

Chiral EFT and Nuclear Forces: Are We in Trouble?

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Abstract

During the past two decades, chiral effective field theory has become a popular tool to derive nuclear forces from first principles. Two-, three-, and four-nucleon forces have been calculated up to next-to-next-to-next-to-leading order ($N^3\text{LO}$) and (partially) applied in nuclear few- and many-body systems — with, in general, a good deal of success. But in spite of these achievements, we are still faced with some great challenges. Among them is the issue of a proper renormalization of the two-nucleon potential, which is highly controversial in the community. Another issue are the subleading many-body forces, where we are faced with an “explosion” of the number of terms with increasing order that no practitioner can ever handle. I will comment on the current status and will provide hints for how to deal with it.

Keywords: *Low-energy QCD; chiral effective field theory; nucleon-nucleon scattering; few-nucleon forces*

1 Introduction

The problem of a proper derivation of nuclear forces is as old as nuclear physics itself, namely, almost 80 years [1]. The modern view is that, since the nuclear force is a manifestation of strong interactions, any serious derivation has to start from quantum chromodynamics (QCD). However, the well-known problem with QCD is that it is non-perturbative in the low-energy regime characteristic for nuclear physics. For many years this fact was perceived as the great obstacle for a derivation of nuclear forces from QCD — impossible to overcome except by lattice QCD.

The effective field theory (EFT) concept has shown the way out of this dilemma. For the development of an EFT, it is crucial to identify a separation of scales. In the hadron spectrum, a large gap between the masses of the pions and the masses of the vector mesons, like $\rho(770)$ and $\omega(782)$, can clearly be identified. Thus, it is natural to assume that the pion mass sets the soft scale, $Q \sim m_\pi$, and the rho mass the hard scale, $\Lambda_\chi \sim m_\rho \sim 1$ GeV, also known as the chiral-symmetry breaking scale. This is suggestive of considering a low-energy expansion arranged in terms of the soft scale over the hard scale, $(Q/\Lambda_\chi)^\nu$, where Q is generic for an external momentum (nucleon three-momentum or pion four-momentum) or a pion mass. The appropriate degrees of freedom are, obviously, pions and nucleons, and not quarks and gluons. To make sure that this EFT is not just another phenomenology, it must have a firm link with QCD. The link is established by having the EFT to observe all relevant symmetries of the underlying theory, in particular, the broken chiral symmetry of low-energy QCD [2].

The early applications of chiral perturbation theory (ChPT) focused on systems like $\pi\pi$ and πN , where the Goldstone-boson character of the pion guarantees that the

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<http://www.ntse-2014.khb.ru/Proc/Machleidt.pdf>.

expansion converges. But the past 20 years have also seen great progress in applying ChPT to nuclear forces (see Refs. [3,4] for recent reviews and find comprehensive lists of references therein). As a result, nucleon-nucleon (NN) potentials of high precision have been constructed, which are based upon ChPT carried to next-to-next-to-next-to-leading order ($N^3\text{LO}$) [5,6], and applied in nuclear structure calculations with great success.

However, in spite of this progress, we are not done. Due to the complexity of the nuclear force issue, there are still many subtle and not so subtle open problems. We will not list and discuss all of them, but just mention two, which we perceive as the most important ones:

- The proper renormalization of chiral nuclear potentials and
- Subleading chiral few-nucleon forces.

I discussed the renormalization issue in my contribution to the NTSE-2013 [7], where the interested reader will also find a broad introduction into the topic of nuclear interactions. In this contribution, I will focus on nuclear many-body forces.

2 The chiral NN potential

In terms of naive dimensional analysis or “Weinberg counting”, the various orders of the low-energy expansion which define the chiral NN potential, are given by:

$$V_{\text{LO}} = V_{\text{ct}}^{(0)} + V_{1\pi}^{(0)}, \quad (1)$$

$$V_{\text{NLO}} = V_{\text{LO}} + V_{\text{ct}}^{(2)} + V_{1\pi}^{(2)} + V_{2\pi}^{(2)}, \quad (2)$$

$$V_{\text{NNLO}} = V_{\text{NLO}} + V_{1\pi}^{(3)} + V_{2\pi}^{(3)}, \quad (3)$$

$$V_{\text{N}^3\text{LO}} = V_{\text{NNLO}} + V_{\text{ct}}^{(4)} + V_{1\pi}^{(4)} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)}, \quad (4)$$

where the superscript denotes the order ν of the expansion. LO stands for leading order, NLO for next-to-leading order, etc. Contact potentials carry the subscript “ct” and pion-exchange potentials can be identified by an obvious subscript.

NN potentials have been constructed at all of the above orders, and it has been shown [5] that at $N^3\text{LO}$ the precision is finally achieved, which is necessary and sufficient for reliable applications in *ab initio* nuclear structure calculations. Thus, the NN problem appears to be under control, at least for the time being.

3 Nuclear many-body forces

The chiral two-nucleon force (2NF) at $N^3\text{LO}$ has been applied in microscopic calculations of nuclear structure with, in general, a great deal of success. However, from high-precision studies conducted in the 1990s, it is well-known that certain few-nucleon reactions and nuclear structure issues require three-nucleon forces (3NFs) for their microscopic explanation. Outstanding examples are the A_y puzzle of N - d scattering and the ground state of ^{10}B . *An important advantage of the EFT approach to nuclear forces is that it creates two- and many-nucleon forces on an equal footing.* In this section, I will now explain in some detail those chiral three- and four-nucleon forces.

3.1 Three-nucleon forces

The order of a 3NF is given by

$$\nu = 2 + 2L + \sum_i \Delta_i, \quad (5)$$

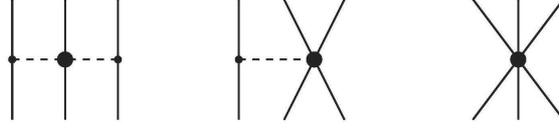


Figure 1: The three-nucleon force at NNLO. From left to right: 2PE, 1PE, and contact diagrams. Solid lines represent nucleons and dashed lines pions. Small solid dots denote vertices of index $\Delta_i = 0$ and large solid dots are $\Delta_i = 1$.

where L denotes the number of loops and Δ_i is the vertex index. We will use this formula to analyze 3NF contributions order by order.

3.1.1 Next-to-leading order.

The lowest possible power is obviously $\nu = 2$ (NLO), which is obtained for no loops ($L = 0$) and only leading vertices ($\sum_i \Delta_i = 0$). As it turns out, the contribution from these NLO diagrams vanishes. So, the bottom line is that there is no genuine 3NF at NLO. The first non-vanishing 3NF appears at NNLO.

3.1.2 Next-to-next-to-leading order.

The power $\nu = 3$ (NNLO) is obtained when there are no loops ($L = 0$) and $\sum_i \Delta_i = 1$; i.e., $\Delta_i = 1$ for one vertex while $\Delta_i = 0$ for all other vertices. There are three topologies which fulfill this condition, known as the two-pion exchange (2PE), one-pion exchange (1PE), and contact graphs (Fig. 1).

The 1PE and contact 3NF terms involve each a new parameter, which are commonly denoted by D and E and which do not appear in the 2N problem. There are many ways to pin these two parameters down. The triton binding energy and the nd doublet scattering length $^2a_{nd}$ have been used for this purpose. But one may also choose the binding energies of ^3H and ^4He , an optimal over-all fit of the properties of light nuclei, or electroweak processes like the tritium β decay. Once D and E are fixed, the results for other 3N, 4N, etc. observables are predictions.

The 3NF at NNLO has been applied in calculations of few-nucleon reactions, structure of light- and medium-mass nuclei [8,9], and nuclear and neutron matter [10–12] with a good deal of success. Yet, the famous ‘ A_y puzzle’ of nucleon-deuteron scattering is not resolved. When only 2NFs are applied, the analyzing power in p - ^3He scattering is even more underpredicted than in p - d . However, when the NNLO 3NF is added, the p - ^3He A_y substantially improves (more than in p - d) [13] — but a discrepancy remains. Furthermore, the spectra of light nuclei leave room for improvement.

To summarize, the 3NF at NNLO is a remarkable contribution: It represents the leading many-body force within the scheme of ChPT; it includes terms that were advocated already some 50 years ago; and it produces noticeable improvements in few-nucleon reactions and the structure of light nuclei. But unresolved problems remain. Moreover, in the case of the 2NF, we have pointed out that one has to proceed to N^3LO to achieve sufficient accuracy. Therefore, the 3NF at subleading order is needed for at least two reasons: for consistency with the 2NF and to hopefully resolve outstanding problems in microscopic nuclear structure and reactions.

3.1.3 Next-to-next-to-next-to-leading order.

At N^3LO , there are loop and tree diagrams. For the loops (Fig. 2), we have $L = 1$ and, therefore, all Δ_i have to be zero to ensure $\nu = 4$. Thus, these one-loop 3NF diagrams can include only leading order vertices, the parameters of which are fixed from πN and NN analysis. The long-range part of the chiral N^3LO 3NF has been tested in the triton [14] and in three-nucleon scattering [15] yielding only moderate effects and

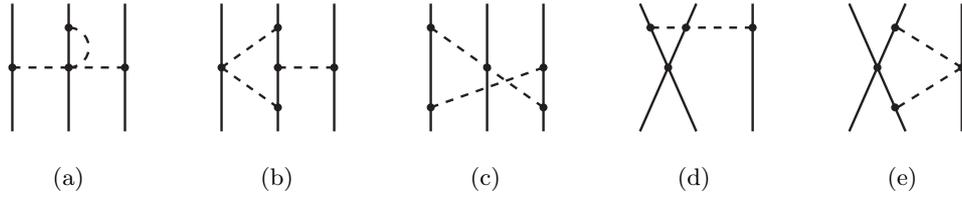


Figure 2: Leading one-loop 3NF diagrams at N^3LO . We show one representative example for each of five topologies, which are: (a) 2PE, (b) 1PE-2PE, (c) ring, (d) contact-1PE, (e) contact-2PE. Notation as in Fig. 1.

no improvement of the A_y puzzle. The long- and short-range parts of this force have been used in neutron matter calculations (together with the N^3LO 4NF) producing surprisingly large contributions from the 3NF [16]. Thus, the ultimate assessment of the N^3LO 3NF is still outstanding and will require more few- and many-body applications. But we expect that, overall, the 3NF at N^3LO is small and will most likely not solve the outstanding problems.

3.1.4 The 3NF at N^4LO .

Because the 3NF at N^3LO is presumably small, it is necessary to move on to the next order of 3NFs, which is N^4LO or $\nu = 5$ (of the Δ -less theory which we have silently assumed so far). The loop contributions that occur at this order are obtained by replacing in the N^3LO loops one vertex by a $\Delta_i = 1$ vertex (with LEC c_i), Fig. 3, which is why these loops may be more sizable than the N^3LO loops. The 2PE, 1PE-2PE, and ring topologies have been evaluated [17]. Note that each diagram in Fig. 3 stands symbolically for a group of diagrams. We demonstrate this for the 1PE-2PE topology, for which we display in Fig. 4 all diagrams for that topology. This applies to each topology and, thus, provides an idea of the ‘‘explosion’’ of 3NF contributions at subleading orders.

In addition to the loops, we have at N^4LO three ‘tree’ topologies (Fig. 5), which include a new set of 3N contact interactions, which have recently been derived by the Pisa group [18]. Contact terms are typically simple (as compared to loop diagrams) and their coefficients are essentially free. Therefore, it would be an attractive project to test some terms (in particular, the spin-orbit terms) of the N^4LO contact 3NF [18] in calculations of few-body reactions (specifically, the p - d and p - 3He A_y) and spectra of light nuclei.

3.2 Four-nucleon forces

For four-nucleon forces (4NFs), the power is given by

$$\nu = 4 + 2L + \sum_i \Delta_i. \quad (6)$$

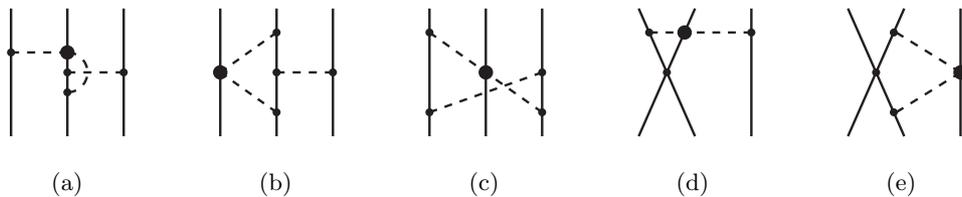


Figure 3: Sub-leading one-loop 3NF diagrams which appear at N^4LO . We show one representative example for each of five topologies, which are: (a) 2PE, (b) 1PE-2PE, (c) ring, (d) contact-1PE, (e) contact-2PE. Notation as in Fig. 1.

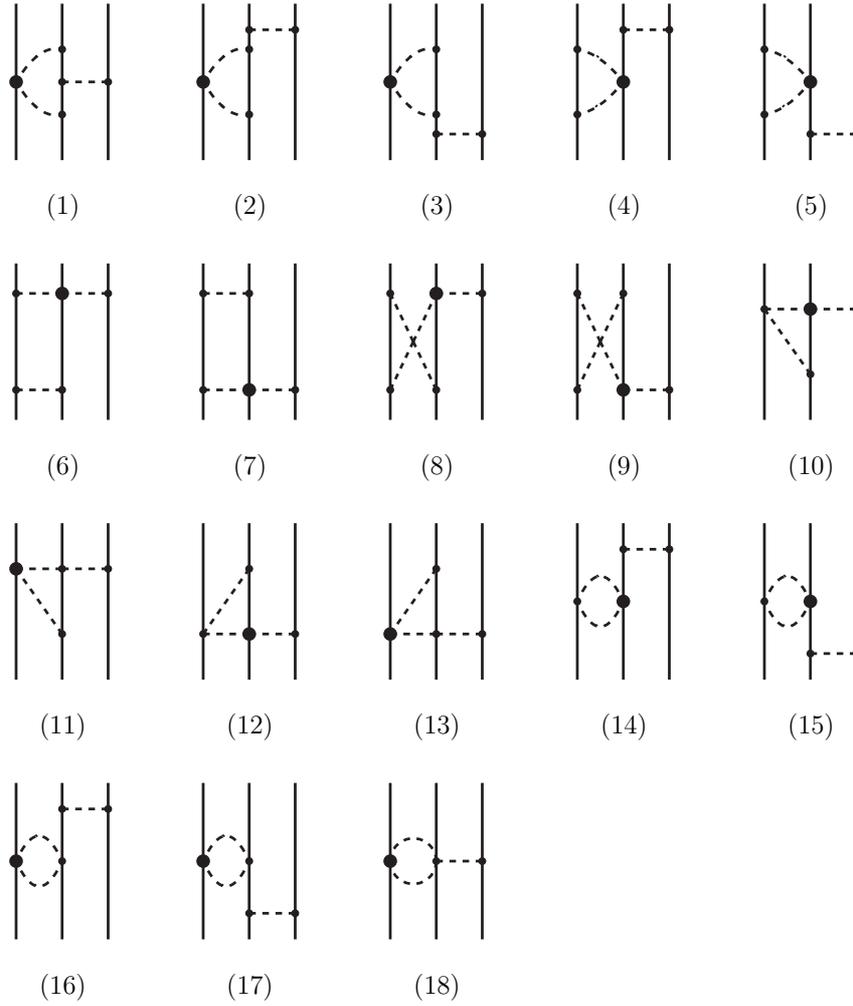


Figure 4: The topology (b) of Fig. 3 (1PE-2PE) in detail. Notation as in Fig. 1.

Therefore, a 4NF appears for the first time at $\nu = 4$ (N^3LO), with no loops and only leading vertices, Fig. 6. This 4NF includes no new parameters and does not vanish [19]. It has been applied in a calculation of the 4He binding energy, where it was found to contribute a few 100 keV [20]. It should be noted that this preliminary calculation involves many approximations, but it provides an idea of the order of magnitude of the 4NF, which is indeed small as compared to the full 4He binding energy of 28.3 MeV.

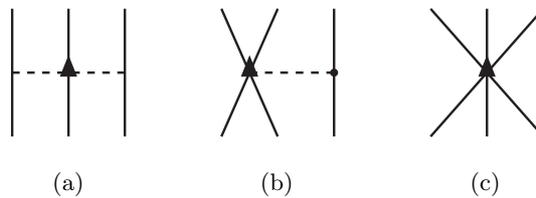
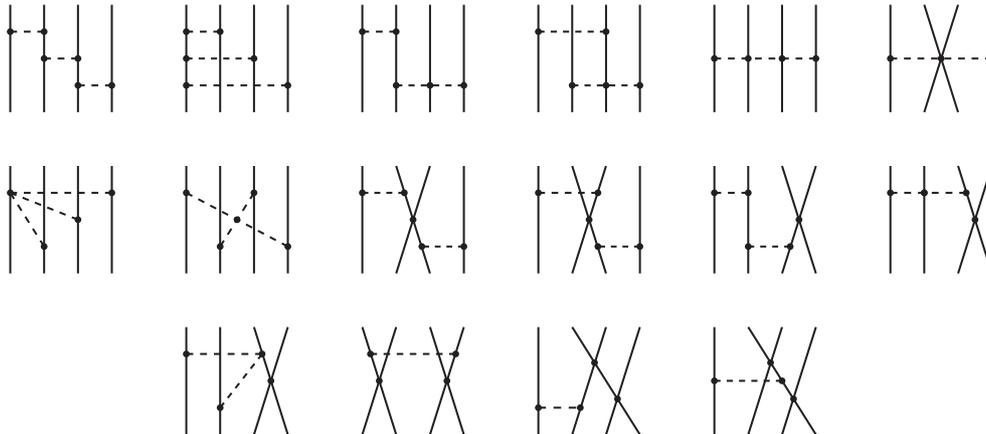


Figure 5: 3NF tree graphs at N^4LO ($\nu = 5$) denoted by: (a) 2PE, (b) 1PE-contact, and (c) contact. Solid triangles represent vertices of index $\Delta_i = 3$. Other notation as in Fig. 1.

Figure 6: Leading four-nucleon force at $N^3\text{LO}$.

4 Conclusions

The past 20 years have seen great progress in our understanding of nuclear forces in terms of low-energy QCD. Key to this development was the realization that low-energy QCD is equivalent to an effective field theory (EFT) which allows for a perturbative expansion that has become known as chiral perturbation theory (ChPT). In this framework, two- and many-body forces emerge on an equal footing and the empirical fact that nuclear many-body forces are substantially weaker than the two-nucleon force is explained naturally.

In this contribution, I have focused mainly on nuclear many-body forces based upon chiral EFT. The 3NF at NNLO has been known for a while and applied in few-nucleon reactions, structure of light- and medium-mass nuclei, and nuclear and neutron matter with some success. However, the famous ‘ A_y puzzle’ of nucleon-deuteron scattering is not resolved by the 3NF at NNLO. Thus, one important open issue are the few-nucleon forces beyond NNLO (“sub-leading few-nucleon forces”) which, besides the A_y puzzle, may also resolve some important outstanding nuclear structure problems. As explained, this may require going even beyond $N^3\text{LO}$. However, as demonstrated, with each higher order, the number of diagrams increases enormously. Thus, practitioners are faced with the problem of how to deal with this explosion of 3NF contributions. My advice is that, for a while, one should not aim at complete calculations at given higher orders. Rather one will have to be selective and try to identify the more important 3NF terms in the “forrest” of diagrams. The $N^4\text{LO}$ 3NF contact terms [Fig. 5(c)] [18] are a promising and manageable starting point.

Finally, let me note that, because of lack of space, I have discussed here only the so-called Δ -less version of ChPT. There is also the Δ -full version (see Ref. [3] for details), in which the number of diagrams is even larger.

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