M$, then

$$|I - I_M| \leq \varepsilon_M \int d\sigma \, |h(\sigma,\Gamma)|$$

$$g(\omega) = \frac{\Gamma}{\pi} \int d\sigma \frac{h(\sigma,\Gamma)}{(\omega-\sigma)^2 + \Gamma^2}$$

N. Nevo-Dinur, N. Barnea, C. Ji, S. Bacca, PRC **89**, 064317 (2014)

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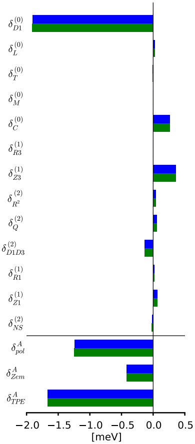
Results

Deuteron

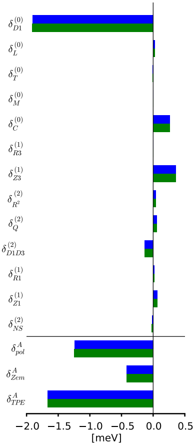


	AV18	χEFT
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.020	-0.021
δ_{pol}^{A}	-1.243	-1.248
δ_{Zem}^{A}	-0.421	-0.423
δ_{TPE}^{A}	-1.664	-1.671

Triton

TPE - Calculations results

Deuteron



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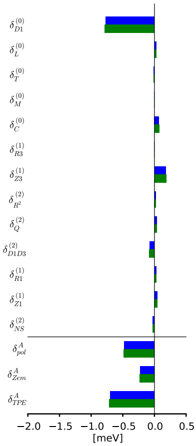
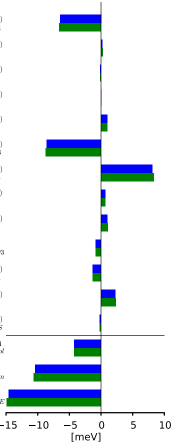


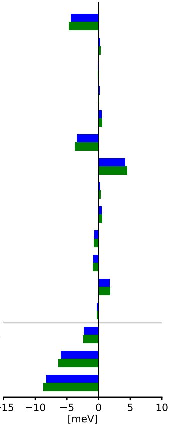
	AV18/UIX	χEFT
$\delta^{(0)}_{D1}$	-0.7669	-0.7848
$\delta^{(0)}_{L}$	0.0285	0.0296
$\delta^{(0)}_{T}$	-0.0128	-0.0132
$\delta^{(0)}_{M}$	0.0007	0.0007
$\delta^{(0)}_{C}$	0.0718	0.0732
$\delta^{(1)}_{R3}$	-	-
$\delta^{(1)}_{Z3}$	0.1778	0.1844
$\delta^{(2)}_{R^2}$	0.0199	0.0206
$\delta^{(2)}_{Q}$	0.0344	0.0358
$\delta^{(2)}_{D1D3}$	-0.0783	-0.0811
$\delta^{(1)}_{R1}$	0.0280	0.0287
$\delta^{(1)}_{Z1}$	0.0453	0.0463
$\delta^{(2)}_{NS}$	-0.0239	-0.0247
δ^{A}_{pol}	-0.4755	-0.4845
δ^{A}_{Zem}	-0.2231	-0.2307
δ^{A}_{TPE}	-0.6986	-0.7152

^{3}He



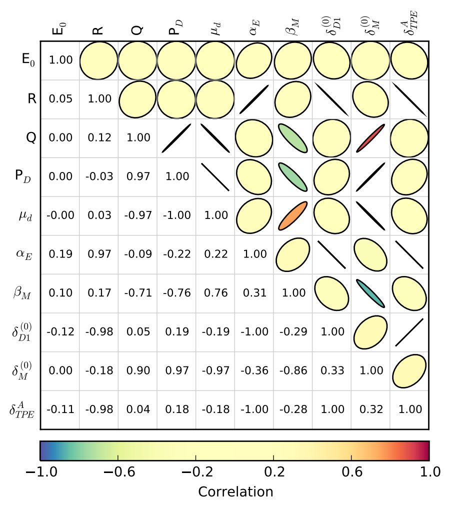
	AV18/UIX	χEFT
$\delta^{(0)}_{D1}$	-6.479	-6.633
$\delta^{(0)}_{L}$	0.232	0.240
$\delta^{(0)}_{T}$	-0.103	-0.107
$\delta^{(0)}_{M}$	0.006	0.006
$\delta^{(0)}_{C}$	1.000	1.020
$\delta^{(1)}_{R3}$	-8.539	-8.711
$\delta^{(1)}_{Z3}$	8.100	8.327
$\delta^{(2)}_{R^2}$	0.632	0.654
$\delta^{(2)}_{Q}$	1.015	1.038
$\delta^{(2)}_{D1D3}$	-0.841	-0.862
$\delta^{(1)}_{R1}$	-1.294	-1.314
$\delta^{(1)}_{Z1}$	2.256	2.291
$\delta^{(2)}_{NS}$	-0.193	-0.199
δ^{A}_{pol}	-4.208	-4.250
δ^{A}_{Zem}	-10.356	-10.618
δ^{A}_{TPE}	-14.564	-14.868

^{4}He

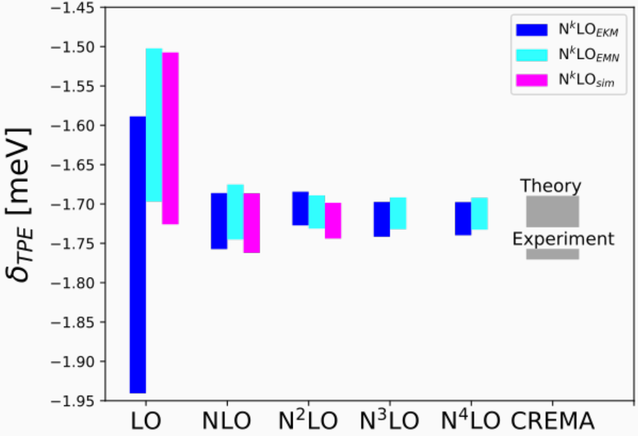


	AV18/UIX	χ^{EFT}
$\delta^{(0)}_{D1}$	-4.418	-4.701
$\delta^{(0)}_{L}$	0.289	0.308
$\delta^{(0)}_{T}$	-0.126	-0.134
$\delta^{(0)}_{M}$	0.157	0.083
$\delta^{(0)}_{C}$	0.512	0.546
$\delta^{(1)}_{R3}$	-3.442	-3.717
$\delta^{(1)}_{Z3}$	4.183	4.526
$\delta^{(2)}_{R^2}$	0.259	0.324
$\delta^{(2)}_{Q}$	0.484	0.561
$\delta^{(2)}_{D1D3}$	-0.666	-0.784
$\delta^{(1)}_{R1}$	-0.848	-0.874
$\delta^{(1)}_{Z1}$	1.753	1.812
$\delta^{(2)}_{NS}$	-0.279	-0.293
δ^{A}_{pol}	-2.299	-2.426
δ^{A}_{Zem}	-5.936	-6.338
δ^{A}_{TPE}	-8.235	-8.764

The μ-D Lamb shift correlation matrix Calculated for N^{2}LO χEFT potential



The evolution of the μ-D Lamb shift with χEFT order



^{4}He		AV18/UIX	χEFT	Difference
binding energy	B_0 [MeV]	28.422	28.343	0.28%
rms radius	R_{rms} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
μ ^{4}He$^+$ nuclear polarizablity	δ_{pol} [meV]	-2.408	-2.542	5.5%

- The uncertainty from nuclear physics: $\left(5.5\%/\sqrt{2}\right)$

  $\delta_{pol} = -2.47$ meV $\pm 4\%(1\sigma)$

Error estimate - $A \geq 3$

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TPE in muonic heliom

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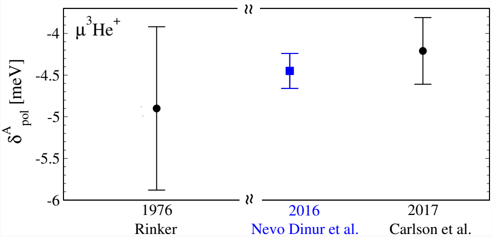
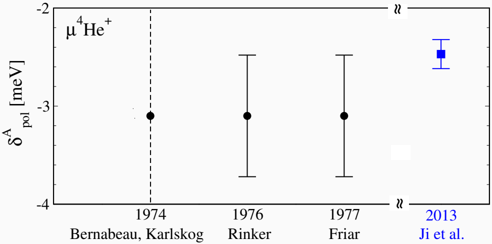
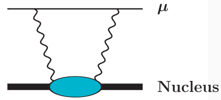




δ_{TPE} in light muonic atoms

$$\delta_{\mathrm{TPE}} = \delta^A_{\mathrm{Zem}} + \delta^A_{\mathrm{pol}} + \delta^N_{\mathrm{Zem}} + \delta^N_{\mathrm{pol}}$$



	δ^A_{Zem}	δ^A_{pol}	δ^N_{Zem}	δ^N_{pol}	δ_{TPE}
$\mu^2\mathrm{H}$	−0.424(03)	−1.245(19)	−0.030(02)	−0.020(10)	−1.727(20)
$\mu^3\mathrm{H}$	−0.227(06)	−0.473(17)	−0.033(02)	−0.031(17)	−0.767(25)
$\mu^3\mathrm{He}^+$	−10.49(24)	−4.17(17)	−0.52(03)	−0.25(13)	−15.46(39)
$\mu^4\mathrm{He}^+$	−6.29(28)	−2.36(14)	−0.54(03)	−0.34(20)	−9.58(38)

Summary

Summary (muonic atoms)

האוניברסיטה העברית בירושלים
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- Lamb shifts in muonic atoms
  - raise interesting questions about lepton symmetry
  - connect nuclear, atomic and particle physics
- ab-initio calculations are key to analyze Lamb shift experiments
- obtaining a few percent accuracy in δ_{TPE}
- Nuclear physics predictions with a sub meV accuracy :)
- more accurate than estimates using experimental data
- light nuclei with $A \leq 4$ are done, what about $A > 4$?

Outlook

האוניברסיטה העברית בירושלים
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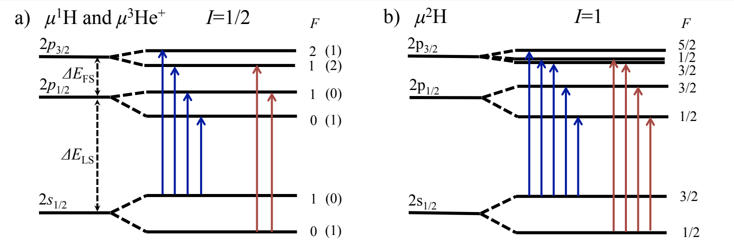
hyperfine splittings (HFS) in muonic atoms $\Longrightarrow$ nuclear magnetic radii



study 2γ exchange contributions to HFS

- magnetic polarizability plays an significant role
- meson-exchange current is important

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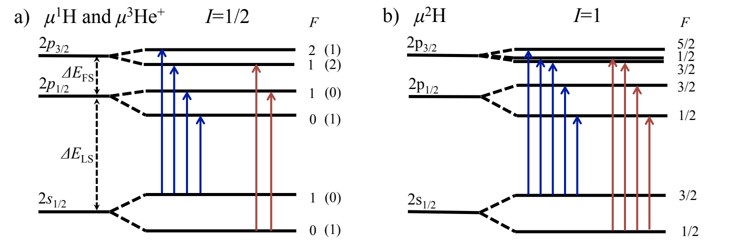

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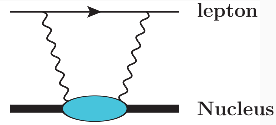
תודה
Dankie
Gracias
Спасибо
شكراً
Merci
Takk
Köszönjük
Terima kasih
Grazie
Dziękujemy
Dėkojame
Ďakujeme
Vielen Dank
Paldies
Kiitos
Täname teid
谢谢
Thank You
Tak
感謝您
Obrigado
Teşekkür Ederiz
감사합니다
Σας Ευχαριστούμ
ขอบคุณ
Bedankt
Děkujeme vám
ありがとうございます
Tack

backup

BACK UP

Non-relativistic limit

- Neglect Coulomb interactions in the intermediate state
- Expand muon matrix element based an expansion in $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

- $|\boldsymbol{R}-\boldsymbol{R}'| \Longrightarrow$ "virtual" distance a proton travels in 2γ exchange
- uncertainty principal $|\boldsymbol{R}-\boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
- $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} \approx 0.17$ for μ^4He^+

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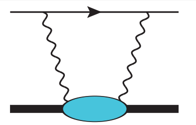

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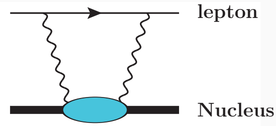

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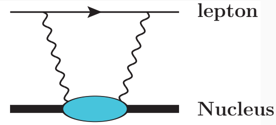

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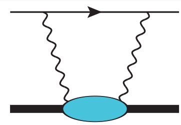

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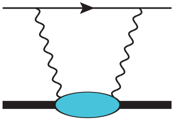

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c.f. Pachucki '11 & Friar '13 (μ D)

- $\delta_{NR}^{(2)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$
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  - $S_{R^2}(\omega)$ (monopole) & $S_Q(\omega)$ (quadrupole)
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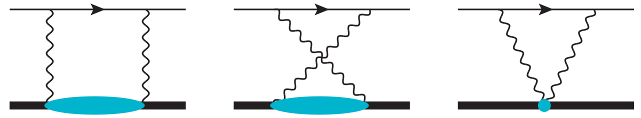
Relativistic dipole polarizability

We use the formalism of forward Compton scattering



- Longitudinal contributions $\delta_L^{(0)}$
  - exchange Coulomb photon
- Transverse contributions $\delta_T^{(0)}$
  - convection current & spin current
  - seagull term: cancels infrared divergence restore gauge invariance
- $\delta_{L(T)}^{(0)}$ are sum rule of dipole response with different weights

$$\delta_{L(T)}^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}(\frac{\omega}{m_r})\, S_{D_1}(\omega)$$

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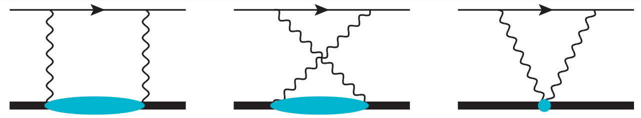

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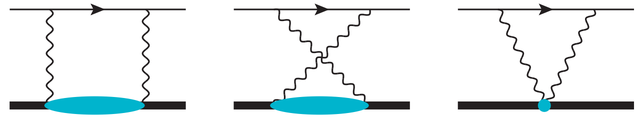

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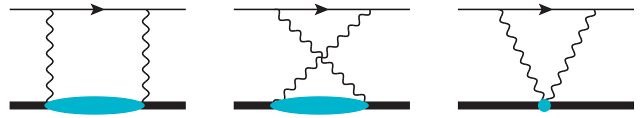

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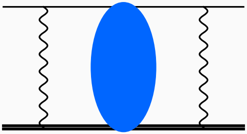
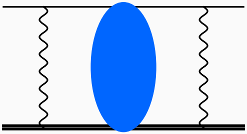


- Non-perturbative Coulomb interaction in intermediate state
  - naive estimation: $\delta_C^{(0)} \sim (Z\alpha)^6$
  - full analysis: logarithmically enhanced $\delta_C^{(0)} \sim (Z\alpha)^6 \ln(Z\alpha)$

Friar '77 & Pachucki '11



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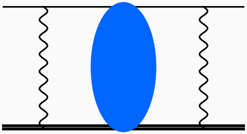
Coulomb distortion



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Friar '77 & Pachucki '11

Finite nucleon size corrections

- In point-nucleon limit

$$\Delta H = -\alpha \sum_{i}^{Z} \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} + \frac{Z\alpha}{r}$$

- consider finite nucleon sizes $\Longrightarrow$ include nucleon charge density

$$\Delta H = -\alpha \sum_{i}^{Z} \int d\boldsymbol{R}' \, \frac{n_p(\boldsymbol{R}' - \boldsymbol{R}_i)}{|\boldsymbol{r} - \boldsymbol{R}'|} - \alpha \sum_{j}^{N} \int d\boldsymbol{R}' \, \frac{n_n(\boldsymbol{R}' - \boldsymbol{R}_j)}{|\boldsymbol{r} - \boldsymbol{R}'|} + \frac{Z\alpha}{r}$$

- low-Q approximations of nucleon form factors

$$G_p^E(q) \simeq 1 - \frac{\langle r_p^2 \rangle}{6} q^2$$

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Finite nucleon size corrections

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$$\Delta H = -\alpha \sum_{i}^{Z} \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} + \frac{Z\alpha}{r}$$

- consider finite nucleon sizes $\Longrightarrow$ include nucleon charge density

$$\Delta H = -\alpha \sum_{i}^{Z} \int d\boldsymbol{R}' \, \frac{n_p(\boldsymbol{R}' - \boldsymbol{R}_i)}{|\boldsymbol{r} - \boldsymbol{R}'|} - \alpha \sum_{j}^{N} \int d\boldsymbol{R}' \, \frac{n_n(\boldsymbol{R}' - \boldsymbol{R}_j)}{|\boldsymbol{r} - \boldsymbol{R}'|} + \frac{Z\alpha}{r}$$

- low-Q approximations of nucleon form factors

$$G_p^E(q) \simeq 1 - \frac{\langle r_p^2 \rangle}{6} q^2$$

$$G_n^E(q) \simeq -\frac{\langle r_n^2 \rangle}{6} q^2$$

results

nuclear polarizability in $\mu\,^4\mathrm{He}^+$

[meV]		AV18/UIX	χEFT★
$\delta^{(0)}$	$\delta^{(0)}_{D1}$	-4.418	-4.701
	$\delta^{(0)}_{L}$	0.289	0.308
	$\delta^{(0)}_{T}$	-0.126	-0.134
	$\delta^{(0)}_{C}$	0.512	0.546
$\delta^{(1)}$	$\delta^{(1)}_{R3pp}$	-3.442	-3.717
	$\delta^{(1)}_{Z3}$	4.183	4.526
$\delta^{(2)}$	$\delta^{(2)}_{R2}$	0.259	0.324
	$\delta^{(2)}_{Q}$	0.484	0.561
	$\delta^{(2)}_{D1D3}$	-0.666	-0.784
δ_{NS}	$\delta^{(1)}_{R1pp}$	-1.036	-1.071
	$\delta^{(1)}_{Z1}$	1.753	1.811
	$\delta^{(2)}_{NS}$	-0.200	-0.210
δ_{pol}		-2.408	-2.542

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$ in a systematic expansion of $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{m_r/M_N} \approx 0.17$
- δ_{pol} with AV18/UIX & χEFT differ: $\sim$ 5.5% (0.134 meV)

★ NN(N^{3}LO)/$3N$(N^{2}LO)

c_D=1, c_E=-0.029

Navrátil, Few-Body Syst 2007

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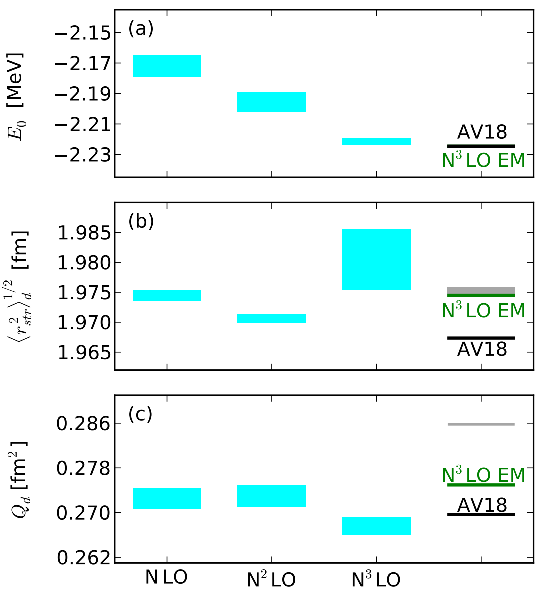
Deuteron ground-state observables

Hernandez, C. Ji, N. Nevo-Dinur, S. Bacca, N. Barnea, PLB **736**, 344 (2014)



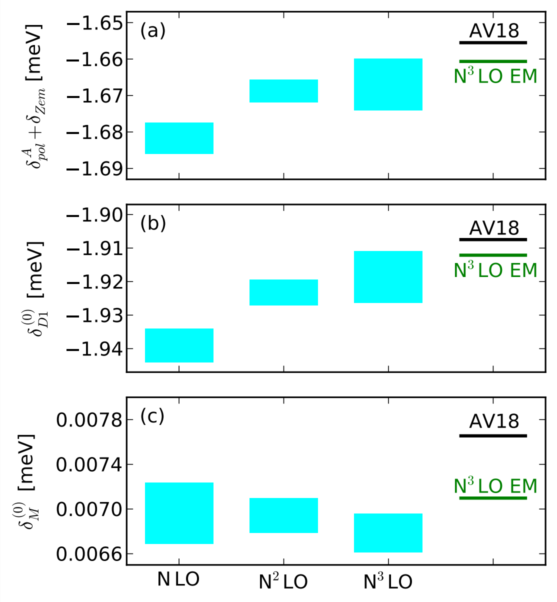
with relativistic corrections & meson-exchange currents

		$\langle r_{str}^2 \rangle_d^{1/2}$ [fm]	Q_d [fm^2]
N^3LO-EM	Our work	1.974	0.2750
	+RC+MEC	1.978[1]	0.285[1]
AV18	Our work	1.967	0.2697
	+RC+MEC	—	0.275[2]
Experiment		1.97507(78)	0.285783(30)

[1]Entem, Machleidt '03
[2]Wiringa, Stoks, Schiavilla '95

Nuclear structure effects in μD

O.J. Hernandez, C. Ji, N. Nevo-Dinur, S. Bacca, N. Barnea, PLB **736**, 344 (2014)



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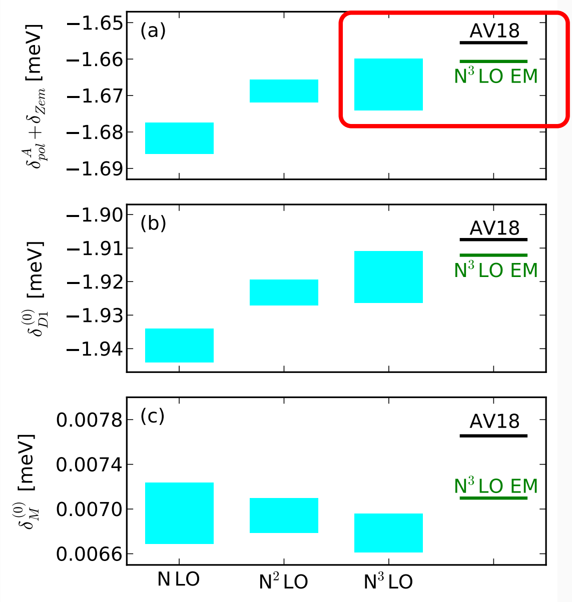

Combine all potentials

$-1.669 \pm 0.007(1\sigma)$ meV

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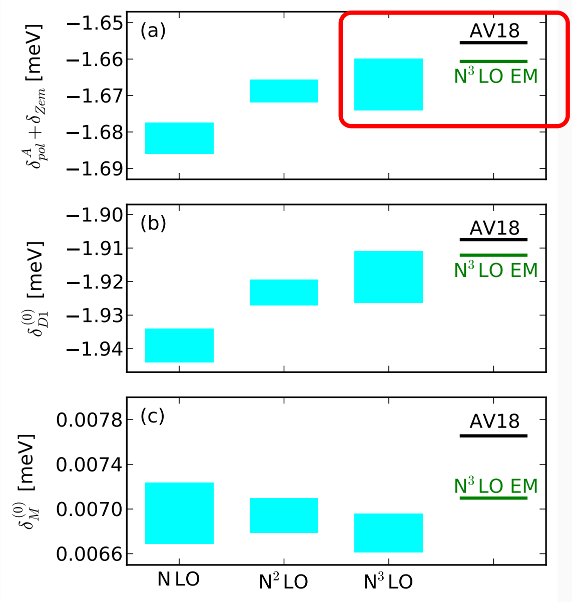

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Other uncertainties

N^2LO$\rightarrow N^3$LO convergence: 0.3%

atomic physics: 1%

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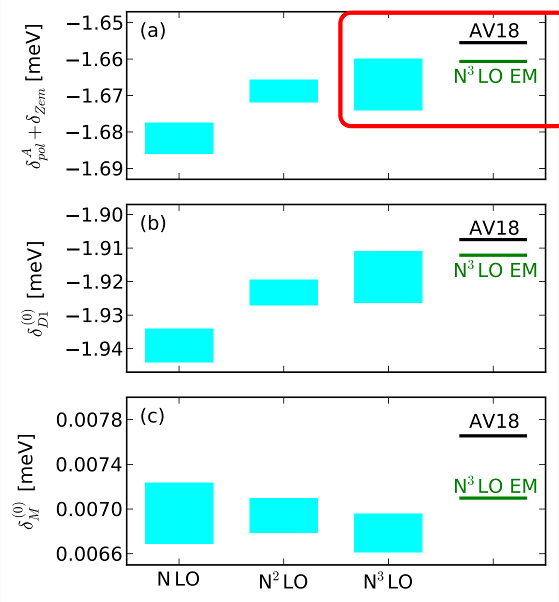

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Other uncertainties

N^{2}LO→N^{3}LO convergence: 0.3%

atomic physics: 1%

Errors in quadrature sum

-1.669 ± 0.018 meV

Muonic ^{3}He and ^{3}H

- Lamb shifts in μ^3He$^+$ will be also measured at PSI (possibility in μ^3H)
- unequal proton and neutron numbers

- We calculate nuclear structure effects in μ^3He$^+$ and μ^3H

N. Nevo-Dinur, C.Ji, S. Bacca, N. Barnea (PLB - accepted for publication)

	μ^3He$^+$		μ^3H	
	AV18/UIX	χEFT	AV18/UIX	χEFT
$\delta^A_{\rm pol}$ [meV]	-4.114(17)	-4.201(8)	-0.4688(6)	-0.4834(4)
$\delta_{\rm Zem}$ [meV]	-10.511(10)	-10.770(3)	-0.2263(1)	-0.2337(1)