ASYMPTOTIC PROPERTIES OF RESONANCE AND WEAKLY-BOUND STATES IN THE SHELL-MODEL CALCULATIONS

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CLUSTERING

STATIC CLUSTERING

Closed channels with low binding energy, alpha-cluster channels first of all are presented in a nucleus (9Be as an example).

DYNAMIC CLUSTERING

Open entrance and exit channels of nuclear reactions induced by composite particle collisions are attributes of these processes (arbitrary fragments shape a channel).

Both these effects are the subjects.

MOTIVATIONS

1. A lot of nuclear states manifest pronounced structuring – formation of two or more substructures (clusters, halo nucleons, etc.). These states shape specific spectra and demonstrate peculiarities in the asymptotic behavior. Therefore it is of value to create methods which give an account of clustering and incorporate them into the shell-model calculations.

2. Progress of shell-model and related high-quality studies of nuclear structure generates a need for accurate approaches involving the results of these studies in treatment of nuclear reactions and decays. Thus one needs to combine the results with the description of certain exit and/or entrance channels.

HOW TO DESCRIBE A CLUSTER CHANNEL ACCURATELY? CLUSTERING IN THE RGM

The wave function of the resonating group model (Wheeler, 1937) is chosen in the form:

$$\Psi_{A_1+A_2} = \hat{A}\{\Psi_{A_1}\Psi_{A_2}\varphi(\vec{\rho})\},$$

where

$$\hat{\mathrm{A}} = \begin{pmatrix} A \\ A_1 \end{pmatrix}^{-1/2} \left(1 + \sum_P (-1)^p \hat{P}\right)$$

The A-fermion Schrödinger equation

$$\hat{H}\Psi_{A_1+A_2} = E\Psi_{A_1+A_2}, \quad \hat{H} = \hat{T} + \hat{V},$$

$$\hat{T} = \sum_{i=1}^{A_1+A_2} \frac{\hat{\vec{p}}_i^{\,2}}{2m}, \qquad \hat{V} = \sum_{i<j=1}^{A_1+A_2} V(\vec{r}_i - \vec{r}_j)$$

being projected results in two-body equation of another type:

$$(\hat{T}_\rho + \hat{V}_\rho - E'\,\hat{N}_\rho)\varphi(\vec{\rho}) = 0\,,$$

$$E' = E - E_1 - E_2\,, \qquad \vec{\rho} = \frac{1}{A_1}\sum_{i=1}^{A_1}\vec{r}_i \; - \frac{1}{A_2}\sum_{j=A_1+1}^{A_2}\vec{r}_j\,,$$

where: $\left\langle \hat{N}_\rho^{1/2}\varphi(\vec{\rho}) \middle| \hat{N}_\rho^{1/2}\varphi(\vec{\rho}) \right\rangle = 1,\ \delta(E-E'),\ \delta(k-k'),\ etc$

$$\begin{pmatrix} \hat{N}_\rho \\ \hat{T}_\rho \\ \hat{V}_\rho \end{pmatrix} \varphi(\rho) \equiv \int \begin{pmatrix} N(\rho',\rho) \\ T(\rho',\rho) \\ V(\rho',\rho) \end{pmatrix} \varphi(\rho')\rho'^2 d\rho'; \qquad \begin{pmatrix} N(\rho',\rho'') \\ T(\rho',\rho'') \\ V(\rho',\rho'') \end{pmatrix} =$$

$$\left\langle \hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_\rho)\right\} \middle| \begin{pmatrix} \hat{1} \\ \hat{T} \\ \hat{V} \end{pmatrix} \middle| \hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho'')Y_{lm}(\Omega_\rho)\right\} \right\rangle.$$

Introducing a new wave function:

$$\phi(\vec{\rho}) = \hat{N}_{\rho}^{1/2}\varphi(\vec{\rho})$$

one can obtain the Schrödinger-like equation with Hermitian Hamiltonian.

$$\left(\hat{N}_{\rho}^{-1/2}\hat{T}_{\rho}\,\hat{N}_{\rho}^{-1/2} + \hat{N}_{\rho}^{-1/2}\hat{V}_{\rho}\,\hat{N}_{\rho}^{-1/2} - E'\right)\phi(\rho) = 0\,,$$

where the habituated orthonormalization conditions take place:

$\left\langle \phi(\vec{\rho}) \middle| \phi(\vec{\rho}) \right\rangle = 1$ - for states of discrete spectra,

$\left\langle \phi_E(\vec{\rho}) \middle| \phi_{E'}(\vec{\rho}) \right\rangle = \delta(E - E'),\ etc.$ - for continuum states.

ALGEBRAIC VERSION OF RGM
(Filippov, Vasilevsky, Okhrimenko, 1980)

In the case that Ψ_{A1} and Ψ_{A2} are oscillator wave functions with one and the same parameter $\hbar\omega$, the following expansion of wave function of the relative motion is useful in the algebraic version of RGM:

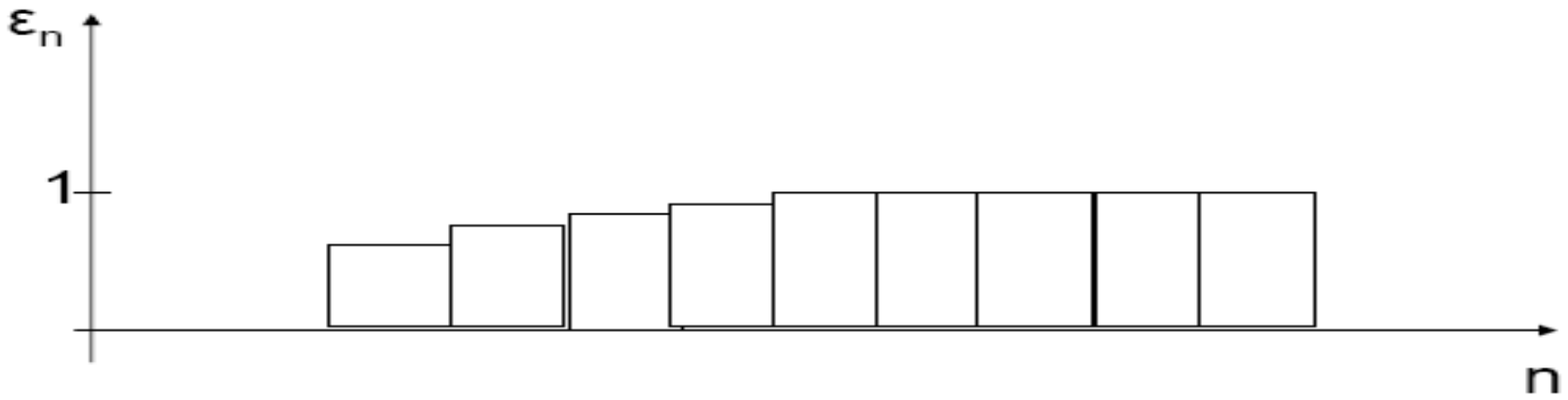
$$\phi_l(\rho) = \sum_n C_{nl}\,\phi_{nl}(\rho)\,,$$

where $\phi_{nl}(\rho)$ - oscillator wave function of the relative motion,

$$C_{nl} = \int \phi_l(\rho)\,\phi_{nl}(\rho)\,\rho^2 d\rho\,.$$

The wave functions $\phi_{nl}(\rho)$ are eigenfunctions of the kernel

$$\hat{N}_\rho\,\phi_{nl}(\rho) = \varepsilon_n\,\phi_{nl}(\rho)\,.$$



with the eigenvalues

$\varepsilon_n = 0$ for forbidden states and $\varepsilon_n \to 1$ at $n \to \infty$

The states related to $\varepsilon_n < 1$ are called semi-forbidden.

ORTHOGONALITY CONDITIONS MODEL

The assumption of the original OCM (Saito,1969) is:

$$\hat{V}(\rho',\rho'') = \left\langle \Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_\rho)\middle|\hat{V}\middle|\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho'')Y_{lm}(\Omega_\rho)\right\rangle =$$

$$\hat{V}(\rho')\delta(\rho'-\rho''),$$

By these means direct (double folding) potential is built. If the forbidden components in the solution are removed somehow then

$$\left(\hat{T}_\rho + \hat{V}(\rho) - E'\right)\tilde{\varphi}(\vec{\rho}) = 0\,,$$

The idea of the approximation is that exchange terms are removed from the matrix elements

$$\hat{\mathrm{A}} = \begin{pmatrix} A \\ A_1 \end{pmatrix}^{-1/2}\left(1 + \sum_P (-1)^p \hat{P}\right)$$

However there is a more elegant way.

PRECISE RGM KINETIC ENERGY OPERATOR

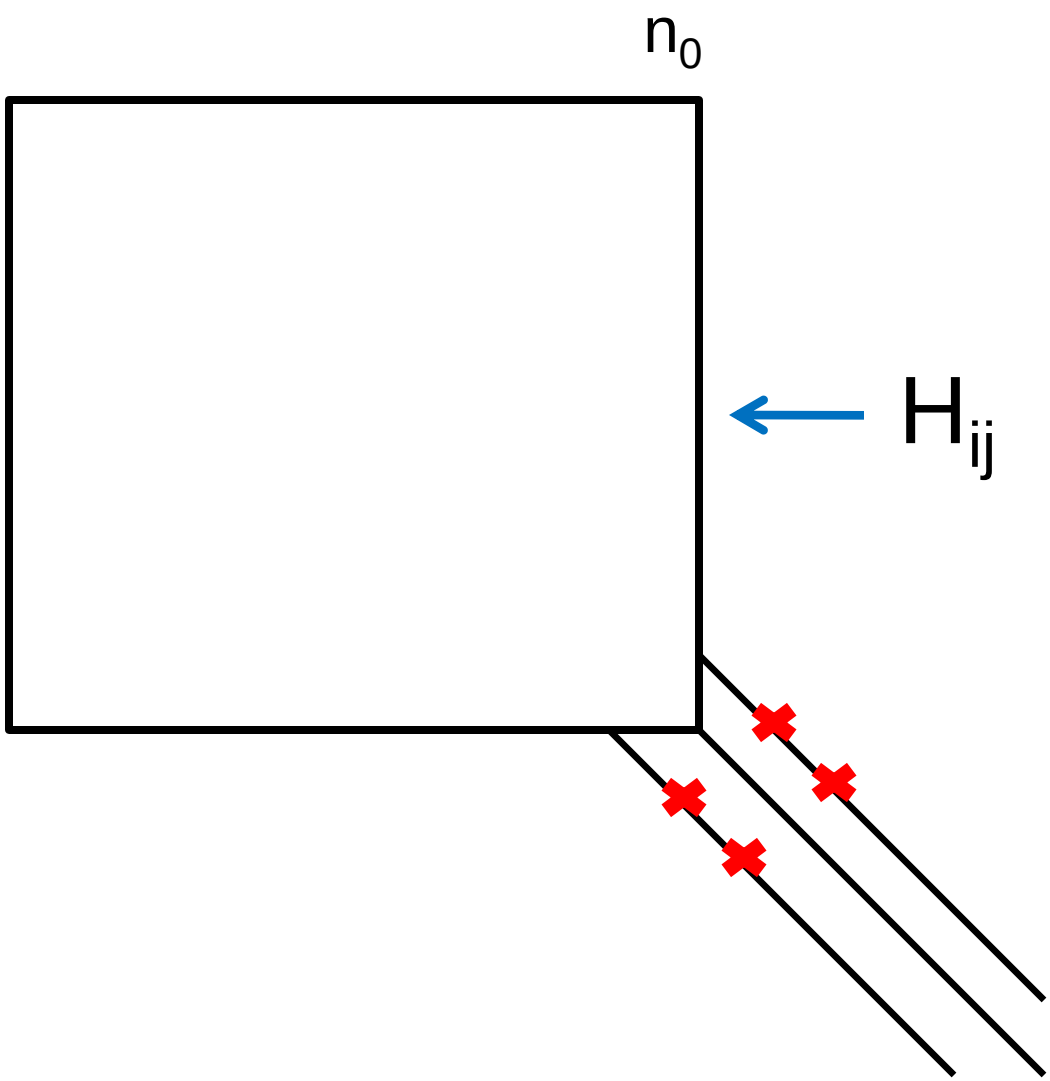
It is known (Horiuchi, 1977) that:

$$\left\langle \phi_{nl}(\rho)\left|\hat{N}_{\rho}^{-1/2}\hat{T}_{\rho}\,\hat{N}_{\rho}^{-1/2}\right|\phi_{n'l}(\rho)\right\rangle = \sqrt{\frac{\varepsilon_{n<}}{\varepsilon_{n>}}}\,T_{nn'},$$

where $n_<$=min(n,n'), $n_>$=max(n,n') and $\hat{T}_{nn'}$ is the matrix element of ordinary two-body kinetic energy operator. So the AV RGM system of equations takes the form:

$$\det\left\| \sqrt{\frac{\varepsilon_{n<}}{\varepsilon_{n>}}}\,T_{nn'} + V_{nn'} - E'\right\| = 0$$

and A-fermion exchange properties of the kinetic energy operator are completely determined by normalization kernel.


n_0
H_{ij}

MEASURE OF CLUSTERING

A long-term concept was that the measure of clustering is so-called spectroscopic amplitude:

$$C^{nl}_{MDC} = < \Psi_M \mid \hat{A}\{\Psi_D \phi_{nl}(\vec{\rho})\Psi_C\} >$$

or the cluster form factor (overlap integral, amplitude of the spectroscopic factor)

$$\Phi_l(\rho) = < \Psi_M \mid \hat{A}\{\Psi_D \frac{1}{\rho^2}\delta(\rho - \rho')Y_{lm}(\Omega_{\rho'})\Psi_C\} >$$

or its norm – cluster spectroscopic factor which can be expressed as:

$$S_{MDC} \equiv \int \Phi(\rho)|^2 \ \rho^2 d\rho = \sum_n \left(C^{nl}_{MDC}\right)^2$$

[H.J. Mang Z. Phys. 148, 556 (1957); V.V. Balashov et al. JETP 37, 1385 (1959); a set of works by SINP MSU and VSU groups].

REDEFINITION OF THE CLUSTERING MEASURES. "NEW" CLUSTER CHARACTERISTICS.

In the paper [T. Fliessbach and H.J. Mang, Nucl. Phys. A 263, 75 (1976)] the habituated view on the clustering measures was thrown doubt. The matter is that a certain matching procedure (point or integral) is required to deduce the amplitude and the width of a cluster channel.

The values of one and the same sense can solely be related (matched, compared).

So the cluster form factor (projection of the shell model wave function)

$$\Phi_l(\rho) = < \Psi_M \mid \hat{A}\{\Psi_D \frac{1}{\rho^2}\delta(\rho - \rho')Y_{lm}(\Omega_{\rho'})\Psi_C\} >$$

must be related to the same projection of the A-nucleon cluster channel WF. Not:

$$\Phi_l(\rho) \not\longleftrightarrow f_l(\rho),$$

$f(\rho)$ – a solution of two-body problem, with the traditional norm, but:

$$\Phi_l(\rho) \longleftrightarrow \Phi'_l(\rho)$$

where:

$$\Phi'_l(\rho) = < \Psi_{D+C} \mid \hat{A}\{\Psi_D \frac{1}{\rho^2}\delta(\rho - \rho')Y_{lm}(\Omega_{\rho'})\Psi_C\} >$$

And the channel wave function:

$$\Psi_{D+C} = \hat{A}\{\Psi_D \varphi(\vec{\rho})\Psi_C\} -$$

microscopic solution of A-nucleon problem which may be RGM, OCM, etc. After the redefinition

$$\phi_l(\rho) = \hat{N}_\rho^{1/2}\varphi_l(\rho),$$

resulting wave function is normalized in the regular way and satisfy the Schrödinger equation with a Hermitian Hamiltonian:

$$\left(\hat{N}_\rho^{-1/2}\hat{T}_\rho\,\hat{N}_\rho^{-1/2} + \hat{N}_\rho^{-1/2}\hat{V}_\rho\,\hat{N}_\rho^{-1/2} - E'\right)\phi_l(\rho) = 0\,.$$

As a consequence:

$$\Phi'_l(\rho) = \hat{N}_\rho \varphi_l(\rho) = \hat{N}_\rho^{1/2} \phi_l(\rho).$$

$$\Phi_l(\rho) \quad \longleftrightarrow \quad \hat{N}_\rho^{1/2} \phi_l(\rho).$$

$$\hat{N}_\rho^{-1/2} \Phi_l(\rho) \quad \longleftrightarrow \quad \phi_l(\rho)$$

$$S'_{MDC} \equiv \int | \hat{N}_\rho^{-1/2} \Phi(\rho) |^2 \, \rho^2 d\rho.$$

So the solution of the Schroedinger equation with the Hermitian Hamiltonian must be related to renormalized cluster form factor. [R. Lovas et al. Phys. Rep. 294, 265 (1998)].

Sometimes the results are drastic.

CLUSTER-NUCLEON CONFIGURATION INTERACTION MODEL AND DESCRIPTION OF EXPERIMENTAL DATA

[A. Volya, Yu.M. T. Phys. Rev. C 91, 044319 ((2015)].

α-clustering in the ground states of (s-d)-shell nuclei

$A_P - A_D$	$S_0^{(\mathrm{exp})}$ [63]	$S_0^{(\mathrm{exp})}$ [64]	$S_0^{(\mathrm{exp})}$ [65]	$S_0^{(\mathrm{old})}$ [24]	$S_0^{(\mathrm{old})}$ this work	$S_0^{(\mathrm{new})}$ this work	
^{20}Ne-^{16}O	1.0	0.54	1	0.18	0.173	0.755	0.23
^{22}Ne-^{18}O			0.37	0.099	0.085	0.481	
^{24}Mg-^{20}Ne	0.76	0.42	0.66	0.11	0.091	0.411	
^{26}Mg-^{22}Ne			0.20	0.077	0.068	0.439	
^{28}Si-^{24}Mg	0.37	0.20	0.33	0.076	0.080	0.526	$S^{(old)}$, Elliott SU(3) model
^{30}Si-^{26}Mg			0.55	0.067	0.061	0.555	
^{32}S-^{28}Si	1.05	0.55	0.45	0.090	0.082	0.911	
^{34}S-^{30}Si				0.065	0.062	0.974	
^{36}Ar-^{32}S				0.070	0.061	0.986	
^{38}Ar-^{34}S			1.30	0.034	0.030	0.997	
^{40}Ca-^{36}Ar	1.56	0.86	1.18	0.043	0.037	1	0.089

SHELL-MODEL FORMALISM FOR AVRGM

A basis wave function of AVRGM multiplied by zero oscillations of the center of mass can be written in the form:

$$\Psi_{A_1+A_2,nl}(R) \equiv \Phi_{00}(\vec{R})\hat{A}\{\Psi_{A_1}\Psi_{A_2}\phi_{nl}(\vec{\rho})\} =$$

$$\sum_{\{n_i l_i\}} \left\langle n_1 l_1\ n_2 l_2 : l \middle| 00\ nl : l \right\rangle \hat{A}\{[\Psi_{A_1}\phi_{n_1 l_1}(\vec{R}_1)][\Psi_{A_2}\phi_{n_2 l_2}(\vec{R}_2)] : l]\}$$

Here the first multiplier is Talmi-Moshinsky coefficient. Each of the products related to one and the same fragment is expressible in terms of so-called cluster coefficients:

$$\Psi_{A_i n_i l_i}\phi(\vec{R}_i) = \sum_{shell\ q.n.} \left\langle \Psi^{shell}_{A_i n_i l_i}(R_1) \middle| \Psi_{A_i}\phi_{n_i l_i}(\vec{R}_i) \right\rangle \Psi^{shell}_{A_i n_i l_i}(R_1)$$

This coefficient determine the weight of the respective cluster internal motion wave function multiplied by the oscillator wave function of the center of mass motion in a certain shell-model configurations. For the lowest s^X configurations of the lightest (X≤4) clusters a simple formula is known (Smirnov, Tchuvil'sky, 1977):

$$< \prod_{i=1}^{X} n_i(n0) : 000 \mid \phi_{(n0)}(R_C)\Psi_C >= X^{-n/2} \left(n!/\prod_{i=1}^{X} n_i! \right)^{1/2} \left(X!\prod_{j=1}^{k} \alpha_i! \right)^{1/2},$$

where α_i is the multiplicity of one and the same n_i in the shell-model configuration.

There are a number of approaches suitable in the cases that the CC of lowest configurations of heavier fragments are calculated.

The methods workable for accurately calculated fragment wave functions also exist. A sufficiently versatile formalism of such a type is based on the boson (oscillator quanta) secondary quantization representation

$$\left\langle \Psi_A \phi_{nl}(\vec{R}) \middle| \Psi^{shell}_{Anl}(R) \right\rangle = (-1)^{(n-l)/2} \sqrt{4\pi}[(n-l)!!(n+l+1)!!]^{-1/2} \times$$

$$\left\langle \Psi^{shell}_{Anl}(R) \middle| (U^\dagger \cdot U^\dagger)^{(n-l)/2} Y_{lm}(U^\dagger) \middle| \phi_{00}(\vec{R}) \Psi_A \right\rangle;$$

where

$$U^\dagger = (1/A)\sum_i u_i^\dagger$$

is the creation operator of the center-of-mass oscillator quantum expressed in terms of one-nucleon oscillator quantum creation operators.

The functions which have just been built are non-normalized and non-orthogonal in the general case. So the next step in construction of the desired basis is their orthonormalization i. e. search for the eigenvalues and the eigenfunctions of the norm kernel of RGM. They can be found by diagonalization of the norm kernel matrix:

$$\| N_{nn'} \| \equiv < \Phi_{00}(\vec{R}) \Psi_{A_1} \Psi_{A_2} \phi_{nl}(\vec{\rho}) \,|\, \hat{A}^2 \,|\, \Phi_{00}(\vec{R}) \Psi_{A_1} \Psi_{A_2} \phi_{n'l}(\vec{\rho}) > .$$

which is easily calculated being expressed in the just presented shell-model form. The eigenfunctions and the eigenvalues resulted in the diagonalization procedure take the following forms

$$f_l^{\,k}(\rho) = \sum_{n=0}^{n_0} B_{nl}^k \phi_{nl}(\rho) \,;$$

$$\varepsilon_k = < \Psi_{A_1} \Psi_{A_2} f_l^{\,k}(\vec{\rho}) \,|\, \hat{A}^2 \,|\, \Psi_{A_1} \Psi_{A_2} f_l^{\,k}(\vec{\rho}) > .$$

Owing to shell-model expansion the kernels of the kinetic and potential energy are calculated by use of ordinary shell-model procedures.

So shell-model formalism for AVRGM calculation is built.

This approach has some merits and disadvantages compared to conventional AVRGM:

A disadvantage is that the basis of the relative motion oscillator wave functions is rather short because the computation of the cluster coefficients for larger values of *n* presents a severe problem.

The advantages of the approach are:

1. More or less standard, applicable for arbitrary light fragment pair, procedures may be explored.
2. The method remains workable for accurately calculated fragment wave functions (superpositions of shell-model configurations).

3. Multi-channel AVRGM (channels are characterized by different wave functions of the cluster internal motion) may be constructed as well.

4. A high-quality nuclear reaction theory ("*A Unified Theory of the Nucleus*"; K. Wildermuth, Y.C. Tang) becomes available in the algebraic approach. Indeed, a hybrid set of cluster channels varying by the internal wave function of fragments may be involved in the basis of A_1+A_2-nucleon problem together with the number of ordinary SM solutions of the same problem. Diagonalization of a realistic Hamiltonian on this basis result in the wave functions

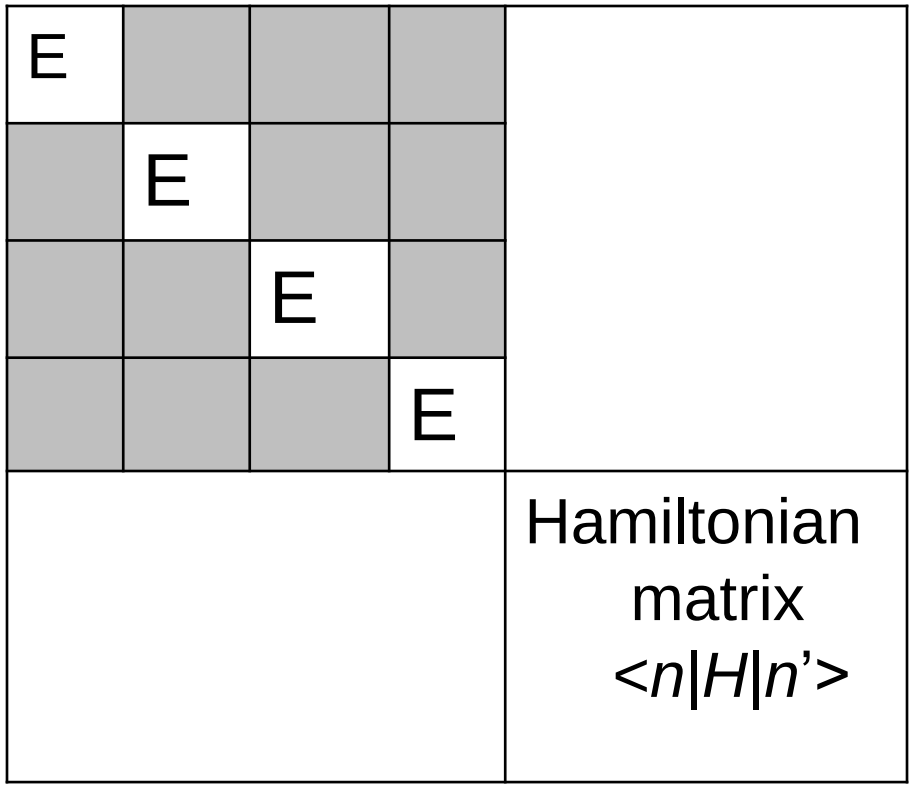
$$\Psi = \sum_{A_1,i,A_2,j,nl} C_{kl}^{\{ij\}} \hat{A}\{\Psi_{A_1}^{i} \Psi_{A_2}^{j} f_l^{\{ij\}k}(\vec{\rho})\,|\} + \sum_p C_p \Psi_{shell}^{p}.$$

HOW TO COME TO THE FAR-DISTANT ASYMPTOTICS?

So for the realistic cluster wave functions the basis of the functions (accessible for the calculations is substantially shorter compared to the basis of the canonical AVRGM. For narrow resonances it may be that the basis is not large enough to achieve the asymptotic area.

In that case one could invoke the following approach. In some distance the inter-cluster interaction is degenerated to a two-body potential V_{cl} (matrix elements of the exchange terms of the antisymmetrizer are annihilated).

In the “A-nucleon” interaction region $n \leq n_0$ the matrix elements of this Hamiltonian are easily removable in algebraic approach. For this purpose the rows and columns $\langle n|H|n' \rangle$ $n_{forb} \langle n,n' \leq n_0$ should be removed from the Hamiltonian matrix and the solutions of the A-nucleon problem should be taken into account. The matrix takes the form:



Forbidden states are considered to removed and all other states related to $n>n_0$ are considered to be allowed.

However for cases in which the asymptotics appears in far distant areas too large basis is required. In that cases it is preferable to solve the problem by the methods of infinite mathematics introducing into a habituated Schrödinger equation a number of additional separable terms:

$$\left(d^2 / dr^2 + E - 2V_{cl}^{ij} \right) \chi^{ij}(r) = V^{sep} \chi^{ij}(r);$$

$$V^{sep} = - \sum_{k,k'=k_{mn}}^{n_0} \left| kl \right\rangle H_{kk'}^{cl} \left\langle k'l \right| + E \sum_{k=k_{mn}}^{n_0} \left| kl \right\rangle \left\langle kl \right|$$

Matrix elements of V^{sep}, on the one hand, cancel with the unneeded initial two-body cluster Hamiltonian matrix elements and, on the other hand add proper matrix elements from. “A-nucleon” interaction region.

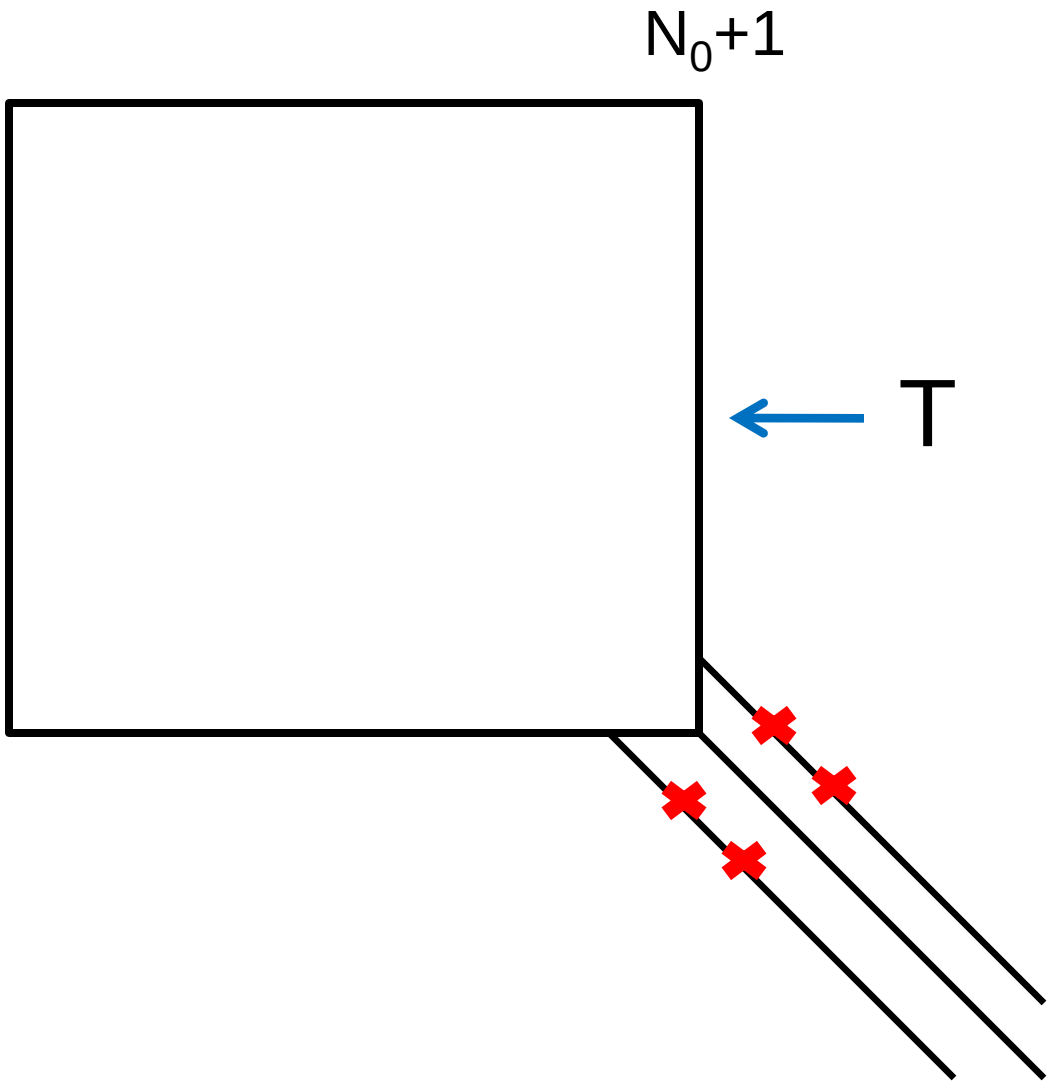
POTENTIAL OF THE GENERAL TYPE CONSIDERING THE FORBIDDEN AND THE SEMI-FORBIDEN STATES

$$V^{sep} = -\sum_{n=0}^{n_0} \sum_{n'=n_0+1}^{\infty} \left(|nl\rangle H_{nn'} \langle n'l| + |n'l\rangle H_{n'n} \langle nl| \right) +$$

$$+ \sum_{n=n_0+1}^{\infty} \left(\sqrt{\frac{\varepsilon_n}{\varepsilon_{n+1}}} - 1 \right) \left(|nl\rangle T_{n,n+1} \langle n+1l| + |n+1l\rangle T_{n+1,n} \langle nl| - \right.$$

$$+ \sum_{k=0}^{n_0+1} B_{n_0}^{k} \left(\sqrt{\frac{\varepsilon_{n_0}}{\varepsilon_{n_0+1}}} - 1 \right) \left(|n_0 l\rangle T_{n_0,n_0+1} \langle n_0 + 1l| + |n_0 + 1l\rangle T_{n_0+1,n_0} \langle n_0 l| - \right.$$

$$- \sum_{k,k'=0}^{n_0+1} |kl\rangle H_{kk'}^{cl} \langle k'l| + E \sum_{k=k_{mn}}^{n_0} |kl\rangle\langle kl|$$


N_0+1
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HIBRID METHOD FOR STUDY OF NARROW RESONANCE STATES

To solve this problem a combination of the algebraic approach with the methods of infinite mathematics is used.

Let us introduce the linearly independent solutions $\chi_{1,2}(r)$ of the equation with local potential V_{cl}:

$$\left(d^2/dr^2 + E - 2V_{cl}\right)\chi(r) = 0$$

The solution of the Schrödinger equation with potential V^{sep}

$$\left(d^2/dr^2 + E - 2V_{cl}\right)\chi(r) = V^{sep}\chi(r)$$

may be written in the form:

$$\chi(r) = C_1\chi_1(r) + C_2\chi_2(r)$$

with the additional condition:

$$C_1'(r)\chi_1(r) + C_2'(r)\chi_2(r) = 0$$

This procedure results in the relationship:

$$\chi''(r) = C_1'(r)\chi_1'(r) + C_2'(r)\chi_2'(r) + C_1(r)\chi_1''(r) + C_2(r)\chi_2''(r)$$

We consider the solution $\chi(r)$ to be regular at $r = 0$ and behaving as $G_l(r)$ in the limit $r \to \infty$.

So let us choose $\chi_1 = \chi_0$ to be regular in the vicinity of zero:

$$\chi_0(0) = 0 \quad r = 0$$

and

$$\chi_2 = \chi_G \quad \chi_G(r) \sim G_l(r) \quad r \to \infty.$$

Functions $C_{1,2}(r)$ satisfy the conditions:

$$\begin{cases} C_1(\infty) = 0 \\ C_2(0) = 0 \end{cases}.$$

Substituting all these conditions into the equation one can obtain:

$$\begin{cases} C_1(\infty) - C_1(r) = -\dfrac{2}{W}\displaystyle\int_r^{\infty} \chi_2(r')\hat{V}^{sep}\chi(r')dr' \\ C_2(r) - C_2(0) = \dfrac{2}{W}\displaystyle\int_0^{r} \chi_1(r')\hat{V}^{sep}\chi(r')dr' \end{cases} ; \quad W = \chi_1\chi_2' - \chi_1'\chi_2 .$$

In other words the proper solution satisfies the following homogeneous integral equation:

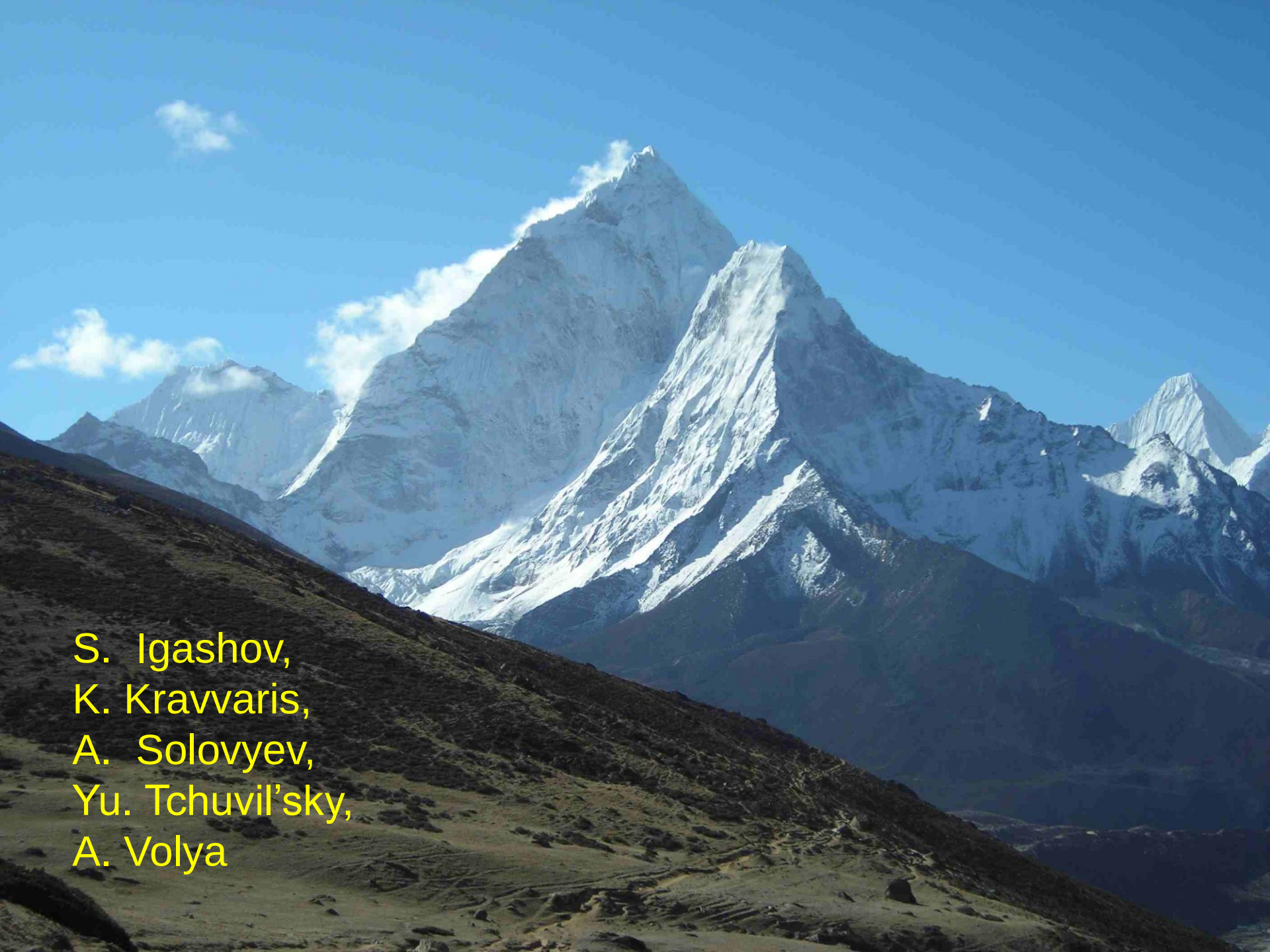
$$\chi(r) = -2\int_0^{\infty} G(r,r')\left[V^{sep}\chi\right](r')dr',$$

$$G(r,r') = \frac{\chi_0(r_<)\chi_G(r_>)}{W}$$

which may have a solution only for the unique values of energy.

CONCLUSIONS

1. Algebraic methods for the description of the clustering phenomena are well-compatible with shell-model approaches.
2. The combined SM-RGM techniques are promising for the theoretical studies of clustered bound states, resonant states, cluster transfer and knock-our reactions.
3. There are methods allowing to change from the algebraic description of nuclear decay and reaction channels to their description in terms of continuous mathematics. By doing so a connection of the results obtained by SM, AVRGM, SM+AVRGM methods and far-distant asymptotics may be established.
4. The discussed schemes can be applied not only for high-quality models but for various approximations of them as well. That makes the area of application of them much more extensive.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THANK YOU FOR ATTENTION!

ALGEBRAIC APPROACH TO THE PROBLEMS OF DISCRETE AND CONTINUOUS SPECTRA

In the algebraic versions of canonic two-body problem, OCM, or RGM the expansion coefficients C_{nl} satisfy the infinite set of linear equations

$$\sum_{n=0}^{\infty}\left(\left\langle \phi_{n'lm}\left|H\right|\phi_{nlm}\right\rangle - E\delta_{n'n}\right)C_{nl}=0, \quad n'=0,1...$$

which is follows from the respective Schrödinger equation.

For an ordinary bound state the eigenvalue problem

$$\det\left\|\widehat{H}\right\|=0, \quad H_{nn'}=\left\langle \phi_{nlm}\left|H\right|\phi_{n'lm}\right\rangle$$

is solved on the truncated basis $n \leq N_{max}$. Here truncation means boundary condition $C_{nl}=0$, $n>N_{max}$ in the n-space.

For states of continuous spectra (including near-barrier resonances) the so-called J-matrix method is applied (Yamani, Fishman, 1975).

$$\sum_{n=0}^{N-1}\left(\left\langle \phi_{0lm}\left|H\right|\phi_{nlm}\right\rangle - E\delta_{0n}\right)C_{nl} = -\sum_{n=N}^{\infty}\left(\left\langle \phi_{0lm}\left|H\right|\phi_{nlm}\right\rangle - E\delta_{0n}\right)C_{nl}^{(as)},$$

.....,

.....,

$$\sum_{n=0}^{N-1}\left(\left\langle \phi_{Nlm}\left|H\right|\phi_{nlm}\right\rangle - E\delta_{Nn}\right)C_{nl} = -\sum_{n=N}^{\infty}\left(\left\langle \phi_{Nlm}\left|H\right|\phi_{nlm}\right\rangle - E\delta_{Nn}\right)C_{nl}^{(as)}.$$

where:

$$C_{nl}^{(as)} = \int f^{(as)}(\rho)\,\phi_{nl}(\rho)\,\rho^2 d\rho,$$

the asymptotic function takes the form

$$f^{(as)}(\rho) \sim [(-1)^{l+1}e^{-ik\rho} + Se^{ik\rho}]$$

and the matrix of the Hamiltonian turns out to be degenerated to the three-diagonal matrix elements of the kinetic energy.

EXAMPLES

ASYMPTOTIC NORMALIZATION COEFFICIENT FOR LOOSELY-BOUND STATE OF ^{17}F NUCLEUS

Asymptotic normalization coefficient D_l is the factor, determining the amplitude of asymptotics of the wave function. The closed channel ^{16}O + p, E_p= -104.94 keV is studied. The asymptotic is expressed through the Wittaker function:

$$\phi_l(\rho) \to D_l W_{-\eta,l+1/2}(2k\rho)/\rho; \quad \eta = Z_1 Z_2 e^2 \mu/\hbar^2 k.$$

The coefficients C_{nl} obtained in an ordinary variational calculation are compared with the asymptotic ones at $n \to \infty$

$$C_n = C_n^{(as)} = \sqrt{r_0}\left[4/(2n+3)\right]^{1/4} W_{-\eta,l+1/2}(2k\rho_n) D_l,$$

where $\rho_n = r_0\sqrt{2n+3}; \quad r_0 = \sqrt{\hbar/\mu_N\omega}$ –

the turning point of the oscillator wave function $\phi_{nl}(\rho)$.

The following potential is used:

$$V(\rho) = -V_0 \left\{1 + \exp\left[\left(\rho - R_0\right)/a\right]\right\}^{-1} + V_c(\rho),$$

$$V_c(\rho) = \begin{cases} (4\alpha_e \hbar c / R_c)(3 - \rho^2 / R_c^2), & \rho < R_c; \\ 8\alpha_e \hbar c / \rho, & \rho > R_c \end{cases}$$

$$R_0 = 3.29\ fm; \quad a = 0.65\ fm; \quad R_c = 3.48\ fm$$

The eigenvalues of the overlap kernel are:

$$\varepsilon_n = 1 + (-1)^n (17n - 1)/16^n,$$

n	0	2	4	6
ε_n	0	1.128906	1.001022	1.000006

The following values of D_l are obtained in the two-body model, OCM with the local potential renormalized by the overlap kernel and OCM without such a renormalization:

Alternative	V_0, MeV	D_l, $fm^{-1/2}$
TBM	49.24	83.33
OCM1	47.61	94.18
OCM2	53.57	92.40

ALPHA-DECAY OF ^{8}Be 91.8 keV 0^{+} RESONANCE

Various versions of two-body, OCM and RGM dynamics of the α-α system bringing the proper resonance energy after fitting of the depth of the potential well were analyzed:

1. Two-body model with forbidden eigenstates of Hamiltinian. Buck potential (no fitting).

$$V(\rho) = V_0 \exp(-b\rho^2) + V_{coul}$$

$$V_0 = 122.6 MeV; b = 0.22\ fm^{-2}$$

E=91.10 keV; Γ=5.5 eV. Experiment - E=91.84 keV; Γ=6.8 eV.

2. The same two-body model with oscillator forbidden states.

$$V_0 = 116.9 MeV; b = 0.22\ fm^{-2}$$

E=91.84 keV; Γ=5.8 eV.

3. RGM. Hasegava-Nagata NN-potential (no fitting).

E=91.84 keV; Γ=4.9 eV.

4. RGM. Gaussian NN-potential.

$$V_{NN}(r_{NN}) = V_0 \exp(-a\rho^2) + V_{coul}$$

$$V_0 = 77.27 MeV; a = 0.943\, fm^{-2}$$

result in the same width of alpha-alpha folding potential as in the case 1.

E=91.85 keV; Γ=3.9 eV.

The depth of the folding potential here is V_0 = 139.4 *MeV* i. e. the exchange terms are repulsive.

5. OCM with RGM-projected kinetic energy operator.

$$V_0 = 136.1 MeV; b = 0.22\, fm^{-2}$$

E=91.84 keV; Γ=4.7 eV.

WIDTH OF THE LOWEST $^{16}O+^{16}O$ RESONANCE STATE

Three alternatives: the OCM with the forbidden states being eigenstates of the two-body Hamiltonian – I, the OCM with the forbidden states being eigenstates of the overlap kernel – II, and the OCM with the forbidden states of such a type and the semi-forbidden states – III; and three sets of the parameters of the local potential

$$V_{cl}(r) = V_{Coul}(r) + V_0 / \left(1 + \exp\left[(r - R)/a\right]\right)^2$$

with different number of the bound states n_0 = 12, 13, 14 from (Y.Kondo. In: Proc. of Int. Conf. “Microscopic Cluster Models of Light Nuclei and Related Topics” Yukawa Inst. Theor. Phys., 1992, p.191) are considered. For the alternatives II and III the depth V_0 of local potential V_{cl} was varied to reproduce the resonance energy of the alternative I).

$n_0 = 12$

Alternative	I	II	III
V_0, MeV	399.2	225.6	422.8
E_{res}, MeV	2.103	2.103	2.102
Γ, MeV	$0.59 \cdot 10^{-27}$	$0.53 \cdot 10^{-28}$	$0.64 \cdot 10^{-35}$

$n_0 = 13$

Alternative	I	II	III
V_0, MeV	668.0	631.0	679.7
E_{res}, MeV	2.981	2.981	2.981
Γ, MeV	$1.9 \cdot 10^{-19}$	$1.6 \cdot 10^{-19}$	$4.1 \cdot 10^{-27}$

$n_0 = 14$

Alternative	I	II	III
V_0, MeV	998.3	993.5	1124.8
E_{res}, MeV	6.233	6.233	6.232
Γ, MeV	$3.2 \cdot 10^{-6}$	$3.2 \cdot 10^{-6}$	$3.1 \cdot 10^{-12}$

OUTLOOK OF THE APPROACH FOR THE THEORY OF ALPHA- AND CLUSTER DECAY

Introduction of the new version of the cluster form-factor (Fliessbasch, Mang, 1976):

$$\Phi_{(new)}(\rho) = \hat{N}_{\rho}^{-1/2} \left\langle \Psi_A \left| \hat{A}\{\Psi_{A_1}\Psi_{A_2}\,\delta(\rho-\rho')\} \right\rangle \right.$$

in contrast to the traditional one (Mang, 1957):

$$\Phi_{(tr)}(\rho) = \left\langle \Psi_A \left| \hat{A}\{\Psi_{A_1}\Psi_{A_2}\,\delta(\rho-\rho')\} \right\rangle \right.$$

result in a significant increasing of alpha- and cluster decay widths. In combination with the enhancement of the processes caused by a configuration mixing (superfluid, etc.) the overestimated widths are widely met. What is the matter? The presented above effect of suppression is not taken into account.

ONE-FERMION + COMPOSITE SYSTEM PROBLEM

The eigenvalues of the overlap kernel for ^{16}O+N pair are:

n	0	2	4	6
ε_n	0	1.128906	1.001022	1.000006

The eigenvalues of the overlap kernel for ^{40}Ca+N pair are:

n	0	2	4	6
ε_n	0	1.050625	1.000064	1.000000

These examples are related to the sharp Fermi surface.
The origin of the superallowed terms is the recoil effect.

For a heavy core the effect is inessential. However this property make it possible to solve a more general problem, namely the fermion system with diffuse Fermi surface. Consider the core (A_1-fermion) Hamiltonian:

$$\hat{H} = \hat{H}_{s.c.} + \hat{H}_{corr.}.$$

In the secondary quantization representation its lower eigenfunction has the form:

$$\Psi_{A_1} = \prod_s \nu_s \hat{a}_s^+ \Psi_0,$$

Where Ψ_0 is the vacuum wave function of the Hamiltonian $H_{s.c}$. In that case the one- fermion wave functions of this Hamiltonian are the eigenfunctions of the overlap kernel of the A_1+F system with the eigenvalues:

$$\varepsilon_s = 1 - \nu_s.$$

So all presented equations are valid for the discussed problem and may be used to describe proton decay and also in neutron and atomic physics.

CONCLUSIONS

1. Properties of the interaction of composite particles are essentially different from the ones of the elementary particles.
2. The main origin of the differences is the exchange effects.
3. Algebraic approaches are convenient tools for taking these effects into account.
4. Developed methods of description of the composite particles interaction are applicable to the calculations of:
   a) near-threshold bound cluster-nucleus states,
   b) phase shifts and cross-section of composite particle interaction including calculations in the optical model,
   c) resonance states of various cluster-cluster pairs,
   d) amplitudes of entrance and exit channels of various reactions.

5. It is required that both clusters should be SU(3)-scalars and one of them should be SU(4)-scalar. Otherwise a channel coupling appears due to antisymmetrization.

6. Interaction of a fermion (nucleon, electron etc.) with a heavy system is an exclusion. Such a system may be treated being non-scalar, possess a non-oscillator WF and diffuse Fermi surface.

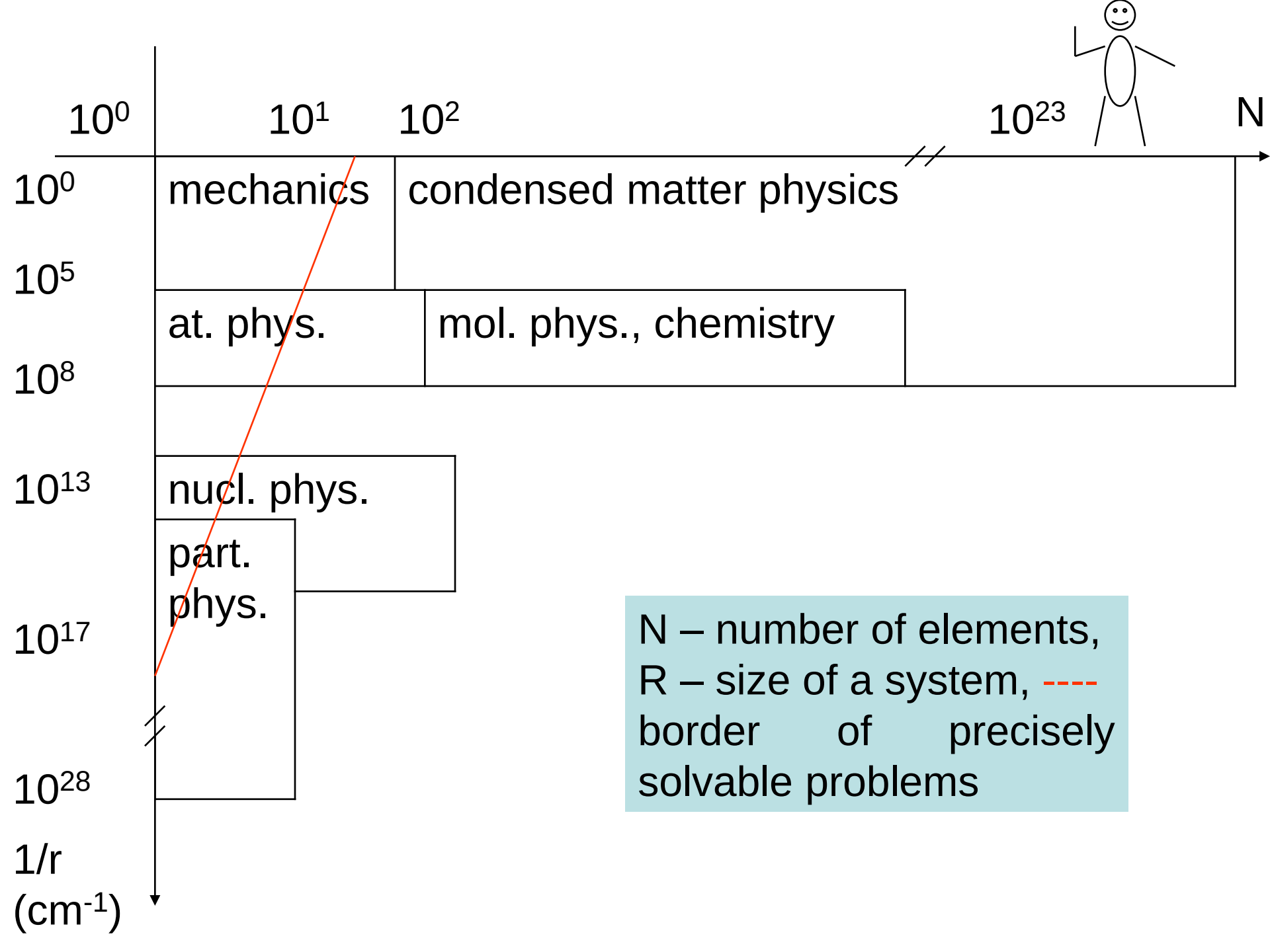
7. The effect of the semi-forbidden states is drastic when the widths of narrow resonances in interaction of a heavy nucleus with the alpha-particle or the interaction of two heavy clusters are calculated.

PREFACE. CRITERIA OF PROMISING BRANCHES OF THEORETICAL NUCLEAR PHYSICS

1. Description of a promising experiments.

2. High-quality justification of previous theoretical results.

3. Development of methods workable beyond the nuclear physics area.


10^0
10^1
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10^0
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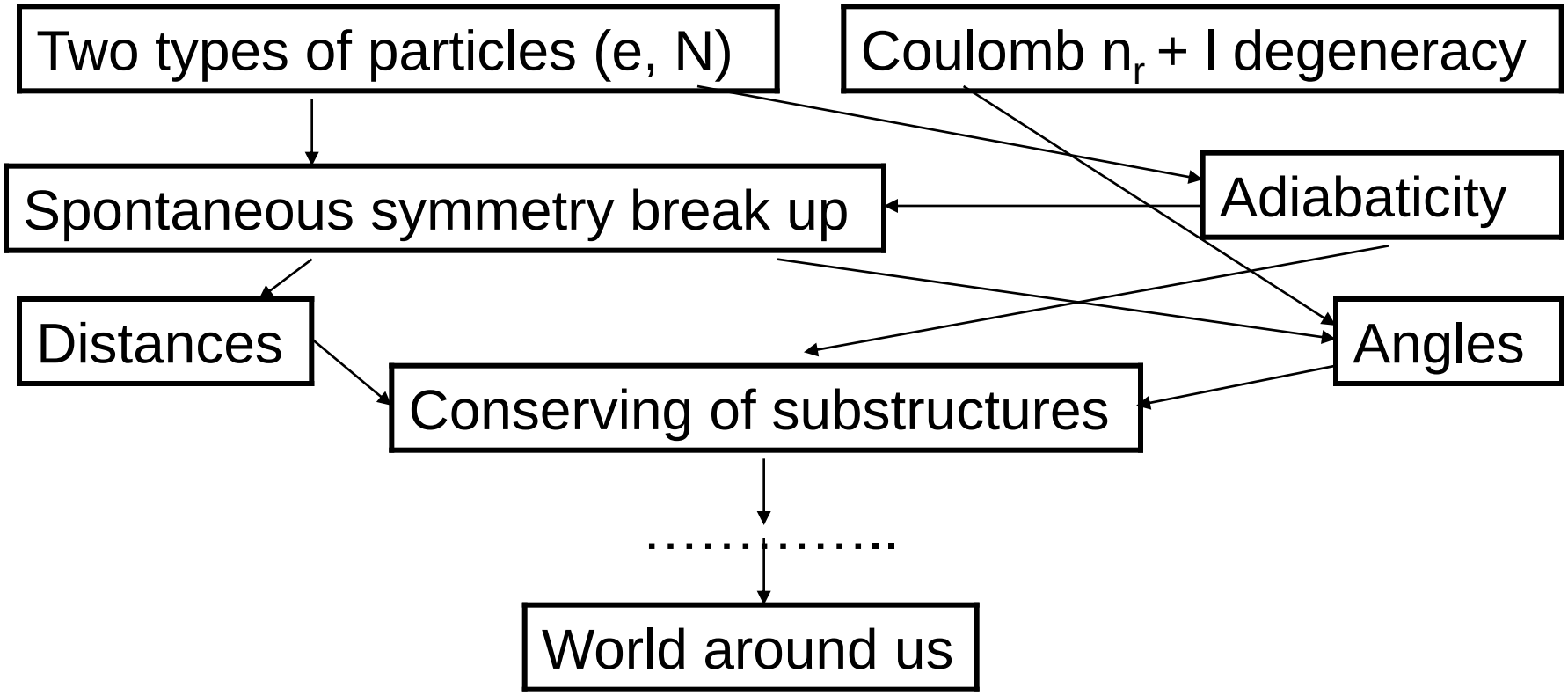
10^8
10^{13}
10^{17}
10^{28}
1/r
(cm^{-1})
mechanics
condensed matter physics
at. phys.
mol. phys., chemistry
nucl. phys.
part. phys.
N – number of elements,
R – size of a system, ---- border of precisely solvable problems

PROBLEM OF STRUCTURING

How to state the problem of substructures (clusters) in two- (or few-) cluster system taking into account indistingvishibility of identical fermions and the strong nuclear interaction?

STRUCTURING OF MOLECULES



ALGEBRAIC VERSION OF THE ORTHOGONALITY CONDITIONS MODEL

The sole other term of AV RGM equation containing fermion exchange operators is potential energy kernel V_ρ. It is just the term which makes RGM a non-universal and overcomlicated model The idea is to consider it phenomenologically via approximating by a local potential. Due to the algebraic original form of the kinetic energy term the approach in which the equation

$$\left(\hat{\tilde{T}}_\rho + \hat{V}(\rho) - E'\right)\phi(\rho) = 0$$

is solved is called AV OCM independently of solution methods which may be algebraic ones or methods of the infinite mathematics. In the oscillator basis the renormalized kinetic energy operator remains three-diagonal form of initial one.

then the width Γ can be extracted from the relation (Kadmensky, Furman, 1985):

$$\chi(R) = \sqrt{\frac{\Gamma k}{2E_{res}}} G_l(\eta, kR)$$

For this type of the resonances the amplitude of the WF in the exterior region is many orders of magnitude smaller than in the interior region therefore in algebraic calculations of the resonant eigenvector χ_{oscill} one can approximately put $C_{nl}=0$, $n>N_{max}$ for them.

The problem of AV OCM is that calculated by this way function correctly describes the interior region does not tend to $G_l(\eta,kr)$ and hence not applicable to calculation of the width. To improve the behavior of WF in the exterior region it is necessary to increase significantly N_{max}, which caused huge numerical difficulties. Thus a delicate problem appears.

RESONANCE STATES IN THE AV OCM

Boundary condition for the resonant state:

$$\chi_{res}(r) \approx G_l(\eta, kr) + iF_l(\eta, kr), \quad as \quad r \to \infty$$

F_l , G_l – regular and irregular Coulomb wave functions. Under Coulomb barrier G_l (η,kr) – increases, F_l (η,kr) – decreases, as $r \to 0$.

Thus for a ***narrow resonance***, in which these tendencies are very strong, E_{res} can be determined by matching of $\chi_0(r)$ and $G_l(\eta,kr)$ under barrier, at the point, where the nuclear part of interaction can be neglected (χ_0 – regular at r=0 solution) and F_l (η,kr) << G_l (η,kr). This approach is rather suitable for local potentials. Width can be calculated as follows. Let the function of resonant state to be normalized to unity in the interior region wide enough for the function turns out to be negligibly small at R:

$$\int_0^R \chi^2(r) dr = 1,$$

$$\begin{pmatrix} < f_n | \\ < f_n | V_{cl} \end{pmatrix} \times \left\{ \begin{array}{l} \chi(r) = 2\sum_{n=0}^{n_0} \int_0^\infty dr' G(r,r') f_n(r') \langle f_n | V_{cl} | \chi \rangle + \\ +2 \left(\begin{array}{l} \int_0^\infty dr' G(r,r') f_{n_0}(r') T_{n_0, n_0+1} + \\ \int_0^\infty dr' G(r,r') f_{n_0+2}(r') \left(1 - \sqrt{\frac{\varepsilon_{n_0+1}}{\varepsilon_{n_0+2}}} \right) T_{n_0+1, n_0+2} \end{array} \right) \langle f_{n_0+1} | \chi \rangle + \\ +2 \sum_{n=n_0+2}^{n_{\max}-1} \left(\int_0^\infty dr' G(r,r') f_{n-1}(r') \left(1 - \sqrt{\frac{\varepsilon_{n-1}}{\varepsilon_n}} \right) T_{n-1,n} + \right. \\ \left. + \int_0^\infty dr' G(r,r') f_{n+1}(r') \left(1 - \sqrt{\frac{\varepsilon_n}{\varepsilon_{n+1}}} \right) T_{n,n+1} \right) \langle f_n | \chi \rangle + \\ +2 \int_0^\infty dr' G(r,r') f_{n_{\max}-1}(r') \left(1 - \sqrt{\frac{\varepsilon_{n_{\max}-1}}{\varepsilon_{n_{\max}}}} \right) T_{n_{\max}-1, n_{\max}} \langle f_{n_{\max}} | \chi \rangle \end{array} \right. \chi_{oscill}$$

where n_{max} determines the limitation of the truncated basis. The simple form of the first term appears due to the property of completeness of the oscillator basis allowing one to express the sum over n' explicitly.

It is convenient to rewrite the integral equation to the algebraic form. Multiplying it by $<f_n|$ and $<f_n|V_{cl}$ from the left one can obtain the set of homogeneous algebraic equations for the unknown coefficients $<f_n|V_{cl}|\chi>$ and $< f_n|\ \chi\ >$. The condition of solvability (zeroing of the determinant) yields E_{res}, after that the coefficients $< f_n|\ \chi\ >$ are calculated. This procedure determines the function $\chi(r)$ and the width of the resonance Γ.

However such a type of numerical calculations of the widths turns out to be unstable at least for very narrow resonances of systems possessing a number of semi-forbidden states with the eigenvalues of the overlap kernel ε_n strongly different from 1. In particular, very high accuracy (ten digits for 2 MeV resonance in 16O+16O system) of the value E_{res} is required to calculate the width reliably.

The way to circumvent this difficulty is the following Consider the mentioned above function obtained by direct algebraic approach:

$$\chi_{oscill}(r) = \sum_{n=n_0+1}^{n_{\max}} C_n f_n(r)$$

where

$$\left\{C_{n_0+1},\ldots, C_{n_{\max}}\right\}$$

is the eigenvector corresponding to the resonance eigenvalue E_{res}. The function χ_{oscill} reproduces precisely the interior behavior of the wave function. It is just one needs to calculate the values $< f_n|V_{cl}|\ \chi >$ and $< f_n|\ \chi >$, due to decreasing of $V_{cl}(r)$ and $f_n(r)$ ($n \le \gamma_0$) as r – increasing. Thus, substituting $\boldsymbol{\chi_{oscill}}$ in the right side of the basic equation one can obtain the solution for all the values of r, including the asymptotic region.

Numerical calculations by means of the proposed method are significantly more stable.