Three-nucleon calculations within the Bethe-Salpeter approach with separable kernel

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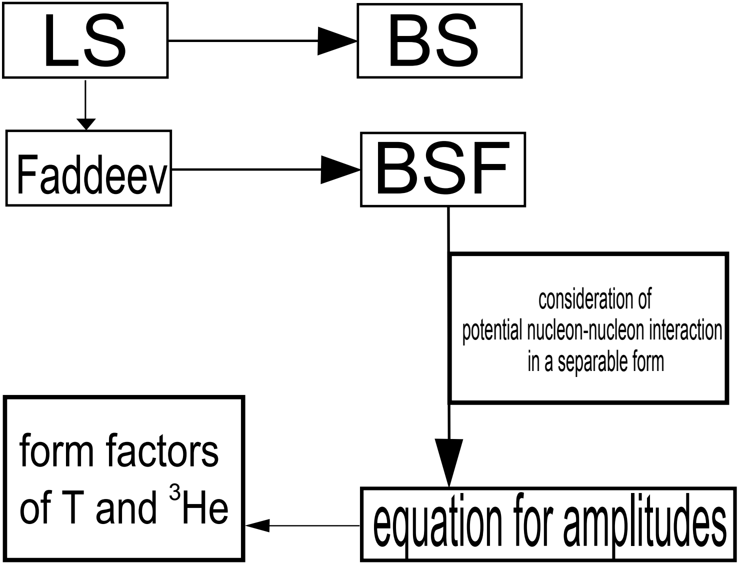
Three-body calculation in nuclear physics are very interesting for describing
three-nucleon bound states (^3He, T)
hadron-deuteron reactions (for example, $pd \to pd$, $pd \to ppn$ and so on)
Reactions at high momentum transfer require to use relativistic methods (such as Bethe-Salpeter formalism).

In this formalism was obtained three-nucleon Faddeev equation

Research in schematic form



Basic approximations:

- equality of masses of nucleons $m_n = m_p$
- scalar propogators for nucleons

$$G(p) = [(p)^2 - m^2]^{-1}$$

- two-particle interaction

$$V(x_1, x_2, x_3) = V(x_1, x_2) + V(x_2, x_3) + V(x_1, x_3)$$

Two-particle Bethe-Salpeter equation has the following form

$$T(p,p^{'};s) = V(p,p^{'}) + \frac{i}{4\pi^3}\int d^4k V(p,k)G(k;s)T(k,p^{'};s)$$

Where

$$G(k;s) = [(\frac{1}{2}P+k)^2 - m^2 + i\epsilon]^{-1}[(P/2-k)^2 - m^2 + i\epsilon]^{-1}$$

is the free two-paticle Green's function
$T(p,p^{'};s)$ two-particle T matrix
$V(p,p^{'})$ potential of the nucleon-nucleon interaction

Separable kernel

The nucleon-nucleon kernel is chosen to be in the separable form.

$$V(p,p^{'}) = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p^{'})$$

For the case of the rank one it has the form:

$$V(p,p^{'}) = \lambda g(p) g(p^{'})$$

Yamaguchi-type functions for the form factors:

$$g_Y(p_0,p) = \frac{1}{-p_0^2 + p^2 + \beta^2 + i\epsilon}.$$

Separable Ansatz for interaction V (rank one)

$$V(p,p') = \lambda g(p) g(p')$$

$$\Downarrow$$

$$T(p,p';s) = \tau(s) g(p) g(p')$$

$$\tau(s) = [\frac{1}{\lambda} - \frac{i}{4\pi^3}\int_{-\infty}^{\infty} dk^0 \int_0^{\infty} k^2 dk g^2(k^0,k) G(k^0,k;s)]^{-1}$$

$$T_L(\bar{p}) = T_L(0,\bar{p},0,\bar{p};s) = \frac{-8\pi\sqrt{s}}{\bar{p}} e^{i\delta_L(\bar{p})} \sin\delta_L(\bar{p})$$

with scattering phase shift $\delta_L(\bar{p})$,
$\bar{p} = \sqrt{s/4 - m_n^2} = \sqrt{\frac{1}{2} m T_{lab}}$.

Rank-one covariant Yamaguchi-function

$$g(p_0, |p|) = \frac{1}{-p_0^2 +^2 + \beta^2 - i\epsilon}$$

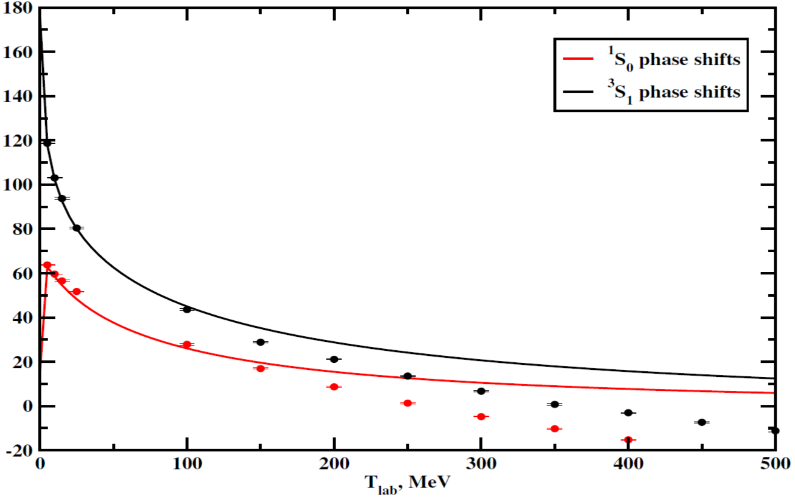
Parameters of the kernels

	$^3S_1$	$^1S_0$
λ (GeV^4)	-3.15480	-1.12087
β (GeV)	0.279731	0.287614

Properties of the proton-neutron scattering and deuteron

	$^3S_1$	exp.	$^1S_0$	exp.
a (fm)	5.424	5.424(4)	-23.748	-23.748(10)
r_0 (fm)	1.775	1.759(5)	2.75	2.75(5)
E_d (MeV)	2.2246	2.224644(46)		

WF



180
160
140
120
100
80
60
40
20
0
-20
0
100
200
300
400
500
T_lab, MeV
¹S₀ phase shifts
³S₁ phase shifts

Approach

$$\begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} - \begin{bmatrix} 0 & T_1 G_1 & T_1 G_1 \\ T_2 G_2 & 0 & T_2 G_2 \\ T_3 G_3 & T_3 G_3 & 0 \end{bmatrix} \begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix}$$

where full T matrix $T = \sum_{i=1}^{3} T^{(i)}$
G_i is the two-particles (j and n) Green function (ijn is cyclic permutation of (1,2,3)):

$$G_i(k_j, k_n) = 1/(k_j^2 - m^2 + i\epsilon)/(k_n^2 - m^2 + i\epsilon),$$

and T_i is the two-particles T matrix which can be written as following

$$T_i(k_1, k_2, k_3; k_1^{'}, k_2^{'}, k_{3^{'}}) = (2\pi)^4 \delta^{(4)}(K_i - K_i^{'}) T_i(k_j, k_n; k_j^{'}, k_n^{'}).$$

Introducing the equal-mass Jacobi momenta

$$p_i = \frac{1}{2}(k_j - k_n), q_i = \frac{1}{3}K - k_i, K = k_1 + k_2 + k_3.$$

we finally have

$$T^{(i)}(p_i, q_i; p_i^{'}, q_i^{'}; s) = (2\pi)^4 \delta^{(4)}(q_i - q_i^{'}) T_i(p_i; p_i^{'}; s) - i \int \frac{dp_i^{''}}{(2\pi)^4}$$

$$\times T_i(p_i; p_i^{''}; s) G_i(k_j^{''}, k_n^{''}) [T^{(j)}(p_j^{''}, q_i^{''}; p_i^{'}, q_i^{'}; s) + T^{(n)}(p_i^{''}, q_i^{''}; p_i^{'}, q_i^{'}; s)]$$

Approach

After partial-wave decomposition:
Amplitude of three-particle state as a projection of T matrix to the bound state:

$$\Psi^{(i)}(p_i, q_i; s) = \langle p_i, q_i | T^{(i)} | M_B \rangle$$

Separable ansatz for two-particles T matrix rank I

$$T_i(p_i; p_i^{'}; s) = g(p_i)\tau(s)g(p_i^{'}),$$

The amplitude can be presented in the form

$$\Psi^{(i)}(p_i, q_i; s) = g(p_i)\tau(s)X(q_i; s).$$

Approach

If consider $L = 0$ and $L_q = 0$ and accordingly two intermediate states ${}^1S_0, {}^3S_1$

$$\Psi^{(i)}(p, p^{'}; s) = \sum_{m=1,2} g_m(p)\tau_m(s)X_m(q; s).$$

$$X_m(q; s) = \sum_{m^{'}=1,2} 2i \int \frac{d^4 q^{'}}{(2p)^4} Z_{mm'}(q, q^{'}; s)S(\frac{1}{3}K - q^{'})\tau_{m'}(s_2)X_{m'}(q; s)$$

$$Z_{mm'}(q, q^{'}; s) = C_{mm'} g_m(-\frac{1}{2}q - q^{'})S(\frac{1}{3}K + q + q^{'})g_{m'}(q + \frac{1}{2}q^{'}),$$

with

$$C_{mm'} = \begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

to take into account spin 1/2 and isospin 1/2 nature of the nucleons.

$$\Phi_j(q_0,q) = -\frac{1}{4\pi^3}\sum_{j'=1}^{2}\int_{-\infty}^{\infty} dq_4^{'}\int_0^{\infty} q^{'2}dq^{'}$$

$$\times Z_{jj'}(q_0,q;q_0^{'},q^{'};s)\frac{\tau_{j'}[(\frac{2}{3}\sqrt{s}+q_0^{'})^2-q^{'2}]}{(\frac{1}{3}\sqrt{s}-q_0^{'})^2-q^{'2}-m^2+i\epsilon}\Phi_{j'}(q_0^{'},q^{'}),$$

where j=$^1S_0, ^3S_1$ is states of sistem,

and Z is the so-called effective energy-dependent potential:

$$Z_{jj'}(q_0,q;q_0^{'},q^{'};s) = C_{jj'}\int_{-1}^{1} d(cos\vartheta_{qq'})$$

$$\times\frac{g_j(-\frac{1}{2}q^0-q^{0'},|\frac{1}{2}\mathbf{q}+\mathbf{q}^{'}|)g_j(q^0+\frac{1}{2}q^{0'},|\mathbf{q}+\frac{1}{2}\mathbf{q}^{'}|)}{(\frac{1}{3}\sqrt{s}+q^0+q^{0'})^2-(|\mathbf{q}+\mathbf{q}'|)^2-m^2+i\epsilon},$$

with $C_{jj'}$ is spin and isospin recoupling-coefficient matrix.

$$\tau(s) = [\frac{1}{\lambda}-\frac{i}{4\pi^3}\int_{-\infty}^{\infty} dk^0\int_0^{\infty} k^2dkg^2(k^0,k)G(k^0,k;s)]^{-1}$$

Singularities

Poles from one-particle propagator

$$q^{0\prime}_{1,2} = \frac{1}{3}\sqrt{s} \mp [E_{|\mathbf{q}'|} - i\epsilon]$$

Poles from propagator in Z-function

$$q^{0\prime}_{3,4} = -\frac{1}{3}\sqrt{s} - q^0 \pm [E_{|\mathbf{q}'+\mathbf{q}|} - i\epsilon]$$

Poles from Yamaguchi-functions

$$q^{0\prime}_{5,6} = -2q^0 \pm 2[E_{|\frac{1}{2}\mathbf{q}'+\mathbf{q}|,\beta} - i\epsilon]$$

and

$$q^{0\prime}_{7,8} = -\frac{1}{2}q^0 \pm \frac{1}{2}[E_{|\mathbf{q}'+\frac{1}{2}\mathbf{q}|,\beta} - i\epsilon]$$

Cuts from two-particle propagator τ

$$q^{0\prime}_{9,10} = \pm\sqrt{q'^2 + 4m^2} - \frac{2}{3}\sqrt{s} \qquad \text{and} \qquad \pm\infty$$

Poles from two-particle propagator τ

$$q^{0\prime}_{11,12} = \pm\sqrt{q'^2 + 4M_d^2} - \frac{2}{3}\sqrt{s}$$

After Wick rotation

$$q_0 \to iq_4$$

equation turn into

$$\Phi_j(q_4, q) = -\frac{1}{4\pi^3} \sum_{j^{'}=1}^{2} \int_{-\infty}^{\infty} dq_4^{'} \int_0^{\infty} q^{'2} dq^{'}$$

$$\times Z_{jj'}(iq_4, q; iq_4^{'}, q^{'}; s) \frac{\tau_{j'}[(\frac{2}{3}\sqrt{s} + iq_4^{'})^2 - q^{'2}]}{(\frac{1}{3}\sqrt{s} - iq_4^{'})^2 - q^{'2} - m^2} \Phi_{j'}(q_4^{'}, q^{'}),$$

without any singularities if $\sqrt{s} \leq 3m_n$.

equation for solve

$$\Phi_j(q_4,q)=\sum_{j'=1}^{2}\int_{-\infty}^{\infty}dq_4'\int_0^{\infty}q'^2dq'\,K_{jj'}(iq_4,q;iq_4',q';s)\Phi_{j'}(q_4',q'),$$

if

$$K=A+iB$$

$$Re\Phi_j(q_4,q)+iIm\Phi_j(q_4,q)=$$

$$\sum_{j'=1}^{2}\int_{-\infty}^{\infty}dq_4'\int_0^{\infty}dq'\,[A_{jj'}(iq_4,q;iq_4',q';s)+iB_{jj'}(iq_4,q;iq_4',q';s)][Re\Phi_j(q_4,q)+iIm\Phi_j(q_4,q)]$$

or

$$Re\Phi_j+iIm\Phi_j=\sum_{j'=1}^{2}\int dx[A_{jj'}+iB_{jj'}][Re\Phi_j+iIm\Phi_j]=$$

$$=[(A_{jj'}Re\Phi_j-B_{jj'}Im\Phi_j)+i(B_{jj'}Re\Phi_j+A_{jj'}Im\Phi_j)]$$

or

$$Re\Phi_j=\sum_{j'=1}^{2}\int dx(A_{jj'}Re\Phi_j-B_{jj'}Im\Phi_j)$$

$$Im\Phi_j=\sum_{j'=1}^{2}\int dx(B_{jj'}Re\Phi_j+A_{jj'}Im\Phi_j)$$

Quadrature method for solving integral equations

$$f(x) = \int_a^b K(x,\alpha) f(\alpha) d\alpha$$

Gauss quadrature for integral

$$\int_{-1}^{1} f(x) dx = \sum_{i=1}^{N} \omega_i f(x_i)$$

$$f(x) = \int_{-1}^{1} K(x,\alpha) f(\alpha) d\alpha = \sum_{i=1}^{N} \omega_i K(x,\alpha_i) f(\alpha_i)$$

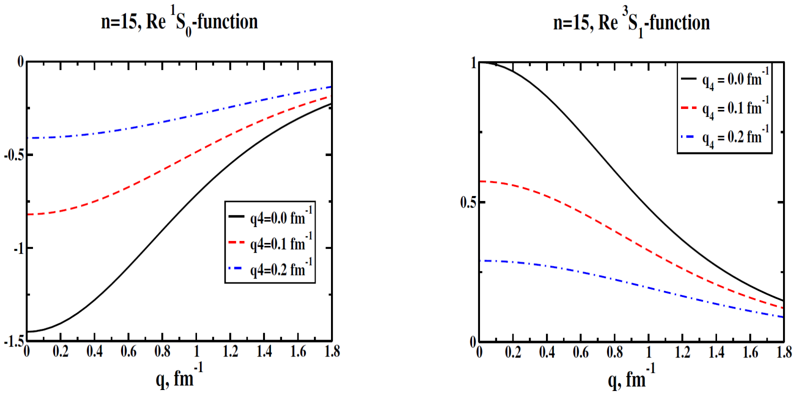
$$\boxed{f(x_j) = \sum_{i=1}^{N} \omega_i K(x_j,\alpha_i) f(\alpha_i)}$$

$$f(x_i, y_j) = \sum_{a,b} \omega_a \omega_b K(x_i, y_j, \alpha_a, \beta_b; s) f(\alpha_a, \beta_b)$$

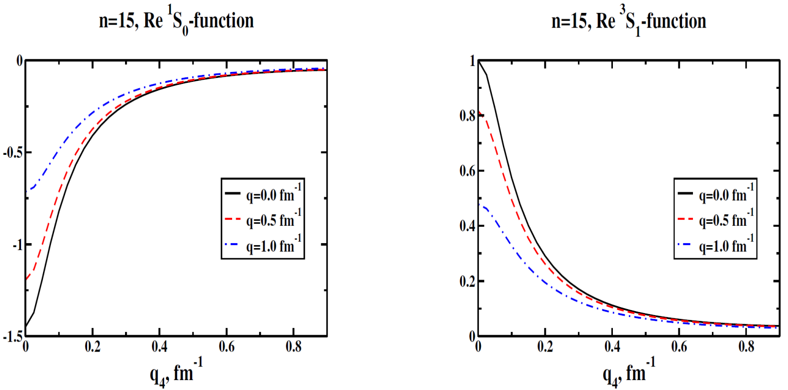
Thus,using quadrature method we converted homogeneous system of integral equations into homogeneous system of linear algebraic equations, which has a solution if the determinant of a matrix is zero.

Solving the equation $det(K(s) - I) = 0$ we can find the binding energy $s = 3M_N - E_{bs}$

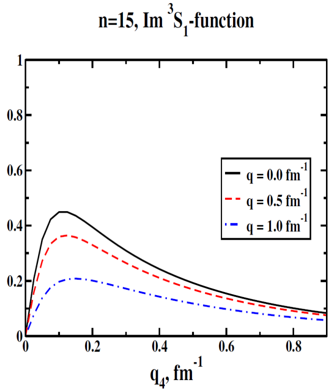
E_{bs} =11.09 MeV (exp. 8.48 MeV)

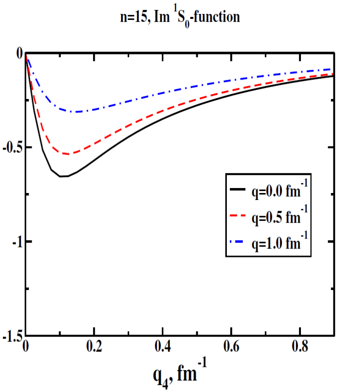

n=15, Re 1S0-function
0
-0.5
-1
-1.5
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
q, fm-1
q4=0.0 fm-1
q4=0.1 fm-1
q4=0.2 fm-1
n=15, Re 3S1-function
1
0.5
0
q, fm-1
q4 = 0.0 fm-1
q4 = 0.1 fm-1
q4 = 0.2 fm-1

WF


n=15, Re 1S0-function
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
q4, fm-1
q=0.0 fm-1
q=0.5 fm-1
q=1.0 fm-1
n=15, Re 3S1-function
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
q4, fm-1
q=0.0 fm-1
q=0.5 fm-1
q=1.0 fm-1

WF


n=15, Im ³S₁-function
q = 0.0 fm⁻¹
q = 0.5 fm⁻¹
q = 1.0 fm⁻¹
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
q₄, fm⁻¹


n=15, Im ¹S₀-function
q=0.0 fm⁻¹
q=0.5 fm⁻¹
q=1.0 fm⁻¹
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
q₄, fm⁻¹

•The relativistic covariant three-nucleon Faddeev equation was obtained in the Bethe-Salpeter formalism

• The separable ansatz was used to solve the homogeneous system of integral equations with two intermediate states ($^1S_0$ and $^3S_1$)

•The system of linear integral equations were solved numerically. The binding energy and amplitudes for two states ($^1S_0$ and $^3S_1$) were obtained.

Plan:

- Calculate amplitudes D and P states of Triton;
- Computation of form factors using the obtained wave functions;
- To extend formalism to multirank separable kernel
- To reformulate formalism in terms of spinor nucleons instead of scalar ones
- Study of collision processes pd $\to$ pd, pd $\to$ ppn and so on.

Thank you for your attention.