Effects of QRPA correlations on nuclear matrix elements of neutrinoless double-beta decay through overlap matrix

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1. Physical issues in neutrino study
2. Neutrinoless double-beta decay
3. Nuclear matrix element
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What is the neutrino mass?

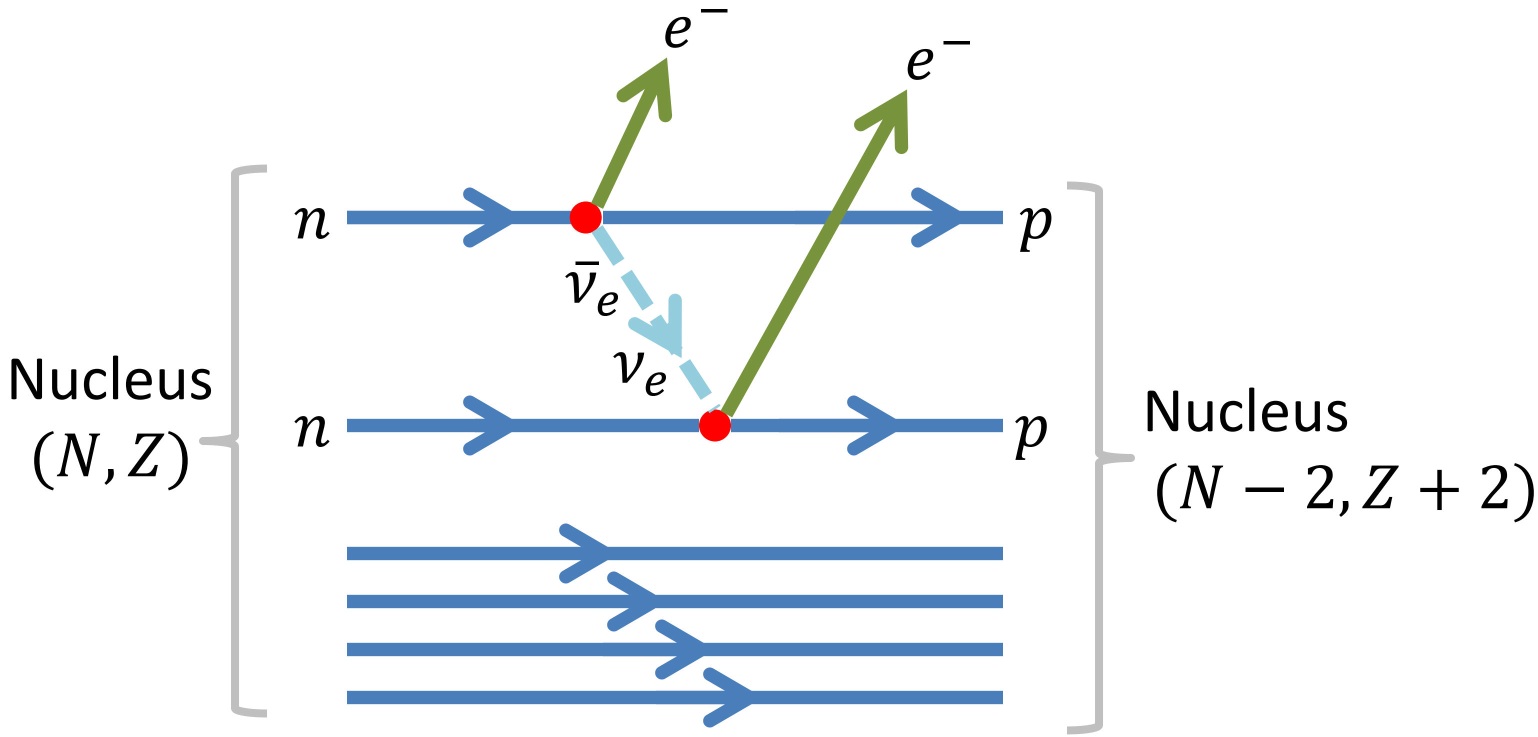
The neutrino is massless in the standard theory.

Other physical issues of neutrino

- Dirac or Majorana particle?
- Breaking of the lepton number conservation?
- How is the right-handed neutrino experimentally observed? Does that neutrino have an interaction?

One of the intensively studied few methods to determine the neutrino mass:

Application of neutrinoless double-beta $(0\nu\beta\beta)$ decay of nuclei



Neutrino assumed to be Majorana particle.

Why nuclei?

Because E(final state) $<$ E(initial state) is necessary.

Other conditions for the nuclei used in the experiments

- Single beta decay is suppressed.
- Energy spectrum of two electrons in $2\nu\beta\beta$ decay is separated from that of $0\nu\beta\beta$.
- Large Q value [E(final state) – E(initial state)].
- The parent nuclei can be produced massively with high purity.

$^{76}Ge \to {}^{76}Se$	$^{130}Te \to {}^{130}Xe$	$^{136}Xe \to {}^{136}Ba$
$^{150}Nd \to {}^{150}Sm$	$^{48}Ca \to {}^{48}Ti$	$^{82}Se \to {}^{82}Kr$
$^{96}Zr \to {}^{96}Mo$	$^{100}Mo \to {}^{100}Ru$	$^{110}Pd \to {}^{110}Cd$
$^{116}Cd \to {}^{116}Sn$	$^{124}Sn \to {}^{124}Te$	and more

The principle to determine the effective neutrino mass using $0\nu\beta\beta$ decay

$$1/T_{0\nu}(0^+ \rightarrow 0^+) = \left|M^{(0\nu)}\right|^2 G_{01} \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Exp. $\rightarrow$ $T_{0\nu}$: half-life of the transition

Cal. $\rightarrow$ $\left\{ \right.$

$M^{(0\nu)}$: transition matrix element of nucleus (nuclear matrix element)

G_{01}: transition matrix element^2 of electron (phase space factor)

$\langle m_\nu \rangle$: effective neutrino mass

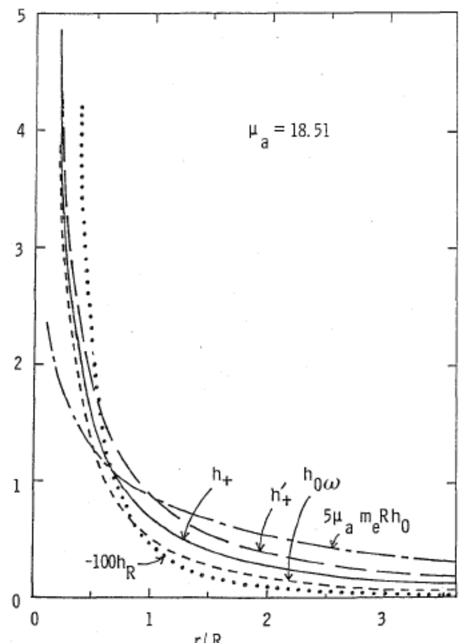
m_e : electron mass

Nuclear matrix element

$$M^{(0\nu)} = \sum_b \sum_{pp'} \sum_{nn'} \langle pp'|V(r_{12},E_b)|nn'\rangle\langle 0_\mathrm{f}^+|c_{p'}^\dagger c_{n'}|b\rangle\langle b|c_p^\dagger c_n|0_\mathrm{i}^+\rangle$$

$$V(r_{12},E_b) \cong h_+(r_{12},E_b)\{-\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2) + g_V^2/g_A^2\}\,\tau^+(1)\tau^+(2)$$

Neutrino potential

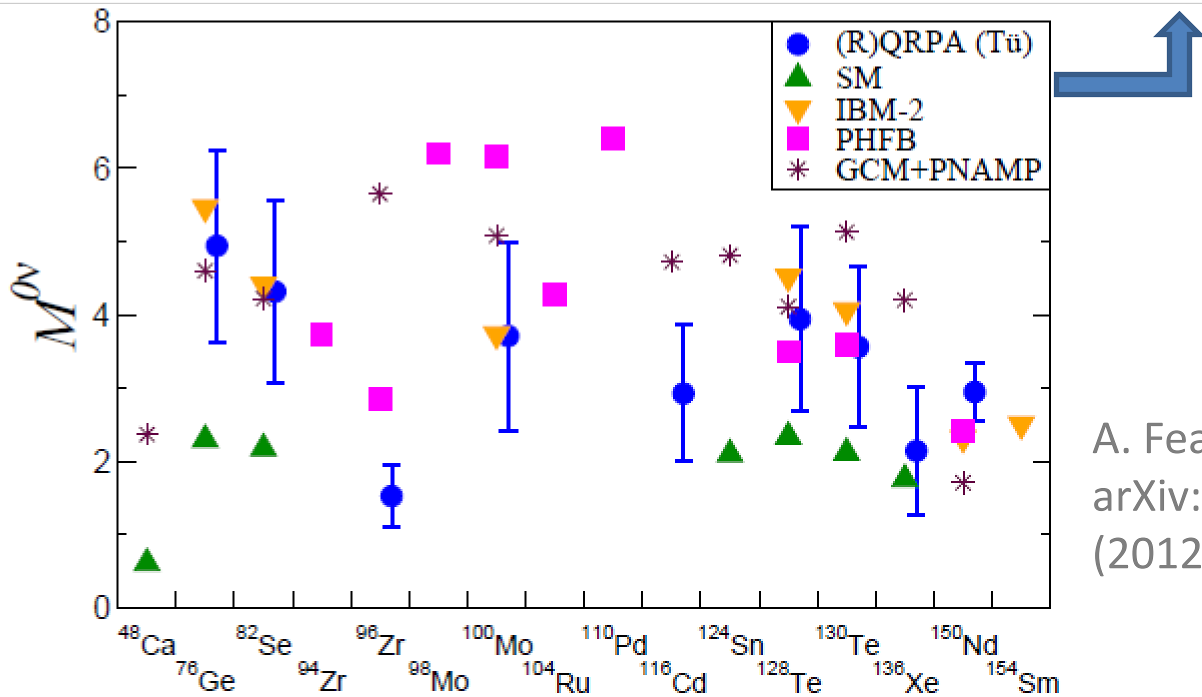


r_{12}/R

M. Doi et al. Prog. Theor. Phys. Suppl. No. 83 (1985) 1

Status

(Relativistic) quasiparticle random-phase approximation
Shell model
Interacting boson model-2
Projected Hartree-Fock-Bogoliubov
Generator-coordinate method + Particle number and angular momentum projection



A. Feassler, arXiv:2103.3648 (2012)

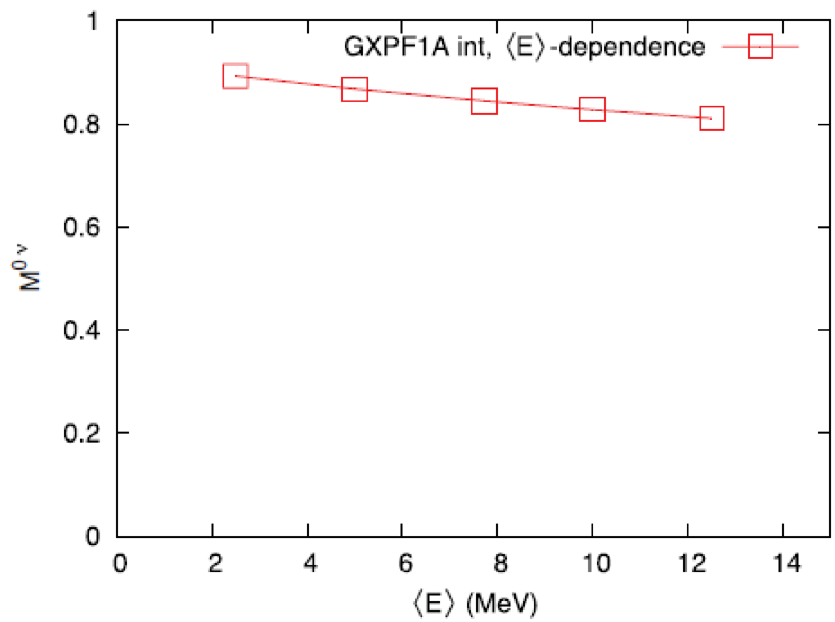
The problem is the discrepancy.

Nuclear matrix element

$$M^{(0\nu)} = \sum_{b} \sum_{pp'nn'} \langle pp'|V(r_{12}, E_b)|nn'\rangle\langle 0_{\mathrm{f}}^{+}|c_{p'}^{\dagger} c_{n'}|b\rangle \langle b|c_{p}^{\dagger} c_{n}|0_{\mathrm{i}}^{+}\rangle$$

$$\cong \sum_{pp'nn'} \langle pp'|V(r_{12}, \bar{E})|nn'\rangle \langle 0_{\mathrm{f}}^{+}|c_{p'}^{\dagger} c_{n'} \sum_{b} |b\rangle\langle b| c_{p}^{\dagger} c_{n}|0_{\mathrm{i}}^{+}\rangle$$

: closure approximation,



$\bar{E}$ dependence of $M^{0\nu}$

M. Horoi et al., PRC 81, 024321 (2010)

Closure approximation is good.

QRPA

Approximation using only two-quasiparticle excitations $a_i^\dagger a_j^\dagger$ and $a_i a_j$ for the elementary mode of excitation

Considering only neutron-neutron and proton-proton quasiparticle pairs : like-particle QRPA

Considering only proton-neutron quasiparticle pairs : proton-neutron QRPA

Application of QRPA to nuclear matrix element

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0^+_{\mathrm{QRPA,f}}|c^\dagger_{p'}c_{n'}c^\dagger_p c_n|0^+_{\mathrm{QRPA,i}}\rangle$$

$$\boxed{1 = |0^+_{\mathrm{QRPA,i}}\rangle\langle 0^+_{\mathrm{QRPA,i}}| + \sum_{b_{\mathrm{i}}:\mathrm{pnQRPA}} |b_{\mathrm{i}}\rangle\langle b_{\mathrm{i}}| + \sum_{b_{\mathrm{i}1}b_{\mathrm{i}2}} |b_{\mathrm{i}1}b_{\mathrm{i}2}\rangle\langle b_{\mathrm{i}1}b_{\mathrm{i}2}| + \sum_{b_{\mathrm{i}1}b_{\mathrm{i}2}b_{\mathrm{i}3}} |b_{\mathrm{i}1}b_{\mathrm{i}2}b_{\mathrm{i}3}\rangle\langle b_{\mathrm{i}1}b_{\mathrm{i}2}b_{\mathrm{i}3}| + \cdots,}$$

Note $c^\dagger_p c_n \sim O^{\mathrm{i}\dagger}_b + O^{\mathrm{i}}_b + O^{\mathrm{i}\dagger}_b O^{\mathrm{i}}_{b'}$, $|b_{\mathrm{i}}\rangle = O^{\mathrm{i}\dagger}_b|0^+_{\mathrm{QRPA,i}}\rangle$

$\langle b_{i1}b_{i2}\cdots|c^\dagger_p c_n|0^+_{\mathrm{QRPA,i}}\rangle = 0$ in QRPA

Application of QRPA to nuclear matrix element

$$M^{(0\nu)} \cong \sum_{b_\mathrm{i} b_\mathrm{f}} \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0^+_{\mathrm{QRPA,f}}|c^\dagger_{p'} c_{n'}|b_\mathrm{f}\rangle\langle b_\mathrm{f}|b_\mathrm{i}\rangle$$
$$\times \langle b_\mathrm{i}|c^\dagger_p c_n|0^+_{\mathrm{QRPA,i}}\rangle$$

: usual equation in the QRPA approach

As long as the closure approximation is used, the application of the QRPA can be justified theoretically.

The overlap of the QRPA states

The QRPA ground state is defined to be the vacuum to the QRPA quasiboson :

$$O_b^{\mathrm{i}}\left|0^+_{\mathrm{QRPA,i}}\right\rangle = 0$$

O_b^{i} : annihilation operator of QRPA state b

$$|0^+_{\mathrm{QRPA,i}}\rangle = \prod_{K\pi} \frac{1}{\mathcal{N}^{K\pi}_{\mathrm{QRPA,i}}} \exp[v_{\mathrm{i}}^{(K\pi)}]|0^+_{\mathrm{HFB,i}}\rangle\,,$$

$$v_{\mathrm{i}}^{(K\pi)} \cong \sum_{\mu\nu\mu'\nu'} \frac{1}{1+\delta_{K0}} \left(Y^{\mathrm{i},K\pi}\frac{1}{X^{\mathrm{i},K\pi}}\right)^{\dagger}_{\mu\nu,\mu'\nu'} a_{\mu}^{\mathrm{i}\dagger} a_{\nu}^{\mathrm{i}\dagger} a_{\mu'}^{\mathrm{i}\dagger} a_{\nu'}^{\mathrm{i}\dagger}$$

$$O_b^{\mathrm{i}\dagger} = \sum_{\mu\nu\mu'\nu'} \left(X^{\mathrm{i},K\pi}_{\mu\nu,b} a_{\mu}^{i\dagger} a_{\nu}^{i\dagger} - Y^{\mathrm{i},K\pi}_{-\mu-\nu,b} a^{\mathrm{i}}_{-\nu} a^{\mathrm{i}}_{-\mu}\right),$$

$$a_{\mu}^{\mathrm{i}}|0^+_{\mathrm{HFB,i}}\rangle = 0.$$

Calculations of the overlap using expansion

$$\langle b_{\mathrm{f}}|b_{\mathrm{i}}\rangle = \frac{1}{\mathcal{N}_{\mathrm{f}}\mathcal{N}_{\mathrm{i}}}\prod_{K_1\pi_1}\langle 0^+_{\mathrm{HFB,f}}|\exp[v_{\mathrm{f}}^{(K_1\pi_1)\dagger}]\ O_b^{\mathrm{f}} O_b^{\mathrm{i}\dagger}$$

$$\times \exp[v_{\mathrm{i}}^{(K_1\pi_1)}]\,|0^+_{\mathrm{HFB,i}}\rangle$$

$$\cong \frac{1}{\mathcal{N}_{\mathrm{f}}\mathcal{N}_{\mathrm{i}}}\Bigg\{\langle 0^+_{\mathrm{HFB,f}}|O_b^{\mathrm{f}} O_b^{\mathrm{i}\dagger}|0^\dagger_{\mathrm{HFB,i}}\rangle$$

$$+ \sum_{K_1\pi_1}\Big(\langle 0^+_{\mathrm{HFB,f}}|v_{\mathrm{f}}^{(K_1\pi_1)\dagger} O_b^{\mathrm{f}} O_b^{\mathrm{i}\dagger}|0^+_{\mathrm{HFB,i}}\rangle$$

$$+ \langle 0^+_{\mathrm{HFB,f}}|O_b^{\mathrm{f}} O_b^{\mathrm{i}\dagger} v_{\mathrm{i}}^{(K_1\pi_1)}|0^+_{\mathrm{HFB,i}}\rangle\Big)$$

$$+ \sum_{K_1\pi_1}\langle 0^+_{\mathrm{HFB,f}}|v_{\mathrm{f}}^{(K_1\pi_1)\dagger} O_b^{\mathrm{f}} O_b^{\mathrm{i}\dagger} v_{\mathrm{i}}^{(K_1\pi_1)}|0^+_{\mathrm{HFB,i}}\rangle\Bigg\}$$

Two methods of QRPA approach

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0^+_{\mathrm{pn,f}}|c^\dagger_{p'}c_{n'}c^\dagger_p c_n|0^+_{\mathrm{pn,i}}\rangle$$

$$\sum_{b_\mathrm{f}:\mathrm{pnQRPA}} |b_\mathrm{f}\rangle\langle b_\mathrm{f}| \qquad \sum_{b_\mathrm{i}:\mathrm{pnQRPA}} |b_\mathrm{i}\rangle\langle b_\mathrm{i}|$$

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0^+_{\mathrm{like,f}}|\underbrace{c^\dagger_{p'}c_{n'}c^\dagger_p c_n}_{-c^\dagger_{p'}c^\dagger_p c_{n'}c_n}|0^+_{\mathrm{like,i}}\rangle$$

$$\sum_{b_\mathrm{f}:\mathrm{likeQRPA}} |b_\mathrm{f}\rangle\langle b_\mathrm{f}| \qquad \sum_{b_\mathrm{i}:\mathrm{likeQRPA}} |b_\mathrm{i}\rangle\langle b_\mathrm{i}|$$

Test calculations of the overlap

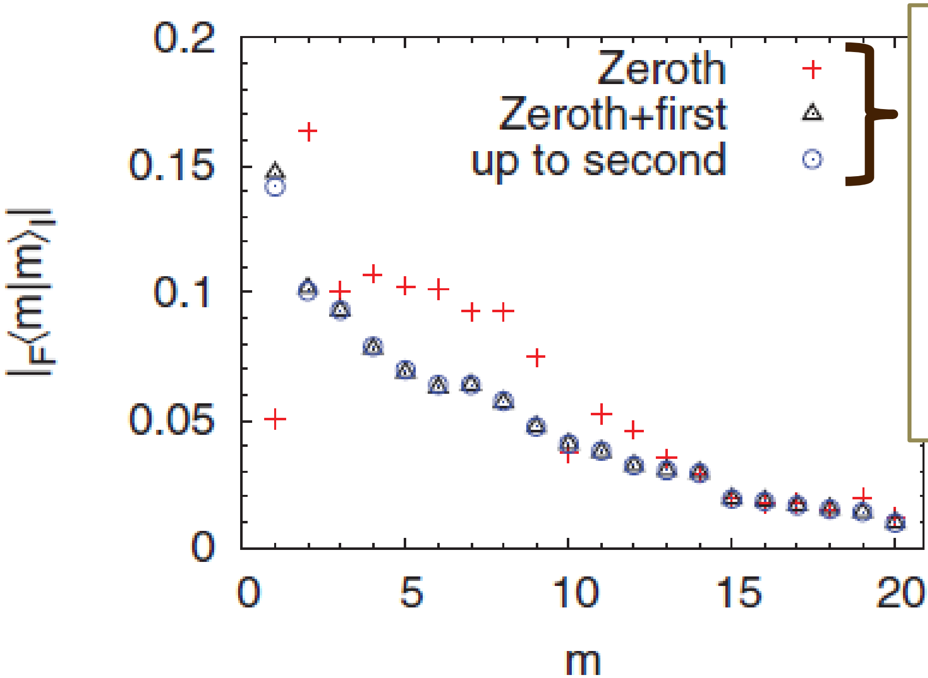
Like-particle QRPA used

^{26}Mg (initial)-^{26}Si (final)

Interaction : Skyrme SkM* and volume delta pairing

J.T. PRC **86**, 021301(R) (2012); **87**, 024316 (2013)

Largest diagonal $|\langle b_\mathrm{f}|b_\mathrm{i}\rangle|$



With respect to $v_{\mathrm{like,f}}^{(K_1\pi_1)\dagger}$ and $v_{\mathrm{like},i}^{(K_1\pi_1)}$, (the converged normalization factor is used.)

Only $(K_1\pi_1) = (0+)$.

Calculation of $M^{(0\nu)}$ of ^{150}Nd-^{150}Sm

- Like-particle QRPA, via ^{148}Nd
- SkM* and volume pairing

Parameters of the HFB calculation

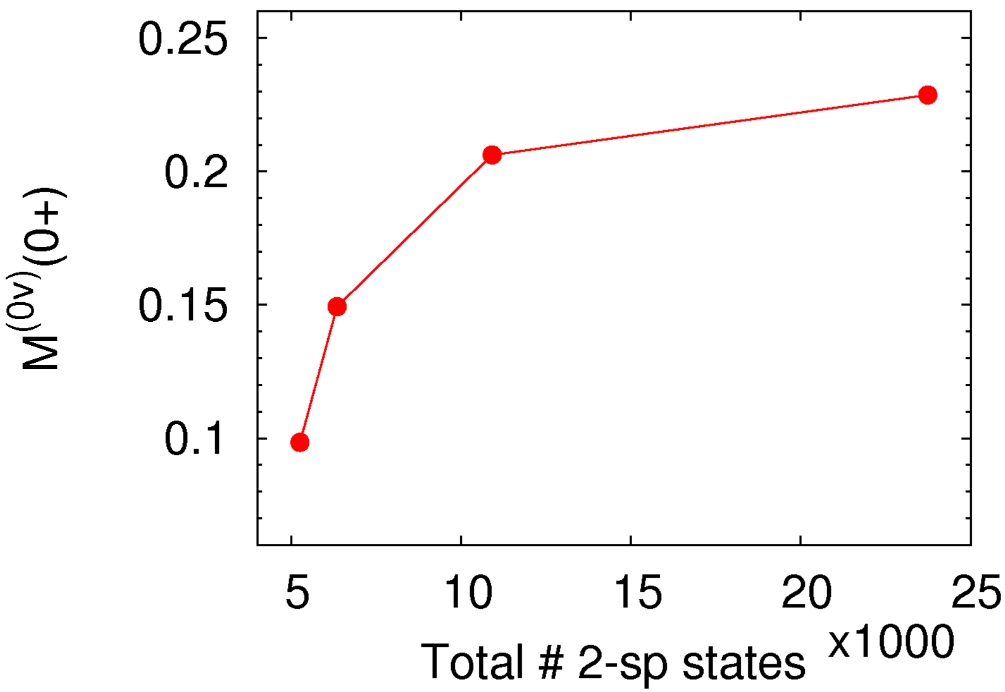
- Size of the cylindrical box; $\rho_{\max} = z_{\max} = 20\mathrm{fm}$.
- Cut-off qp energy $60\mathrm{MeV}$.

Parameters of the like-particle QRPA calculation

- Number of qp states $\cong 1700$ (proton),
  $\cong 2500$ (neutron), $(K^{\pi} = 0^{+})$.
- Number of 2-qp states $\cong 58\,000$ $(K = 0)$,
  $\cong 25\,000$ $(K = 2)$
- $\mathrm{Max}\, K = 8$.

Setup of calculation of nuclear matrix element

- Only GT and Fermi operators
- No proton-neutron pairing interaction
- Average energy of the intermediate state = $10\ \mathrm{MeV}$
- All QRPA solutions are used for the intermediate states. (truncation possible)
- $\mathrm{Max}\ K = 8$.
- No *short-range correlation correction* ─ the raw 0vββ transition operator is used.
- No *quenching* ; $g_A = 1.0, g_V = 1.25.$ (quenching will also be used).



in the matrix of the 0vββ transition operator

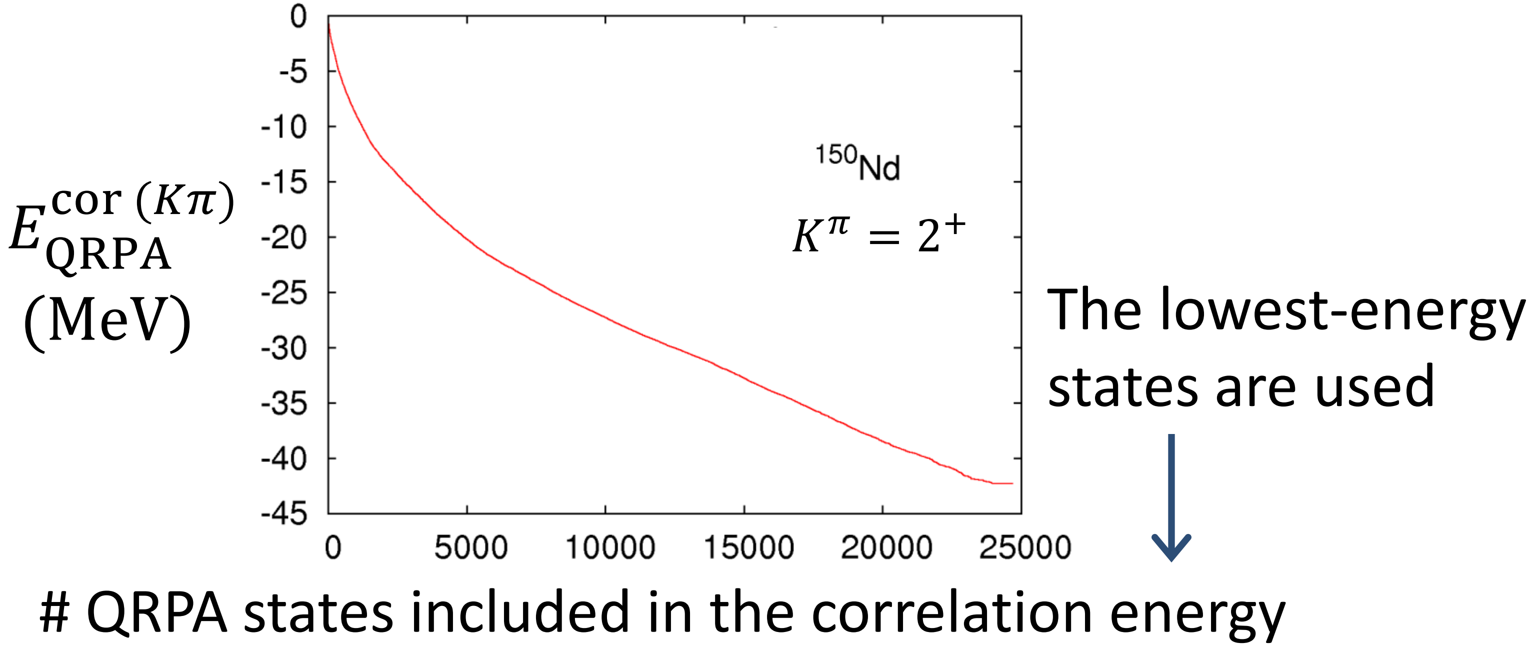
We use as large wave function spaces as possible without the effective methods to compensate for the space truncations.

Result of the first attempt

$$\left|M^{(0\nu)}\right| \sim 0.08$$

2.5 – 3.5 (QRPA , Tübingen),
1.8 – 3.5 including various approaches (the previous figure)

The QRPA correlations are too large; it is known that the correlation energy diverges in the Skyrme QRPA.



My prescription

To pick up the QRPA solutions having the largest backward amplitudes so as to get

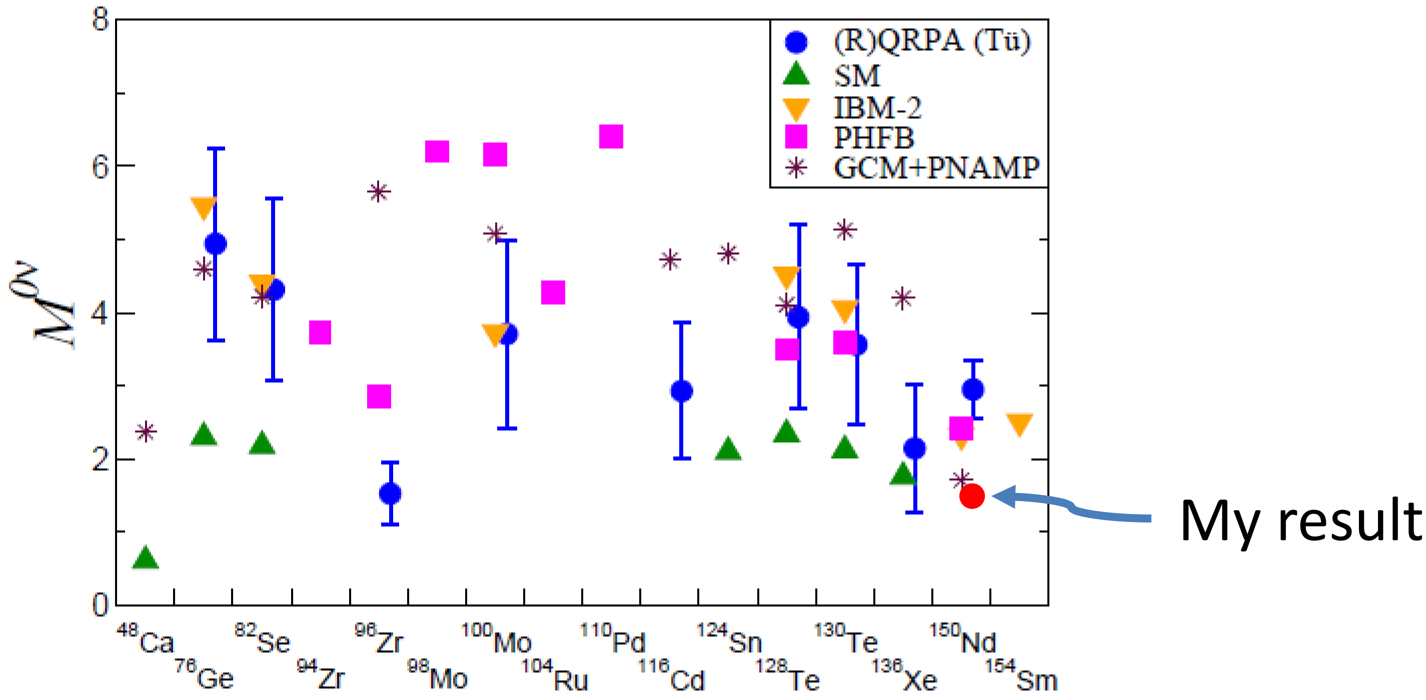
$$E_{\mathrm{QRPA}}^{\mathrm{cor}} = E_{\mathrm{exp}} - E_{\mathrm{HFB}}$$

and use only these states for calculating $v_{\mathrm{like,i}}^{(K\pi)}$, $v_{\mathrm{like,f}}^{(K\pi)}$, i.e. for calculating the QRPA ground states.

27 like-particle QRPA states for $E_{\mathrm{QRPA}}^{\mathrm{cor}} = -1.71\mathrm{MeV}$ (^{150}Nd) and 79 like-particle QRPA states for $E_{\mathrm{QRPA}}^{\mathrm{cor}} = -3.68\mathrm{MeV}$ (^{150}Sm) were picked up.

$E_{\mathrm{cor}} = E_{\mathrm{exp}} - E_{\mathrm{HFB}}$:
-1.70 MeV (^{150}Nd)
-3.66 MeV (^{150}Sm)

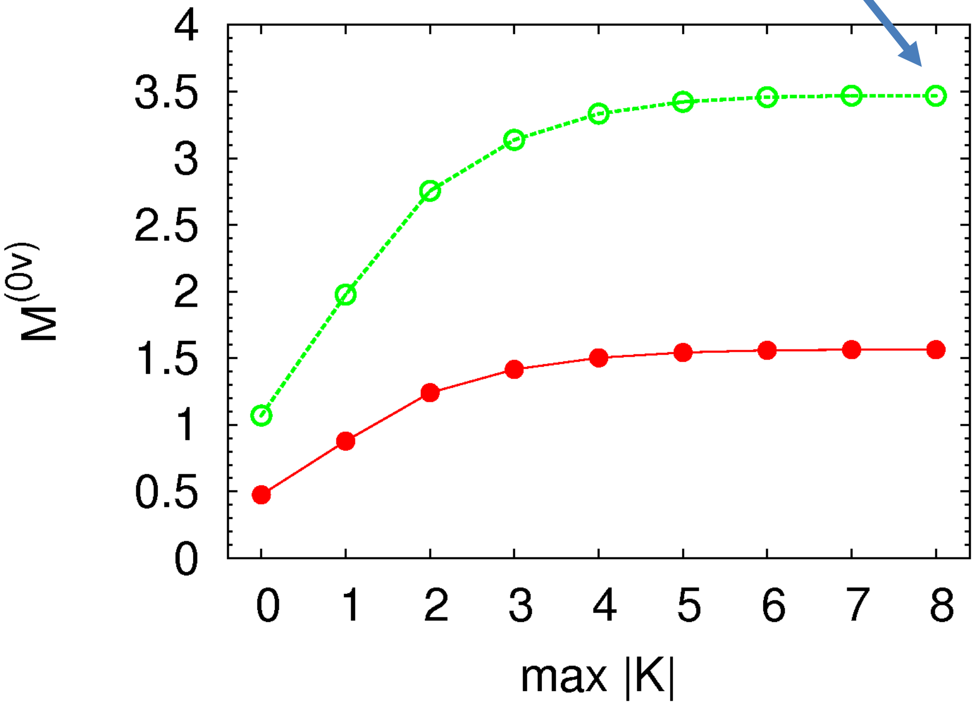
Result



A. Feassler, arXiv:2103.3648 (2012)

Result

HFB gs is used instead of QRPA gs in the overlap calculations.

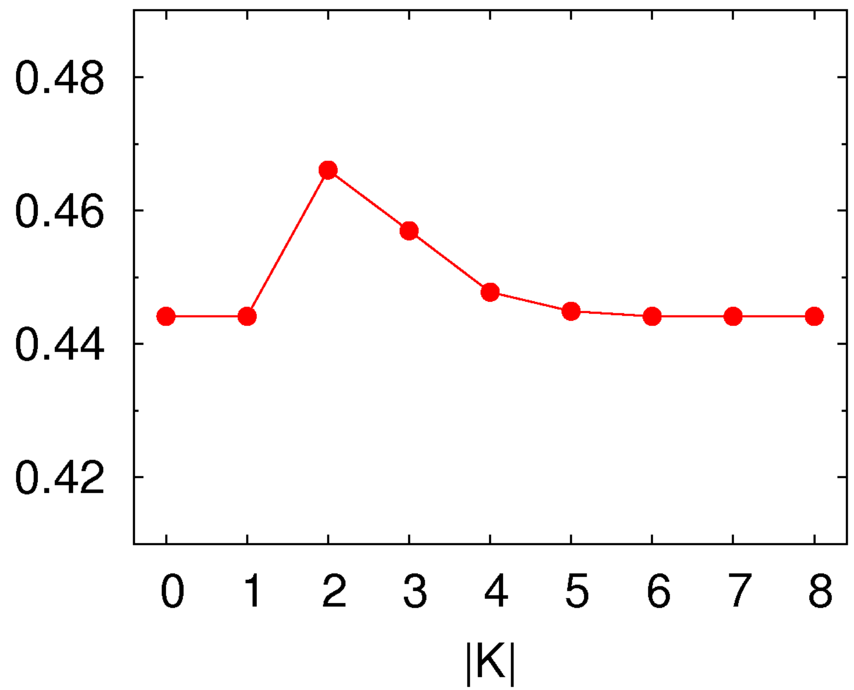


Our best value

$$M^{(0\nu)} = \sum_{K'=-\mathrm{max}K}^{\mathrm{max}K} \sum_{\pi} M^{(0\nu)}(K'\pi)$$

Result

$$\frac{\left|M^{(0\nu)}(|K|)\right|}{\left|M^{(0\nu)}(|K|)\right|_{\mathrm{unc}}}$$

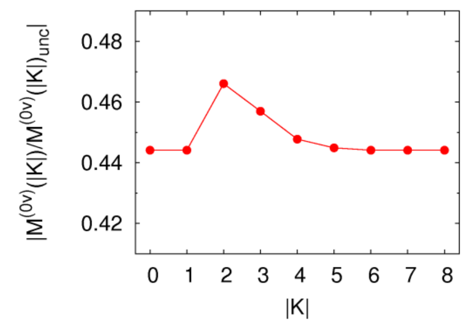


$$M^{(0\nu)}(|K|) \stackrel{\mathrm{def}}{=} \sum_{\pi} M^{(0\nu)}(K,\pi) \times \begin{cases} 1, K = 0 \\ 2, K \neq 0 \end{cases}$$

$M^{(0\nu)}(|K|)_{\mathrm{unc}}$ is defined in the same way but for the HFB gs instead of QRPA gs in the overlap.

Note the equation of the overlap:

$$\frac{1}{\mathcal{N}_{\mathrm{like,f}}\mathcal{N}_{\mathrm{like,i}}}\langle 0^+_{\mathrm{HFB,f}}|\prod_{K_1\pi_1}\exp[v^{(K_1\pi_1)\dagger}_{\mathrm{like,f}}]\,O^{\mathrm{like,f}}_{bK\pi}O^{\mathrm{like,i}\dagger}_{bK\pi}$$
$$\times\exp[v^{(K_1\pi_1)}_{\mathrm{like,i}}]|0^+_{\mathrm{HFB,i}}\rangle$$
$$\cong\frac{1}{\mathcal{N}_{\mathrm{like,f}}\mathcal{N}_{\mathrm{like,i}}}\langle 0^+_{\mathrm{HFB,f}}|\exp[v^{(K\pi)\dagger}_{\mathrm{like,f}}]O^{\mathrm{like,f}}_{bK\pi}O^{\mathrm{like,i}\dagger}_{bK\pi}$$
$$\times\exp[v^{(K\pi)}_{\mathrm{like,i}}]\,|0^+_{\mathrm{HFB,i}}\rangle$$



For *K*=0, 1, 6, 7, and 8, this is equal to

$$\frac{1}{\mathcal{N}_{\mathrm{like,f}}\mathcal{N}_{\mathrm{like,i}}}\underbrace{\langle 0^+_{\mathrm{HFB,f}}|O^{\mathrm{like,f}}_{bK\pi}O^{\mathrm{like,i}\dagger}_{bK\pi}|0^+_{\mathrm{HFB,i}}\rangle}_{\text{“uncorrelated overlap”}}$$

Conclusion

- The first value of nuclear matrix element in my new method was obtained; relatively low without effective methods to lower the nuclear matrix element.

- The QRPA correlations have the effect to reduce the nuclear matrix element through the normalization factor in the overlap.