北京大学物理学院
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CGSM and MBPT calculations with realistic nuclear forces

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Outline

I. Core Gamow Shell Model (CGSM) with CD Bonn

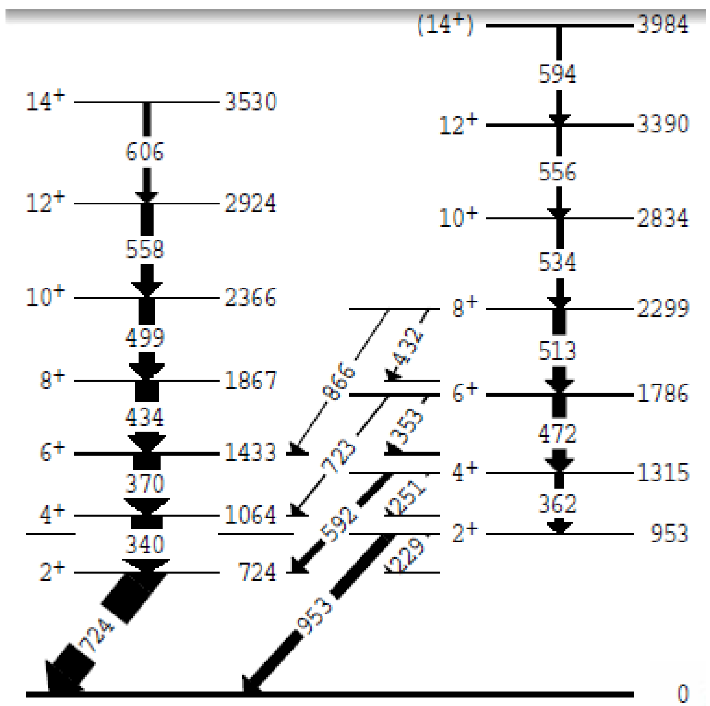
(resonance + continuum)

II. *Ab-initio* MBPT with N^3LO (LQCD)

NTSE–2016, Khabarovsk, 19-23 September, 2016, Khabarovsk, Russian

γ-ray spectra

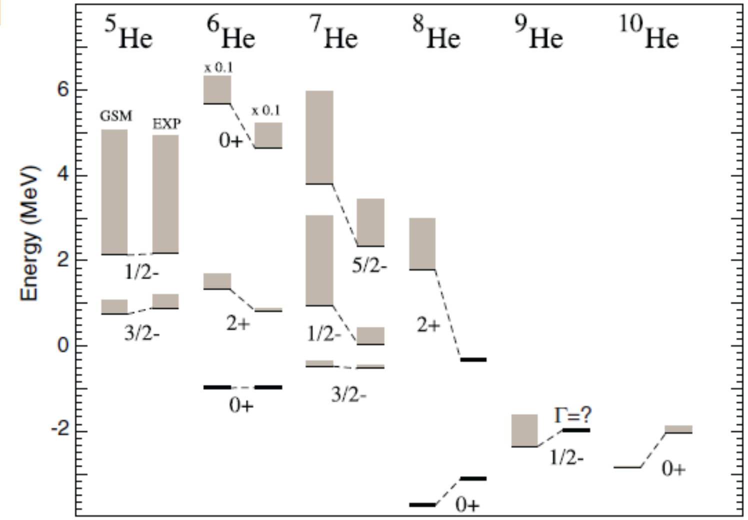
^{188}Pb: prolate and oblate bands

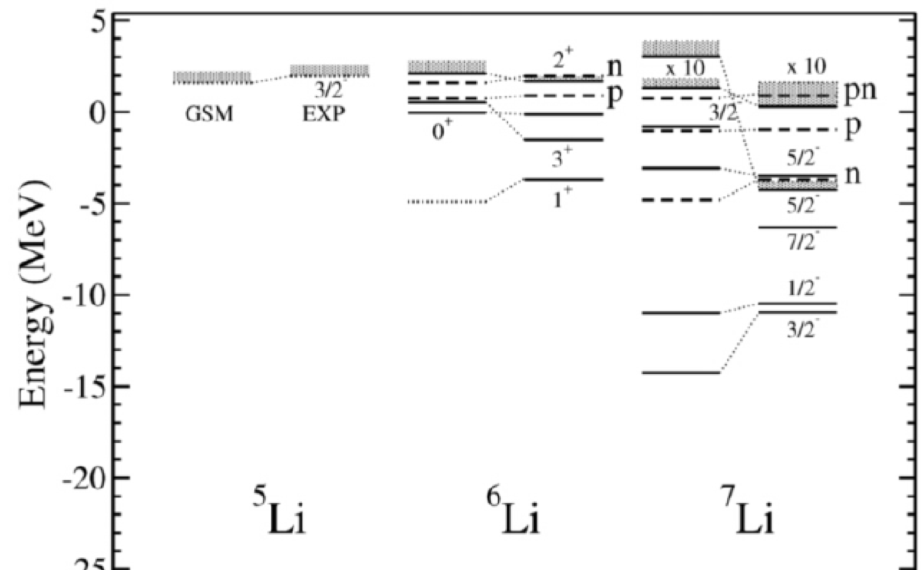


J. Pakarinen et al., PRC 72, 011304(R) (2005)

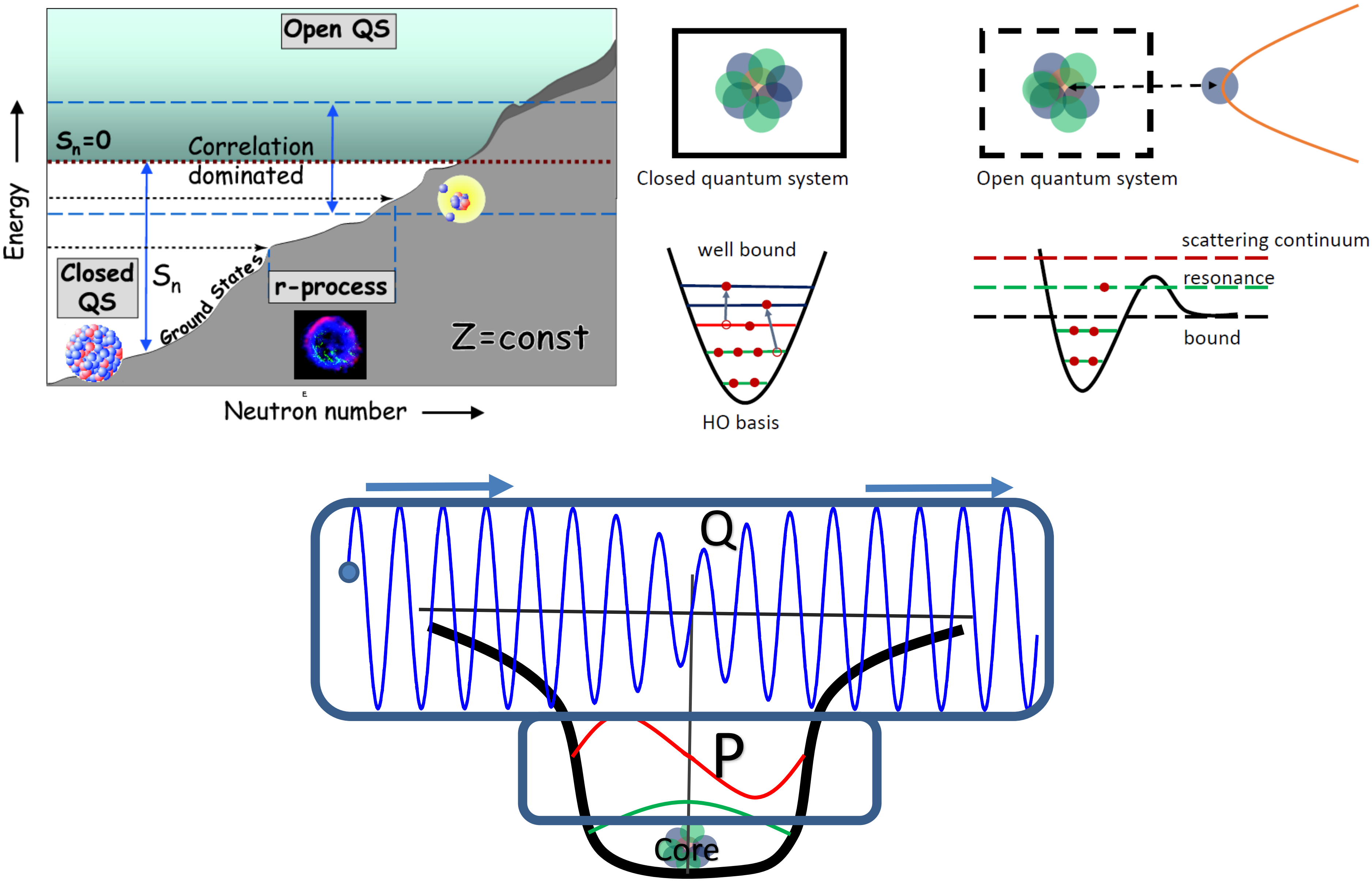
Spectra of resonance states

Energies and lifetimes or widths (against particle emissions)





N. Michel, W. Nazarewicz, J. Okolowicz, M. Ploszjczak, Nucl. Phys. A 752, 335c (2005)



Open QS
S_n=0
Correlation
dominated
Closed
QS
S_n
Ground States
r-process
Z=const
Energy
Neutron number
Closed quantum system
Open quantum system
well bound
HO basis
scattering continuum
resonance
bound
Q
P
Core

T. Berggren, Nucl. Phys. A109 (1968) 265

Single-particle Berggren basis in complex-*k* plane, describing bound, resonance and scattering on equal footing.

The radial wave function $u(r)/r$

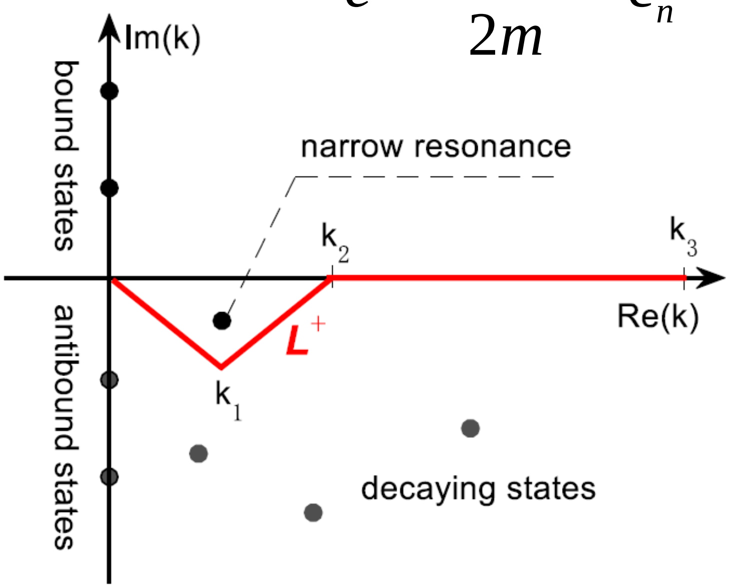
$$\frac{d^2u(k,r)}{dr^2} = \left(\frac{\ell(\ell+1)}{r^2} + \frac{2m}{\hbar^2}V(r) - k^2\right)u(k,r)$$

boundary conditions

$$u(0) = 0,$$

$$u(a)O_l'(ka) - u'(a)O_l(ka) = 0$$

$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$



$$O_l(kr) \sim e^{i(kr - l\pi/2)}$$

Outgoing solution at large distance

Orthogonality and Completeness

$$\delta(r - r') = \sum_n w_n(r, k_n) w_n(r', k_n) + \frac{1}{\pi}\int_{L^+} dq\, u(r,q) u(r',q)$$

Discretized

R.J. Liotta *et al*., PLB 367, 1 (1996)…

Used Berggren basis to describe single-particle resonance in nuclei;

later for two-particle resonance (Betan *et al*., PRL 89, 042601 (2002)

Using phenomenological potential

For many-body systems

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} v_{ij}^{NN} - \frac{\boldsymbol{P}^2}{2Am}$$

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + U + \sum_{i<j=1} \left(v_{ij}^{NN} - U - \frac{p_i^2}{2Am} - \frac{\boldsymbol{p}_i \boldsymbol{p}_j}{Am} \right)$$

$$= H_0 + V.$$

$$E = E_n - i \frac{\Gamma_n}{2}$$

U is the Woods-Saxon potential

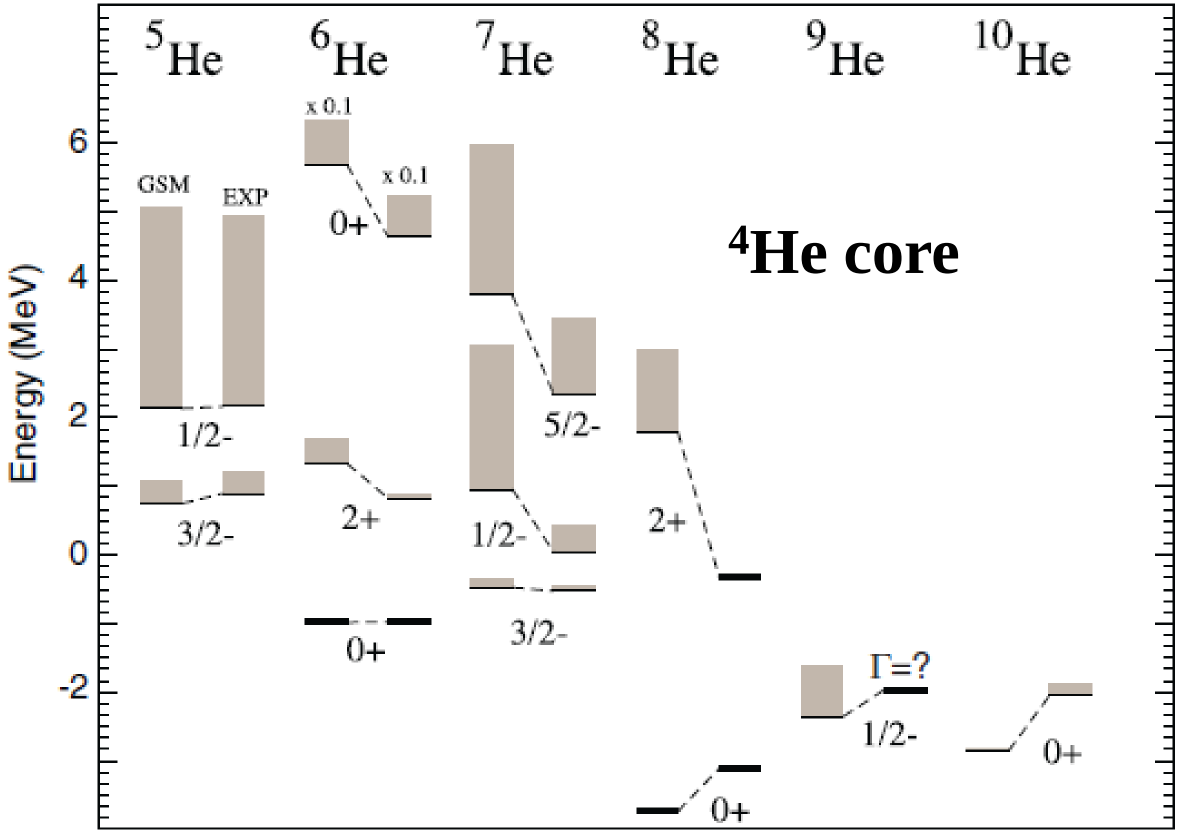
Michel, Nazarewicz, Ploszajczak, Rotureau *et al.*, 2003--

$$V = V_{WS} + V_{J,T}(\vec{r}_1, \vec{r}_2)$$

phenomenological potential

$$V(\mathbf{r}_i, \mathbf{r}_j) = -V_{\mathrm{SGI}}^{(J,T)} \exp\left[-\left(\frac{\mathbf{r}_i - \mathbf{r}_j}{\mu}\right)^2\right] \delta(r_i + r_j - 2R_0)$$

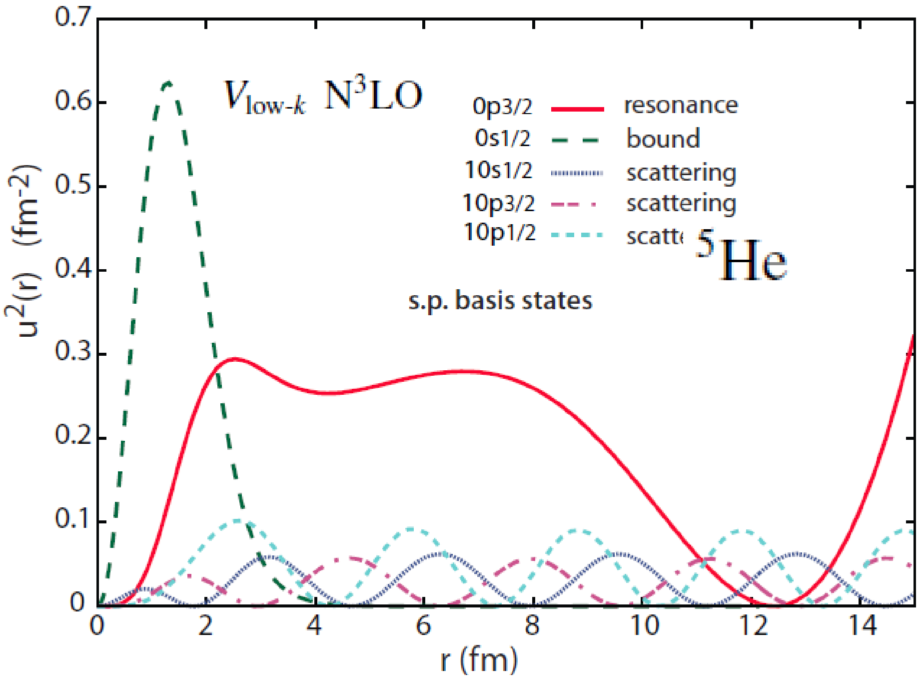
$V_{\mathrm{SGI}}^{(J)}$ is the strength in the JT channel

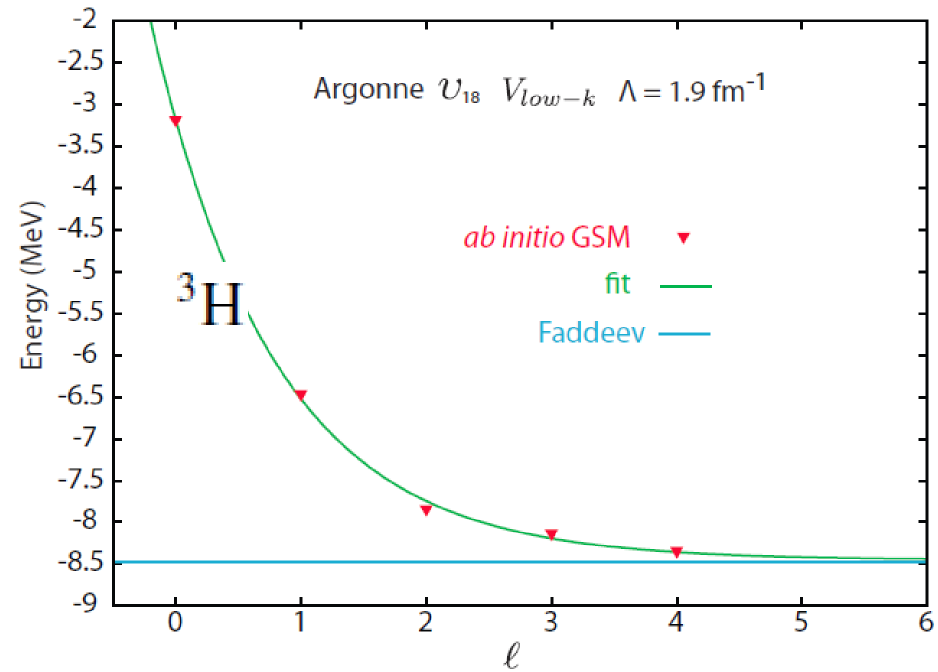


Michel, Nazarewicz, Płoszajczak, Vertse, Phys. G: Nucl. Part. Phys. 36 (2009) 013101

Hagen, Hjorth-Jensen *et al.*, PRC 73, 064307 (2006): Core GSM with realistic nuclear forces, but no Q-box, limited to 2 valence particles; later Tsukiyama Hjorth-Jensen, Hagen, PRC 80, 051301 (R) (2009) improved by using Q-box but not folded diagrams, limited to 2 or 3 valence particles

Papadimitriou *et al.* , Phys. Rev. C 88, 044318 (2013): realistic nuclear forces
***Ab initio* no-core Gamow shell model calculations for light nuclei**





Our Gamow shell model with an inert core

1. **To start from realistic nuclear forces;**
2. **No limitation on the number of valence particles;**

   Q-box + folded diagrams

3. **To calculate level energies + resonance widths of states.**

Realistic CGSM with CD-Bonn

Realistic nuclear forces ⇨ Gamow shell model calculations

Taking a doubly closed core

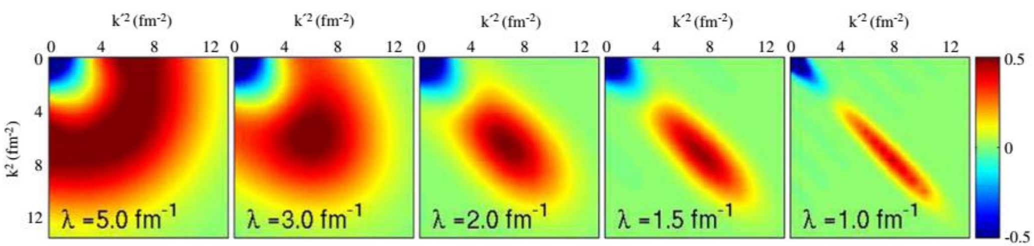
Bare forces: Strong repulsion, slow convergence

$V_{low\ k}$ or SRG

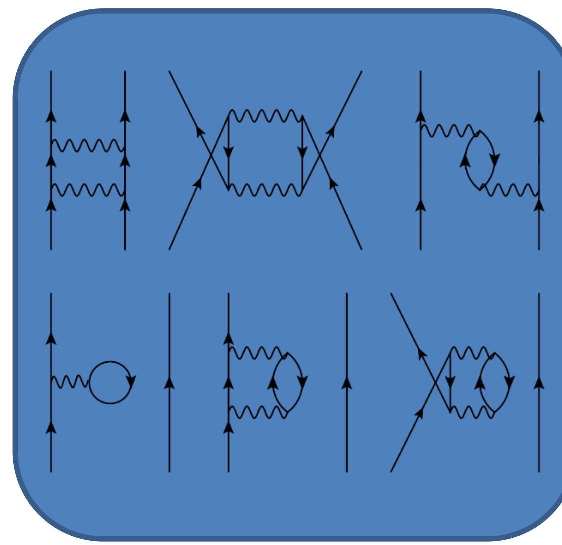
To remove hard core, but still keep good descriptions of NN scattering phase shifts

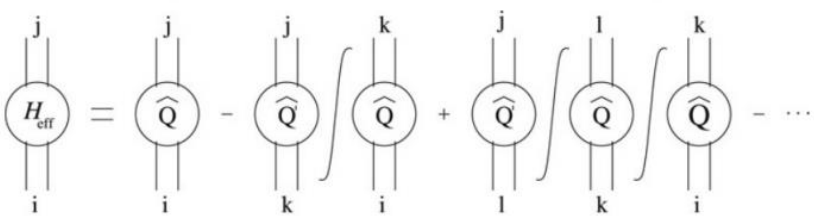
$$\langle \alpha_P | \tilde{H}_{\text{eff}} | \alpha_{P'} \rangle = \sum_{\alpha_{P''}} \sum_{\alpha_{P'''}} \sum_{kk'k'' \in \mathcal{K}} \langle \alpha_P | \tilde{k}'' \rangle \langle \tilde{k}'' | \alpha_{P''} \rangle \langle \alpha_{P''} | \tilde{k} \rangle E_k \langle \tilde{k} | \alpha_{P'''} \rangle \langle \alpha_{P'''} | \tilde{k}' \rangle \langle \tilde{k}' | \alpha_{P'} \rangle$$

$$\frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T_{\text{rel}}, H_\lambda], H_\lambda]$$



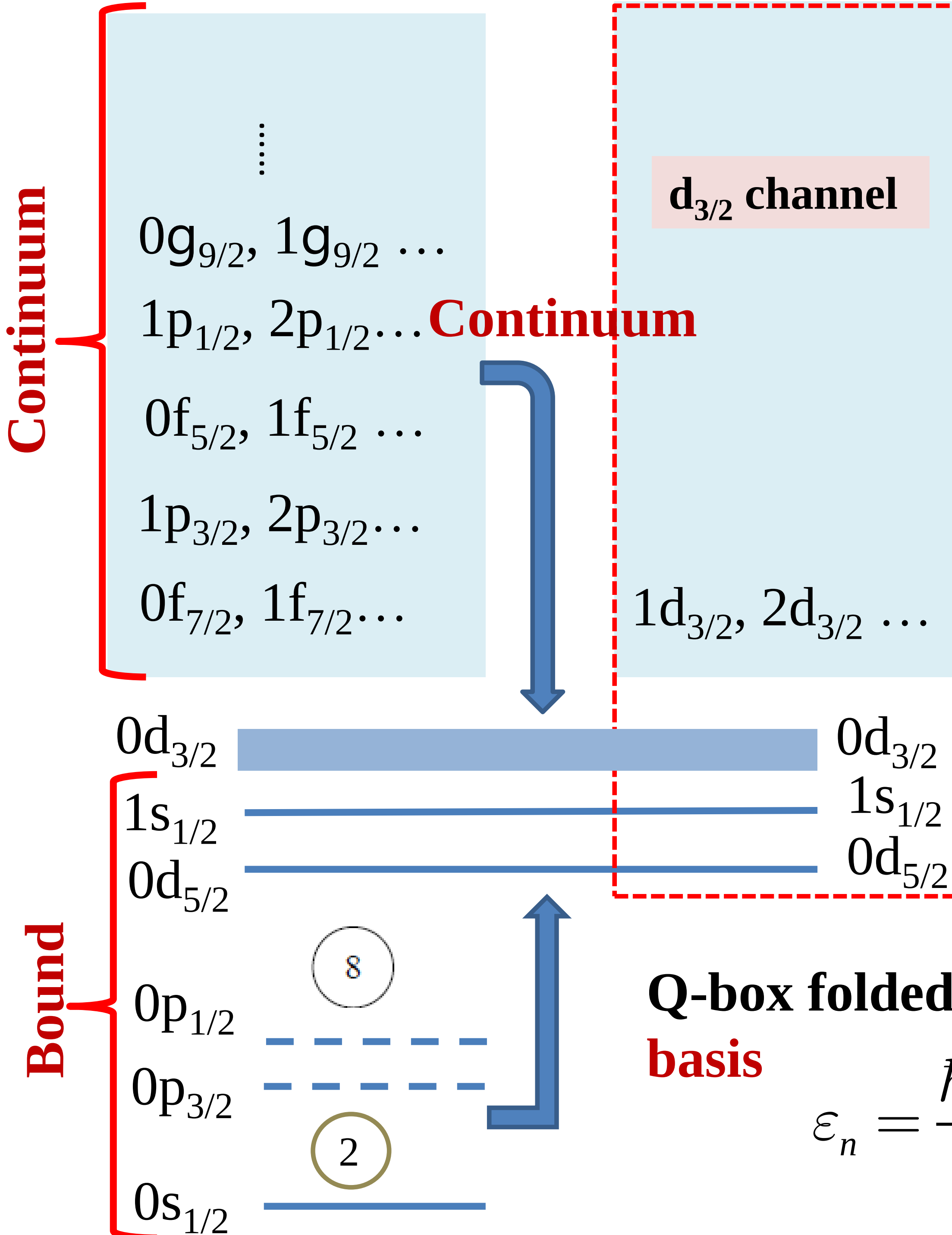
Q





$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP,$$

Nondegenerated extended Kuo-Krenciglowa folded-diagram method (EKK) by Takayanagi, NPA 852, 61 (2011)



Model space

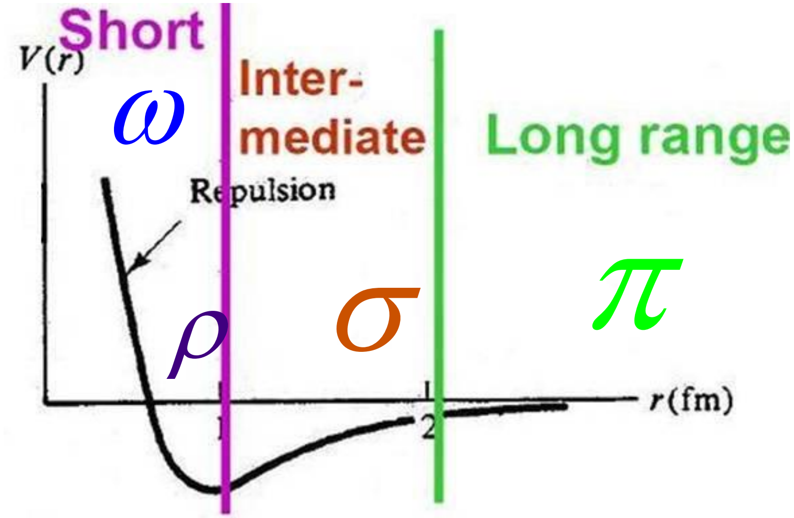
For a given partial wave (a given channel)

Berggren Completeness Relation

$$\sum_n |u_n\rangle\langle\widetilde{u_n}| + \int_{L_+} |u_k\rangle\langle\widetilde{u_k}|dk = 1$$

Q-box folded diagram in complex-k basis

$$\varepsilon_n = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$



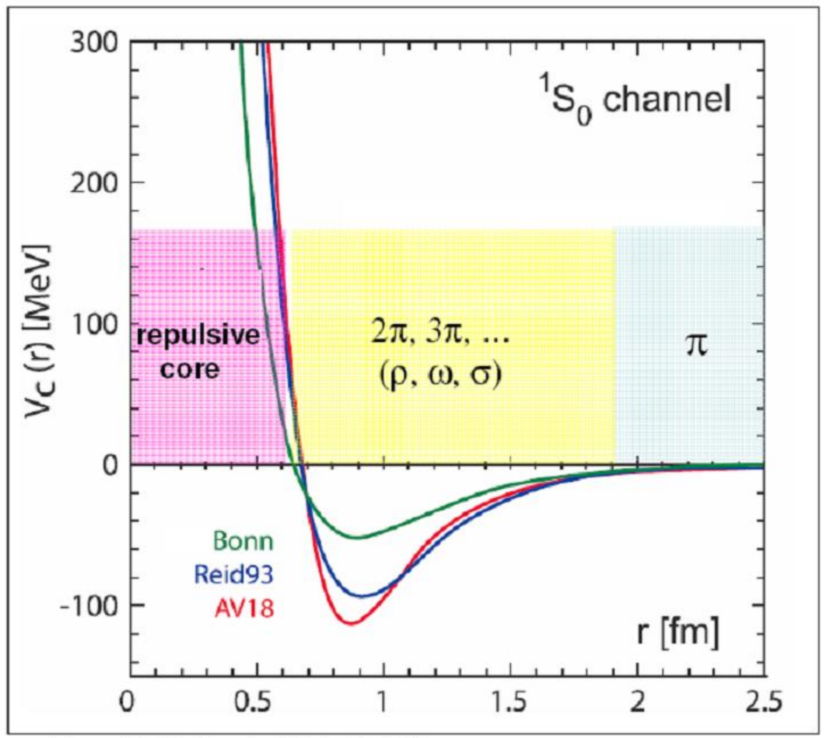
Meson-exchange potential

QCD-based Chiral effective field theory

(Chiral EFT)

Symmetries:

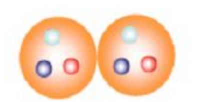
1. parity
2. spin
3. isospin



From T. Hatsuda (Oslo 2008)

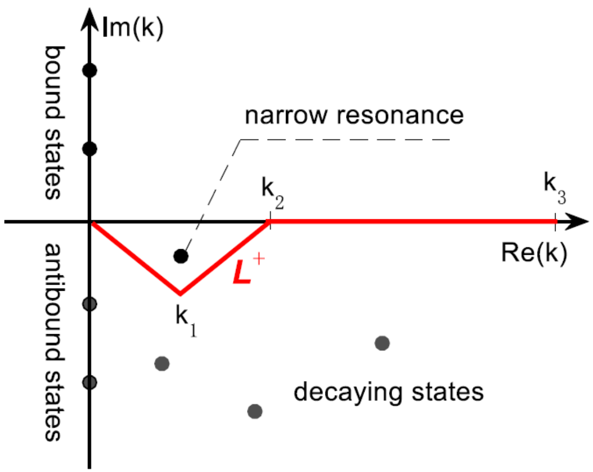
One-pion exchange by Yukawa (1935)

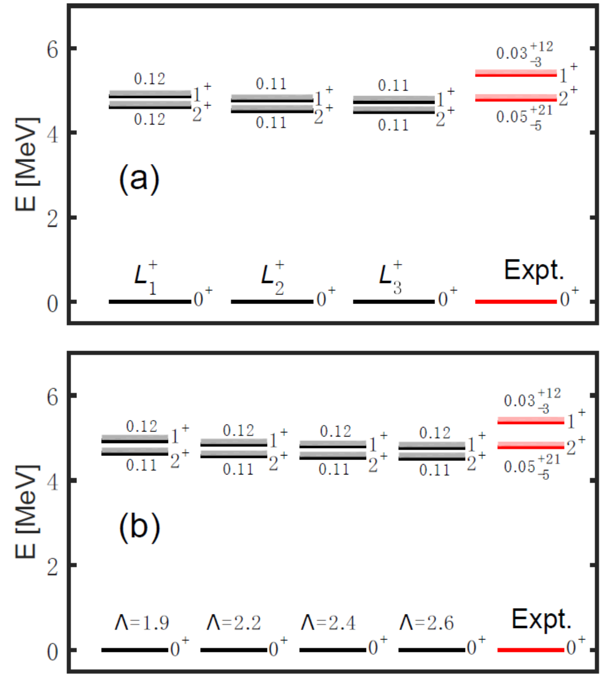
Multi-pions by Taketani (1951)



Repulsive core by Jastrow (1951)

Convergences





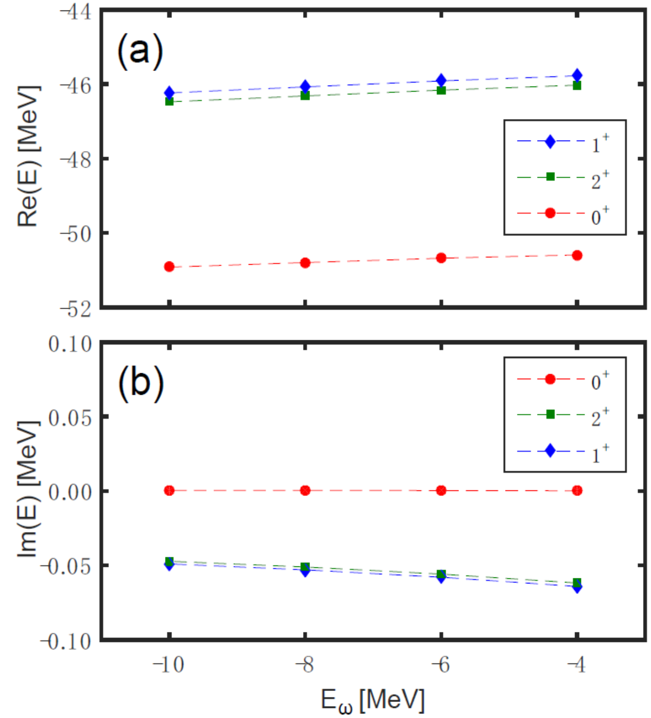
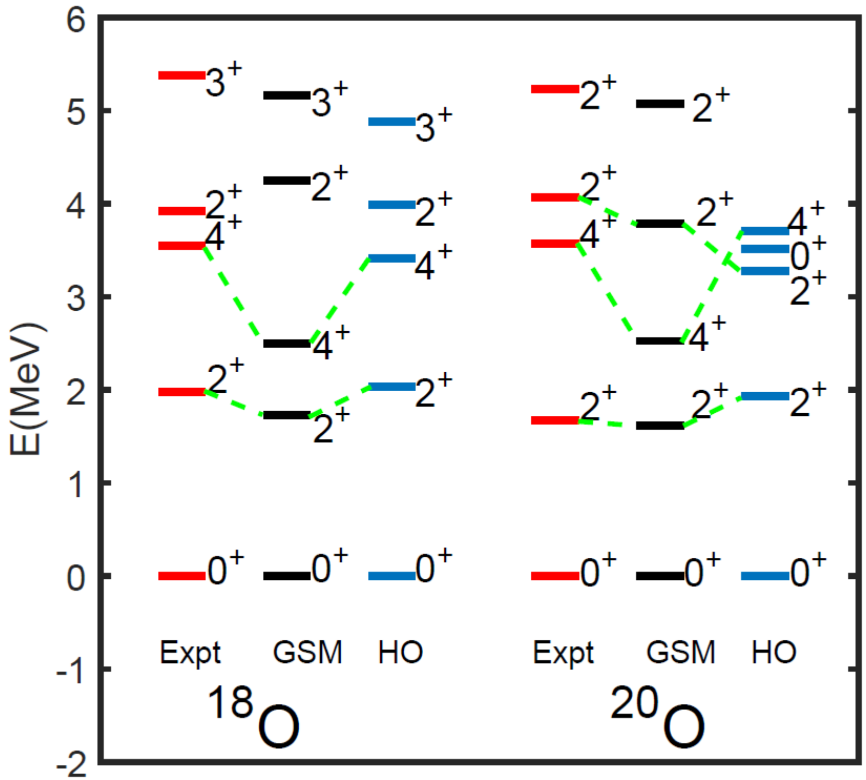
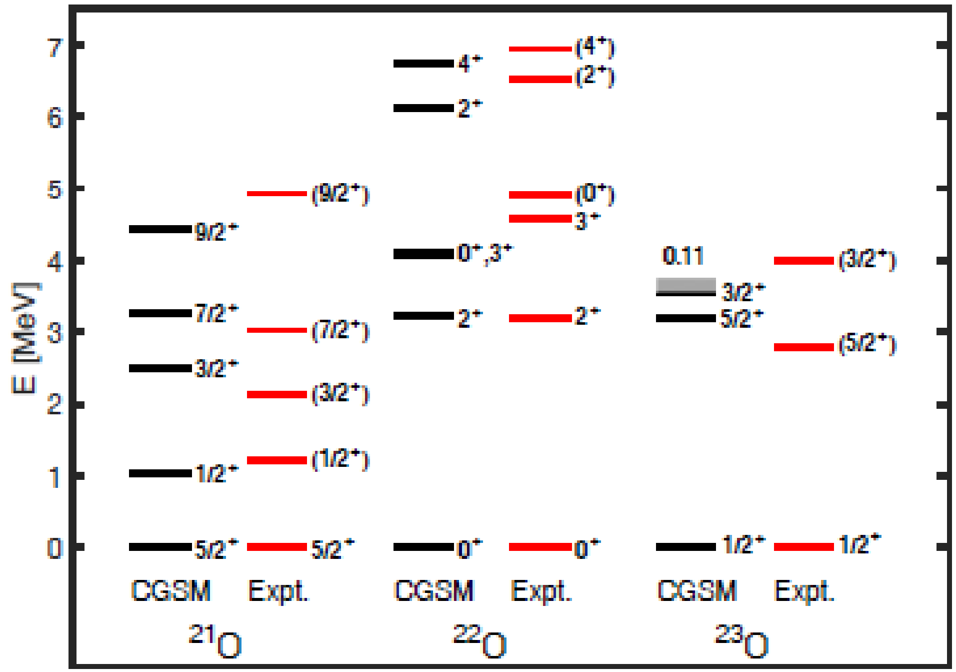


TABLE I. The convergences of the ^{24}O state energies $\tilde{E}_n = E_n - i\Gamma/2$(in MeV) calculated with different discretization point numbers N_L on the $d_{3/2}$ continuum contour $L^+ = \{0.0 \to (0.48 - 0.20i) \to 0.62 \to 2.2\}$ (see Fig.1 for the definition of the contour L^+). $\Lambda = 2.6$ fm^{-1} is taken.

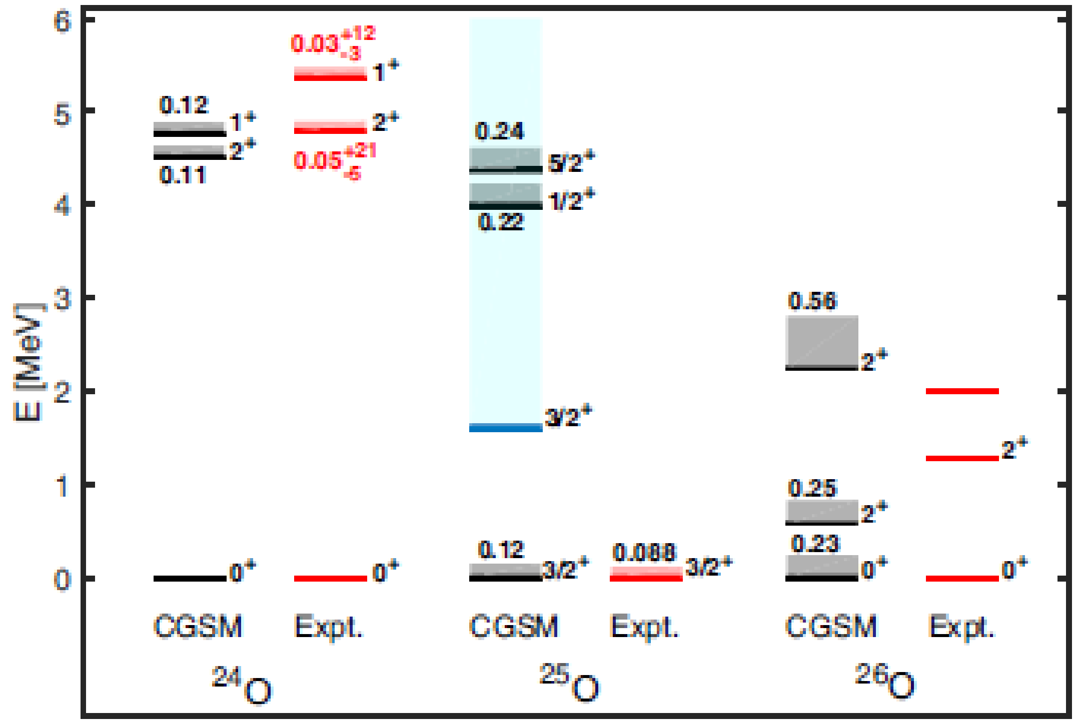
N_L	0^+	2^+	1^+
16	$-50.642 + 0.013i$	$-46.172 - 0.004i$	$-45.922 - 0.009i$
18	$-50.716 + 0.002i$	$-46.262 - 0.046i$	$-46.017 - 0.049i$
20	$-50.711 - 0.001i$	$-46.219 - 0.054i$	$-45.976 - 0.056i$
22	$-50.712 + 0.000i$	$-46.218 - 0.053i$	$-45.974 - 0.056i$

CD-Bonn CGSM, compared with conventional H.O. SM



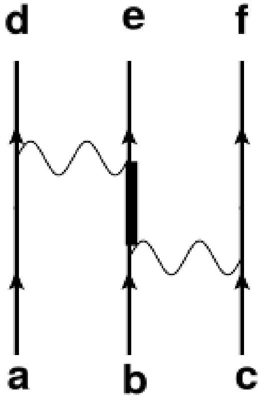


(Λ=2.6 fm^{-1})



(Λ=2.6 fm-1)

No 3-body force



Summary for CGSM calculations

1. **Starting from the realistic nuclear force, CD-Bonn**
2. $V_{\text{low-k}}$
3. **Full Q-box folded-diagrams to build a realistic effective interaction for the defined non-degenerated model space**
4. **GSM with a doubly magic core (resonance & continuum)**
5. **Well describe the spectra of bound, weakly-bound and unbound nuclei.**

II. *ab-initio* calculations with MBPT

$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} \quad ; \qquad H_{\rm int} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am} \quad , \qquad \vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

a) **First we did HF calculations (in HO basis);**
b) **The HF state is chosen as a reference state.**
c) **In the HF basis, we make MBPT corrections up to 3rd order using *j-j* coupling:**

$$H_0 = \sum_{l_1 l_2} h^{HF}_{l_1 l_2} a^\dagger_{l_1} a_{l_2}$$

$$\hat{H} = \hat{H}_0 + (\hat{H} - \hat{H}_0) = \hat{H}_0 + \hat{V}$$

The exact solutions of the A-nucleon system are,

$$\hat{H}\Psi_n = E_n \Psi_n, \qquad n = 0, 1, 2, \ldots$$

The zero-order part is,

$$\hat{H}_0 \Phi_n = E_n^{(0)} \Phi_n, \qquad n = 0, 1, 2, \ldots$$

Perturbation (MBPT)

For the ground state:

$$\chi_0 = \Psi_0 - \Phi_0$$

$$\Delta E = E_0 - E_0^{(0)}$$

$$\Psi_0 = \sum_{m=0}^{\infty} \left[\hat{R}_0(E_0^{(0)})(\hat{V} - \Delta E)\right]^m \Phi_0$$

$$\Delta E = \sum_{m=0}^{\infty} \langle \Phi_0 | \hat{V} \left[\hat{R}_0(E_0^{(0)})(\hat{V} - \Delta E)\right]^m | \Phi_0 \rangle$$

where $\hat{R}_0 = \sum_{i \neq 0} \frac{|\Phi_i\rangle\langle\Phi_i|}{E_0^{(0)} - E_i^{(0)}}$ is called the resolvent of $\hat{H}_0$

Rayleigh-Schrodinger method

$$E_0 = E_0^{(0)} + E_0^{(1)} + E_0^{(2)} + E_0^{(3)} + \ldots$$

HF energy $=< \phi_0 | H | \phi_0 >$

$$E_0^{(1)} = \langle \Phi_0 | \hat{V} | \Phi_0 \rangle$$

$$E_0^{(2)} = \langle \Phi_0 | \hat{V} \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

$$E_0^{(3)} = \langle \Phi_0 | \hat{V} \hat{R}_0 (\hat{V} - \langle \Phi_0 | \hat{V} | \Phi_0 \rangle) \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

$$\Psi_0 = \underline{\Phi_0} + \Psi_0^{(1)} + \Psi_0^{(2)} + \ldots$$

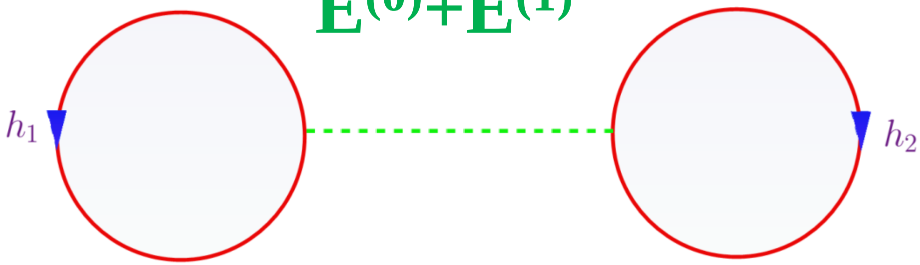
HF

$$\Psi_0^{(1)} = \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

$$\Psi_0^{(2)} = \hat{R}_0 (\hat{V} - E_0^{(1)}) \hat{R}_0 \hat{V} | \Phi_0 \rangle$$

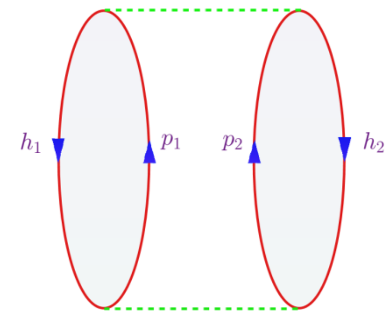
Anti-Symmetrized Goldstone (ASG) diagram expansion

$E^{(0)}+E^{(1)}$



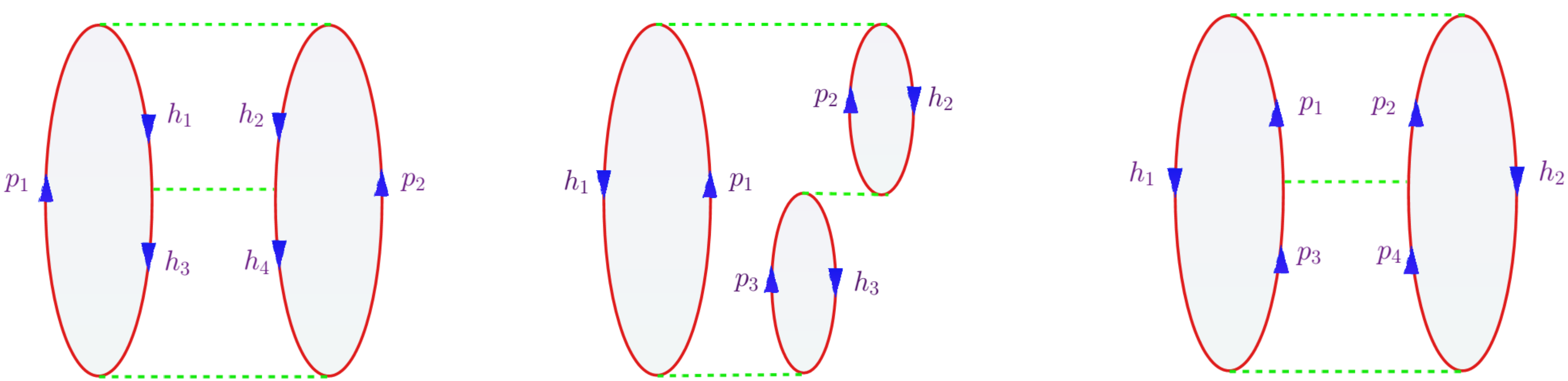
HF energy

$E^{(2)}$



$$\frac{1}{4}\sum_{h_1,h_2<\varepsilon_F}\;\sum_{p_1,p_2>\varepsilon_F}\frac{|\langle p_1p_2|\hat{\mathrm{H}}_{int}|h_1h_2\rangle|^2}{\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2}}$$

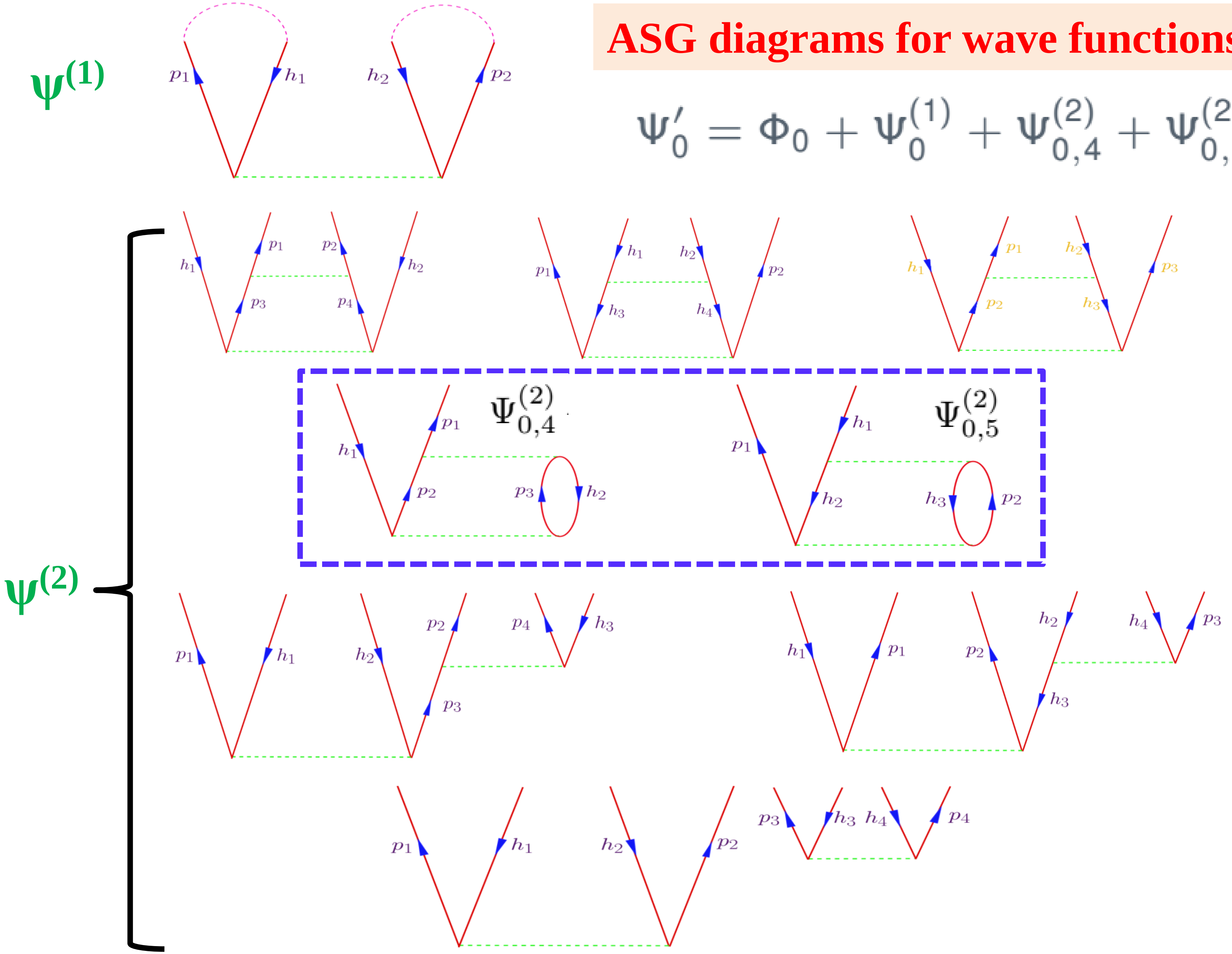
$E^{(3)}$



2p4h = $$\frac{1}{8}\sum_{h_1,h_2,h_3,h_4<\varepsilon_F}\;\sum_{p_1,p_2>\varepsilon_F}\frac{\langle p_1p_2|\hat{\mathrm{H}}_{int}|h_3h_4\rangle\langle h_3h_4|\hat{\mathrm{H}}_{int}|h_1h_2\rangle\langle h_1h_2|\hat{\mathrm{H}}_{int}|p_1p_2\rangle}{(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2})(\varepsilon_{h_3}+\varepsilon_{h_4}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

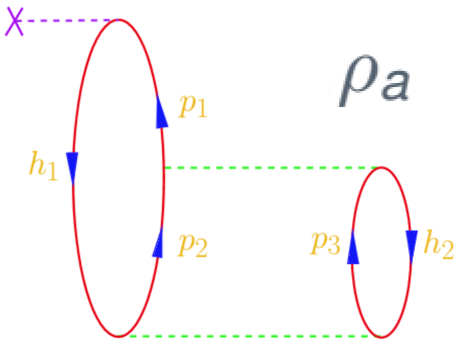
ASG diagrams for wave functions

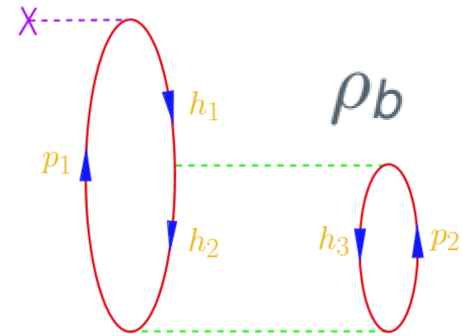
$$\Psi_0' = \Phi_0 + \Psi_0^{(1)} + \Psi_{0,4}^{(2)} + \Psi_{0,5}^{(2)}$$



$$\rho(\vec{r}) = \langle\Phi_0|\rho(\vec{r})|\Phi_0\rangle + \underline{\langle\Phi_0|\rho(\vec{r})|\Phi_0\rangle\langle\Psi_0^{(1)}|\Psi_0^{(1)}\rangle} + \underline{2\rho_a + 2\rho_b + \rho_{c1} + \rho_{c2}} + \dots$$

HF | **2^{nd} order** | **2^{nd} order**

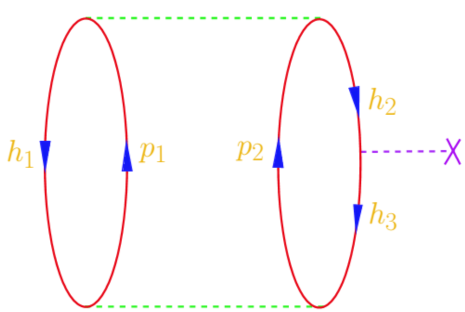




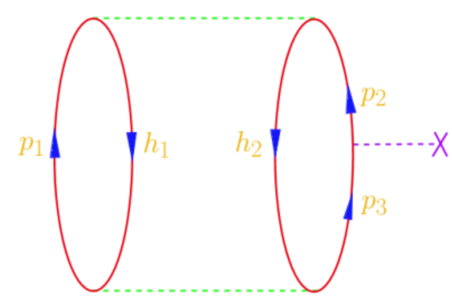
$$\frac{1}{2}\sum_{h_1,h_2<\varepsilon_F}\sum_{p_1,p_2,p_3>\varepsilon_F}\frac{\langle h_1h_2|\hat{H}|p_2p_3\rangle\langle p_2p_3|\hat{H}|p_1h_2\rangle\langle h_1|\rho|p_1\rangle}{(\varepsilon_{h_1}-\varepsilon_{p_1})(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_2}-\varepsilon_{p_3})}$$

$$-\frac{1}{2}\sum_{h_1,h_2,h_3<\varepsilon_F}\sum_{p_1,p_2>\varepsilon_F}\frac{\langle p_1p_2|\hat{H}|h_2h_3\rangle\langle h_2h_3|\hat{H}|h_1p_2\rangle\langle h_1|\rho|p_1\rangle}{(\varepsilon_{h_1}-\varepsilon_{p_1})(\varepsilon_{h_2}+\varepsilon_{h_3}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

ρ_{c1}



ρ_{c2}



$$-\frac{1}{2}\sum_{h_1,h_2,h_3<\varepsilon_F}\sum_{p_1,p_2>\varepsilon_F}\frac{\langle h_1h_2|\hat{H}|p_1p_2\rangle\langle p_1p_2|\hat{H}|h_1h_3\rangle\langle h_3|\rho|h_2\rangle}{(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2})(\varepsilon_{h_1}+\varepsilon_{h_3}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

$$\frac{1}{2}\sum_{h_1,h_2<\varepsilon_F}\sum_{p_1,p_2,p_3>\varepsilon_F}\frac{\langle p_1p_3|\hat{H}|h_1h_2\rangle\langle h_1h_2|\hat{H}|p_1p_2\rangle\langle p_2|\rho|p_3\rangle}{(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_3})(\varepsilon_{h_1}+\varepsilon_{h_2}-\varepsilon_{p_1}-\varepsilon_{p_2})}$$

NCSM

S.K. Bogner *et al.*, arXiv0708.3754v2 (2007)

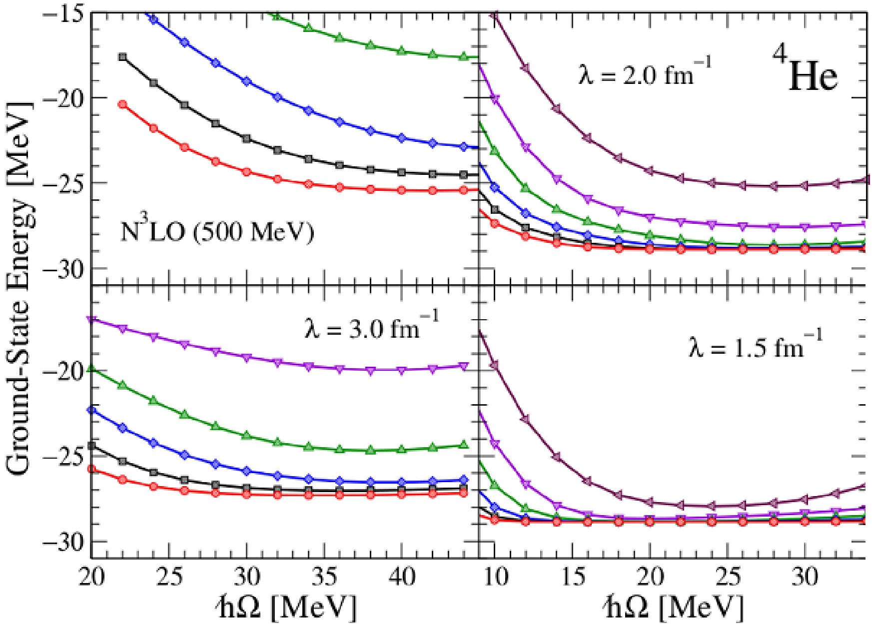
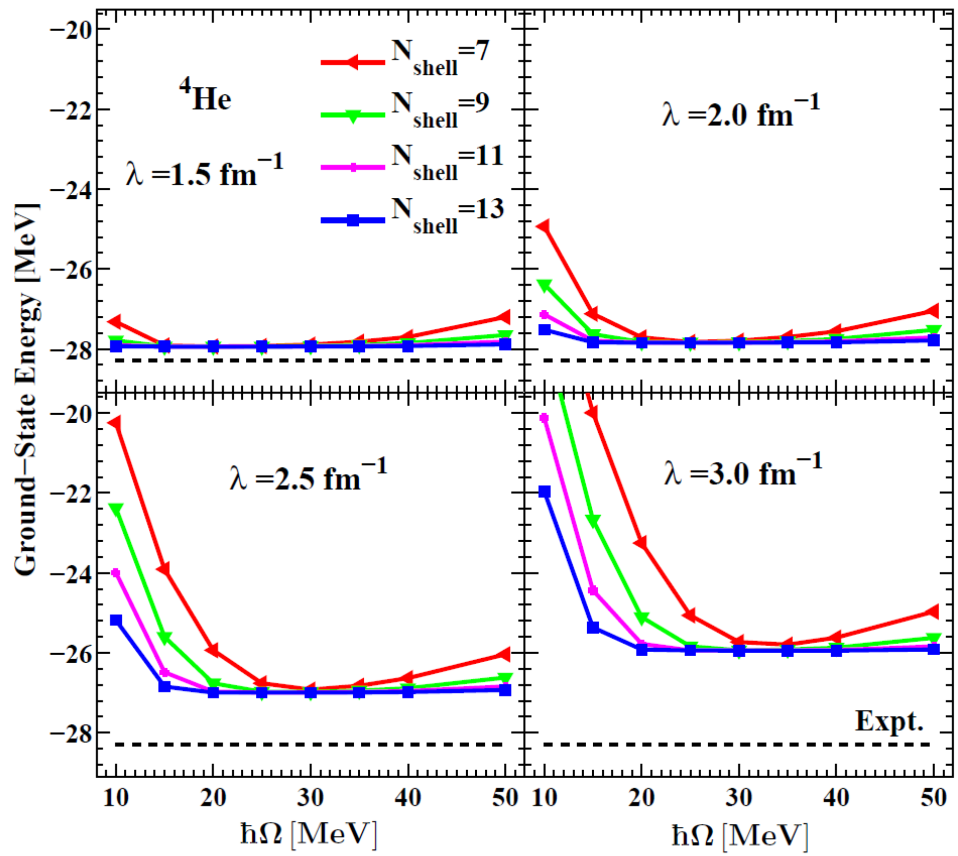


Fig. 3. Ground-state energy of ^{4}He as a function of $\hbar\Omega$ at four different value (∞, 3, 2, 1.5 fm^{-1}). The initial potential is the 500 MeV N^{3}LO NN-only pot from Ref. [13]. The legend from Fig. 1 applies here.

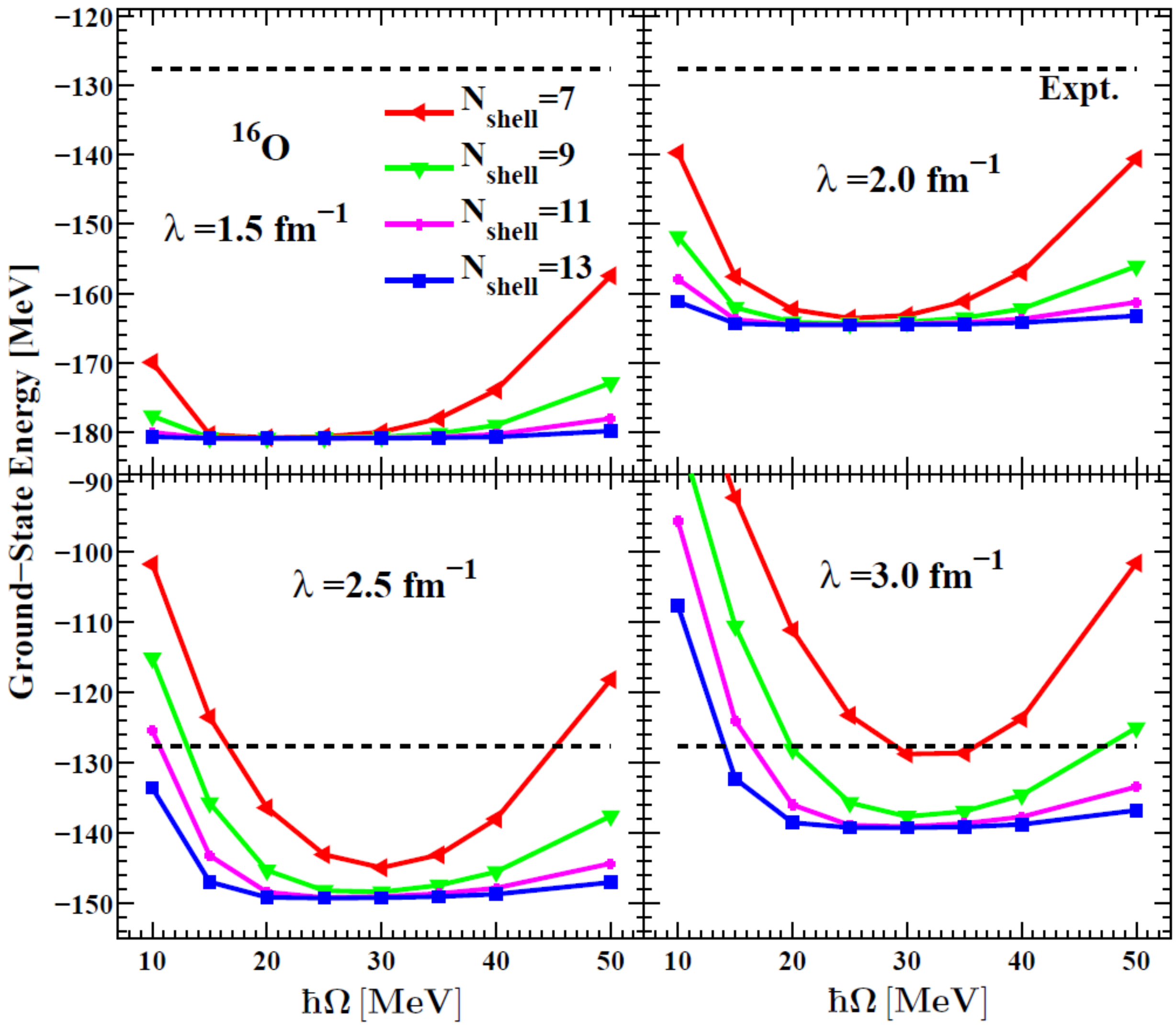
^{4}He

Our MBPT calculations

N^{3}LO+SRG without 3NF

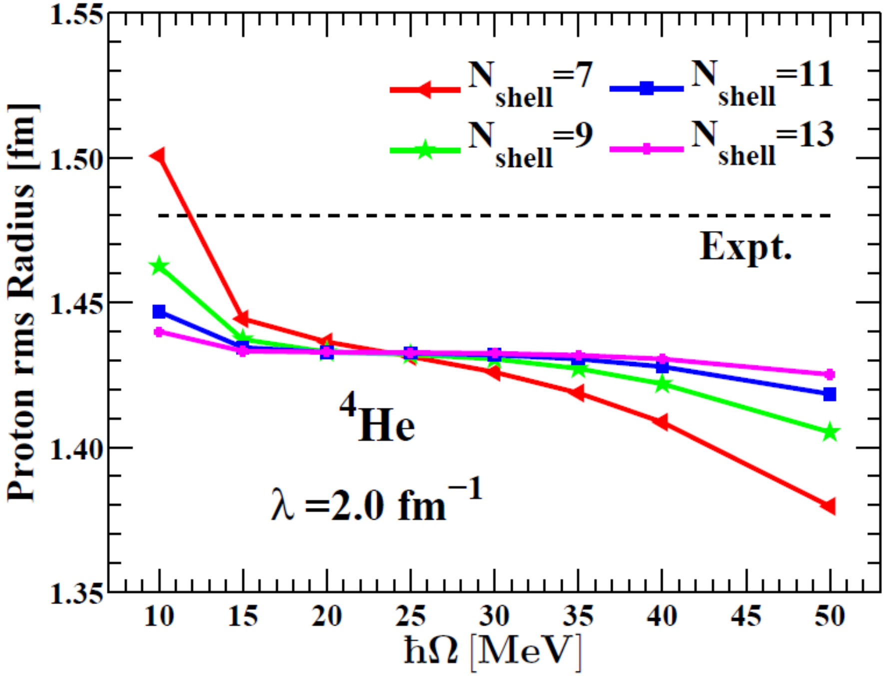


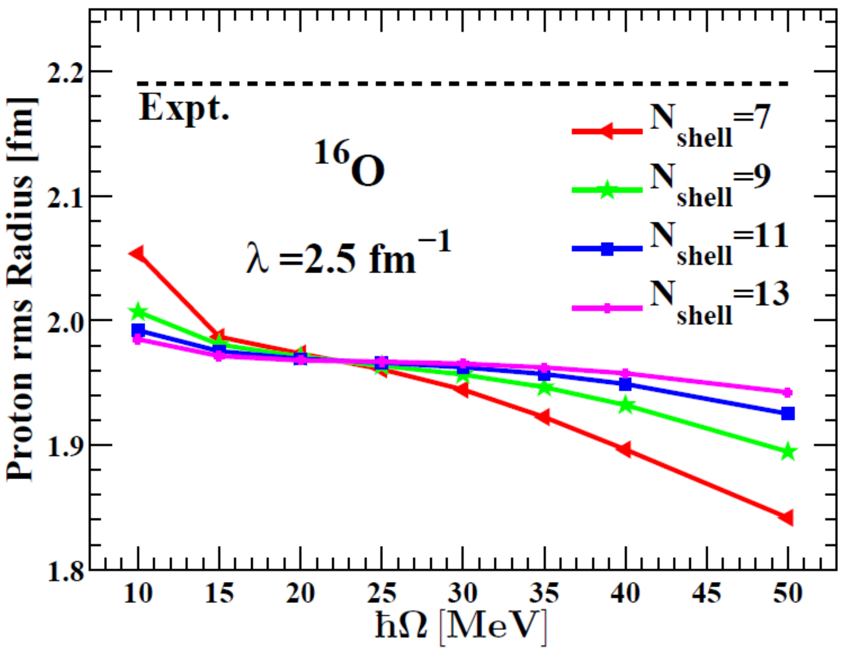
Our MBPT: N^3LO+SRG without 3NF



^{16}O

Our MBPT calculations with N^3LO+SRG: convergence in radius





R. Roth *et al*. (2006) PRC 73, 044312

AV18, UCOM, corrections to 3rd order in energy, 2nd order in radius

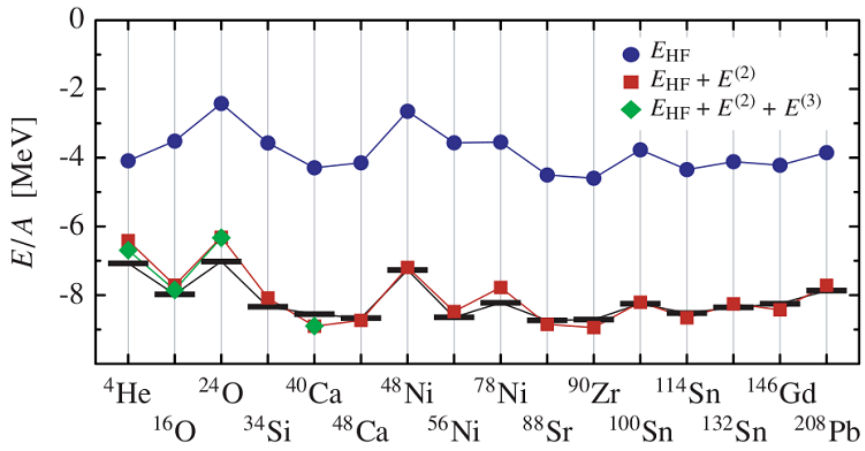


FIG. 5. (Color online) Ground-state energies for selected closed-shell nuclei in HF approximation and with added second- and third-order MBPT corrections. The correlated AV18 potential with $I_\vartheta = 0.09\,\mathrm{fm}^3$ was used. The bars indicate the experimental binding energies [31].

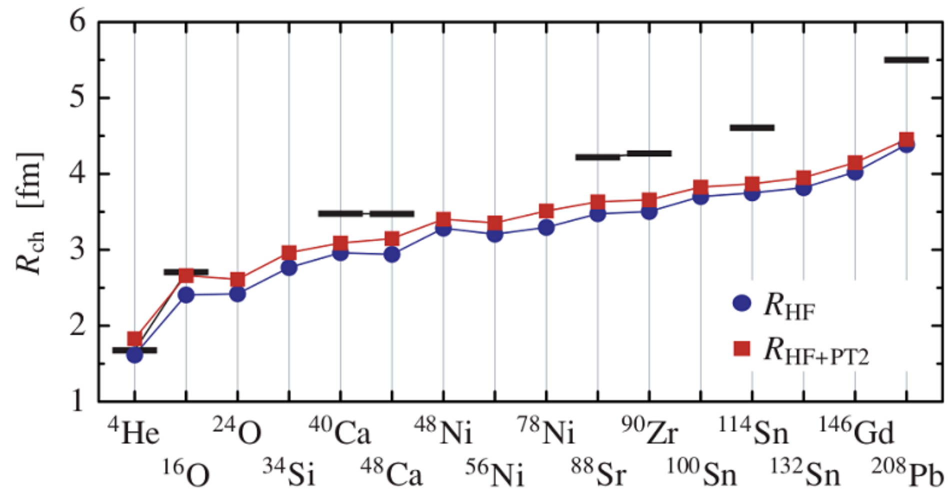


FIG. 8. (Color online) Charge radii for selected closed-shell nuclei in the HF approximation and with added second-order MBPT corrections. The correlated AV18 potential with $I_\vartheta = 0.09\,\mathrm{fm}^3$ was used. The bars indicate experimental charge radii [32].

HF-MBPT calculations for ^{4}He with N^{3}LO-SRG, N_{shell}=13, ħΩ=35 MeV

Binding energy

	SRG flow parameter λ (fm^{-1})			
	1.5	2.0	2.5	3.0
Expt. [60]	-28.296	-28.296	-28.296	-28.296
NCSM [61]	-28.20	-28.41	-27.43	-26.80
SHF	-25.754	-21.864	-15.854	-10.278
PT2	-1.788	-5.088	-9.652	-13.783
PT3	-0.391	-0.899	-1.523	-1.953
SHF+PT2+PT3	-27.933	-27.850	-27.029	-26.013

r_p(NCSM)=1.418 fm with N_{max}=10

Point-proton rms radius

	SRG flow parameter λ (fm^{-1})			
	1.5	2.0	2.5	3.0
Expt.	1.477	1.477	1.477	1.477
SHF	1.677	1.652	1.714	1.816
PT2	0.007	0.001	-0.021	-0.065
$\Delta r_{c.m.}$	-0.226	-0.222	-0.227	-0.235
SHF+PT2+$\Delta r_{c.m.}$	1.458	1.431	1.466	1.516

HF-MBPT calculations for ^{16}O with N^3LO-SRG, N_{shell}=13, $\hbar\Omega$=35 MeV

Binding energy

	SRG flow parameter λ (fm^{-1})			
	1.5	2.0	2.5	3.0
Expt. [60]	-127.619	-127.619	-127.619	-127.619
SHF	-169.968	-133.169	-85.173	-44.102
PT2	-10.132	-29.497	-59.617	-88.326
PT3	-0.794	-1.931	-4.630	-7.339
SHF+PT2+PT3	-180.893	-164.597	-149.419	-139.767

3NF important!

Point-proton rms radius

	SRG flow parameter λ (fm^{-1})			
	1.5	2.0	2.5	3.0
Expt.	2.581	2.581	2.581	2.581
SHF	2.098	2.096	2.201	2.345
PT2	0.011	0.011	-0.006	-0.042
$\Delta r_{\mathrm{c.m.}}$	-0.067	-0.067	-0.070	-0.073
SHF+PT2+$\Delta r_{\mathrm{c.m.}}$	2.042	2.040	2.125	2.230

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Ab initio nuclear many-body perturbation calculations in the Hartree-Fock basis

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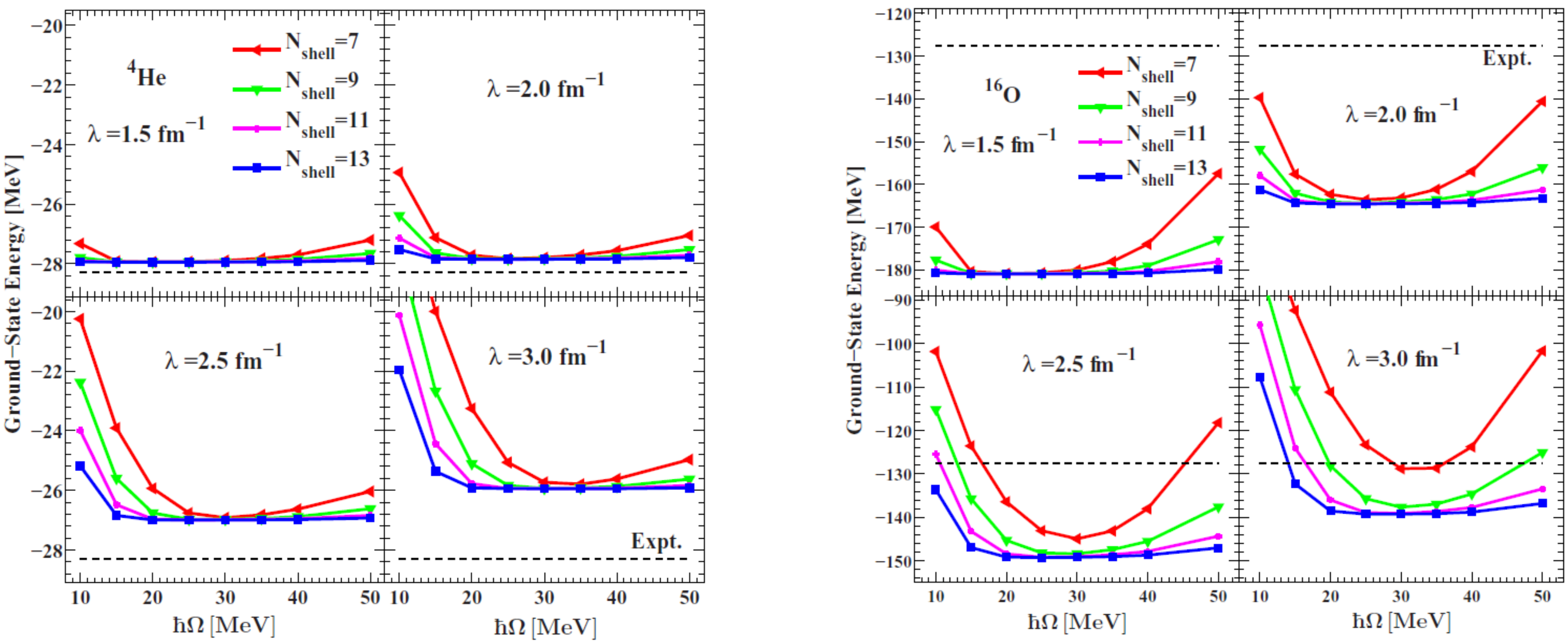


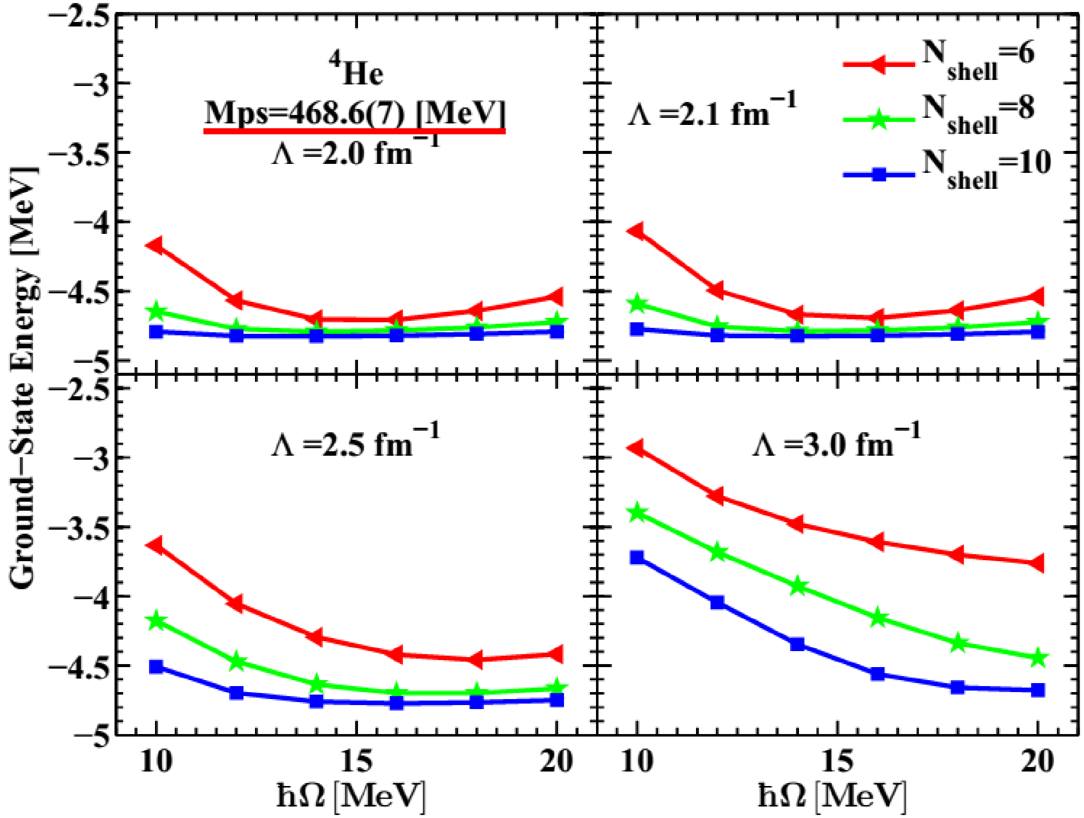
Chiral potential N^{3}LO MBPT calculations

LQCD ⟹ MBPT calculations

LQCD was provided by Aoki and Inoue

We renormalize it using $V_{\text{low-k}}$

Preliminary



E^{expt} = -28.3 MeV

FIG. 6. Ground-state energy of ^4He in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different V_{lowk} interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

LQCD + MBPT

Preliminary

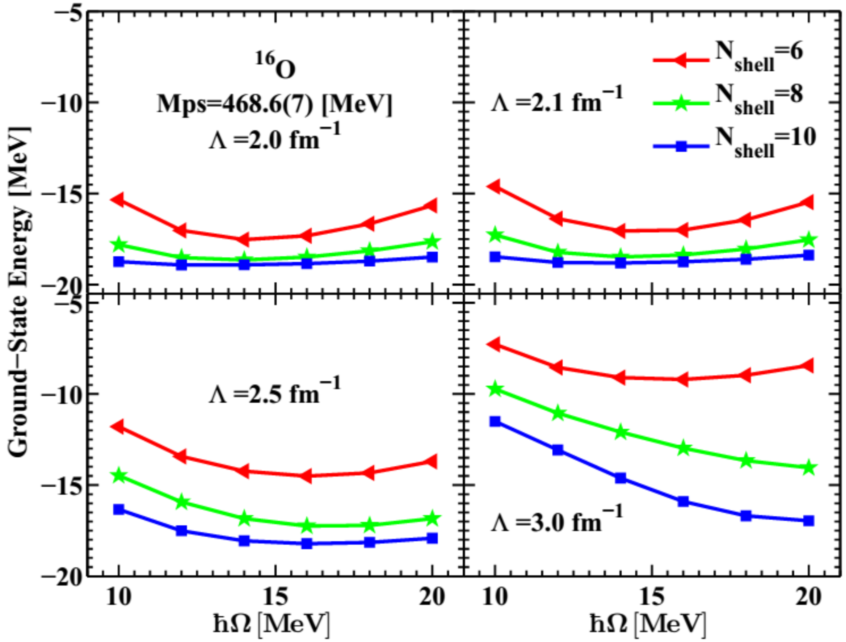


FIG. 7. Ground-state energy of ^{16}O in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different V_{lowk} interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

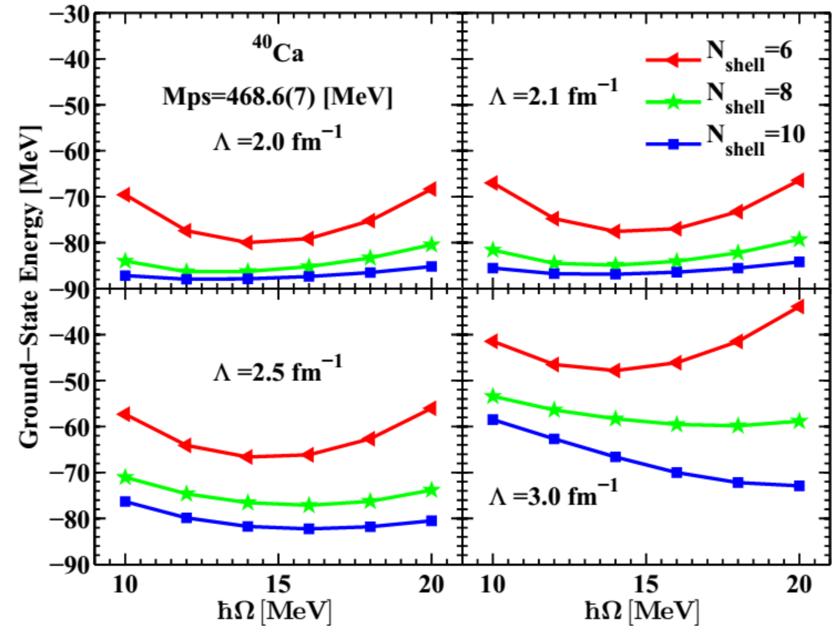


FIG. 8. Ground-state energy of ^{40}Ca in MBPT calculation as a function of oscillator parameter $\hbar\Omega$ for different V_{lowk} interactions with $\Lambda = 2.0, 2.1, 2.5, 3.0\ fm^{-1}$. The initial interaction is the lattice QCD simulations [1–4].

E^{expt} = -127.6 MeV

E^{expt} = -342.0 MeV

III. Summary

Starting with realistic nuclear forces

I. CGSM with CD Bonn + $V_{low\ k}$ +folded-diagram for weakly-bound and unbound nuclei (resonance & continuum), Oxygen isotopes

II. MBPT with N^3LO (LQCD) + SRG for close-shell nuclei

Advantages of *ab-initio* calculations:

i) To understand the nature of nuclear forces;

ii) To understand many-body correlations;

……

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Thank you for your attention

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