STUDY OF CLUSTER REACTIONS IN ADVANCED SHELL MODEL APPROACHES

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MOTIVATIONS

1. A large body of experimental information concerning cluster decay widths of resonance states is accumulated.

2. Redefinition of the cluster spectroscopic characteristics has changed the view on clustering significantly.

3. Supercomputing era came. Advanced approaches to nuclear structure producing wave functions of nuclei which make it possible to describe nuclear spectra, moments, electromagnetic transitions, etc. with rather high quality are created.

INTENSIONS

A global intension is to create the theory of clustering suited to the requirement of supercomputing era.

A particular program is to build techniques for description of the cluster observables for the wave functions of such a type in the case that they are representable in the form of the oscillator expansion.

Contrary to the modern approaches to clustering concentrating attention on the strongly clustered states we try to consider all states as the objects.

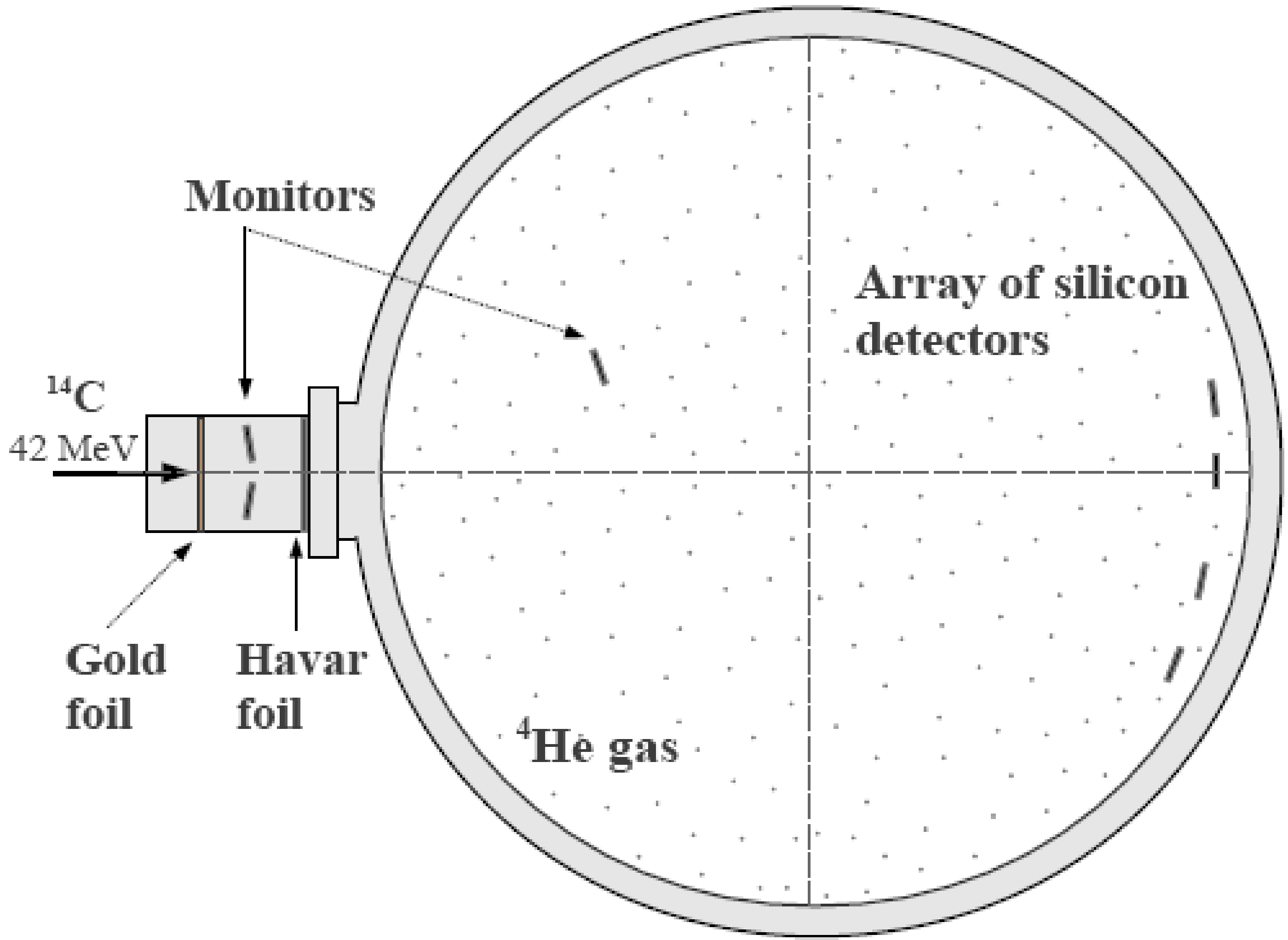
NUCLEAR REACTIONS AND MANIFESTATION OF CLUSTERING

I. Spontaneous cluster decay.
II. Cluster transfer reactions.
III. Cluster knock-out.

IV. RESONANCE SCATTERING OF COMPOSITE PARTICLES.

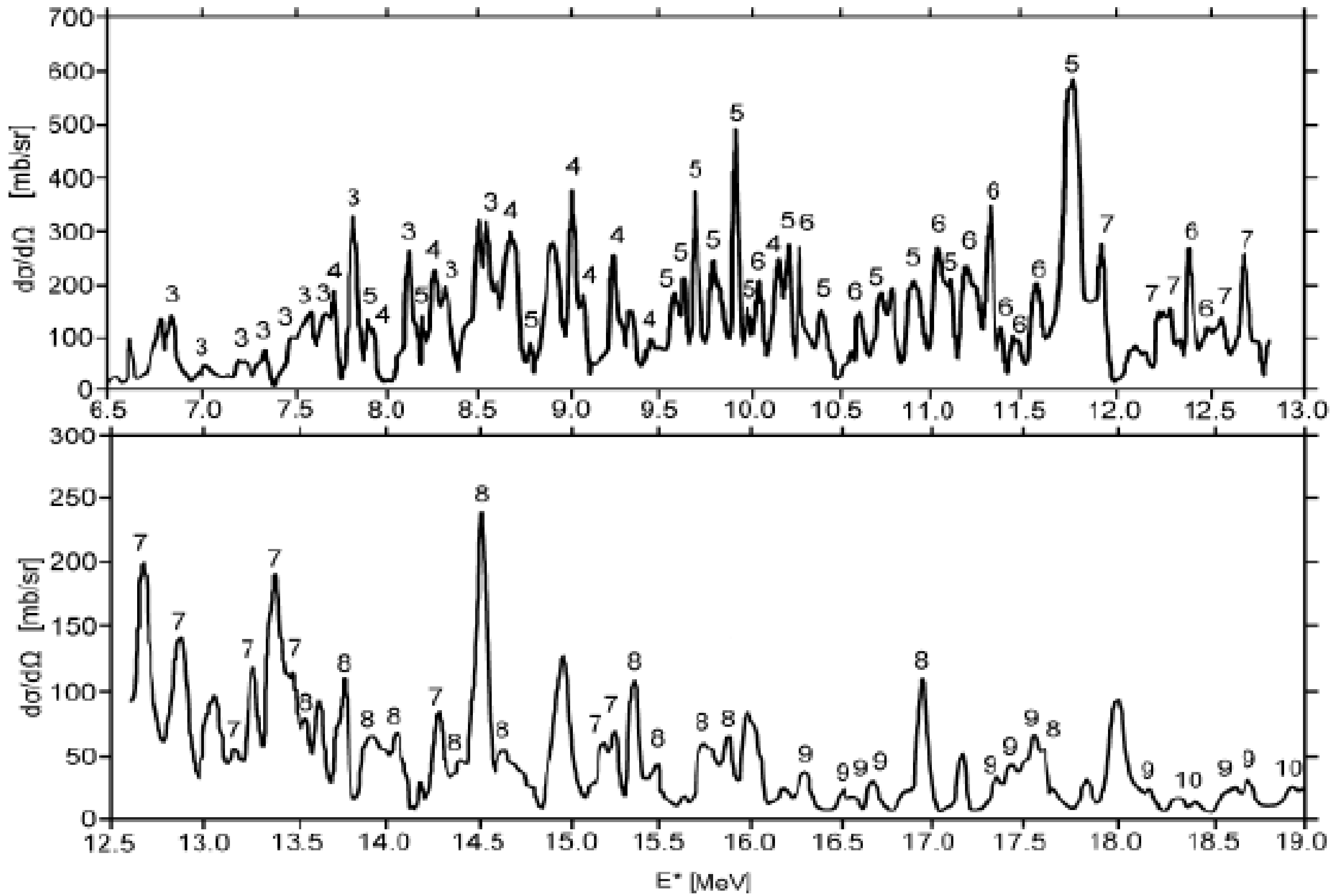
The investigations are:
1. Modern, being in progress, promising.
2. Providing broad and rich cluster spectra.

An objective of this work is to build a theory of such spectra and to study various nuclear processes.



Monitors
Array of silicon detectors
^{14}C
42 MeV
Gold foil
Havar foil
^{4}He gas

ALPHA-PARTICLE LEVEL DENSITY PUZZLE



Alpha-particle states of ^{32}S nucleus (K.-M.Källman et al.).

CLUSTERING IN SHEL-MODEL APPROACHES

MATHEMATICS OF CLUSTERING

I. Translationally- invariant shell model (TISM)

Cluster fractional parentage coefficient (FPC) is defined as:

$$F_{MDC}^{nl} = < \Psi_M \mid \hat{A}\{\Psi_D \phi_{nl}(\vec{\rho})\Psi_C\} >$$

where: $\phi_{nl}(\rho) -$ wave function (WF) of the relative motion,

$\hat{A} -$ the antisymmetrizer, $\Psi_M, \Psi_D, \Psi_C -$

internal translationally-invariant wave functions (WFs) of the mother, daughter nuclei and the cluster respectively. The formalism of translationally-invariant shell model [I.V. Kurdiumov, et al. A 145, 593 (1970)] is, however, too cumbersome for actual calculations.

II. Traditional shell model

Multi-nucleon fractional parentage coefficient of the X-nucleon configuration Ψ_{XN} is defined as:

$$F_{MD}^{R}(XN) = <\Psi_M(R_M)|\hat{A}\{\Psi_D(R_D)\Psi_{XN}\}>$$

where the notation: $\Psi_{M(D)}(R_{M(D)})$ stands for the a WF of the traditional shell model containing the redundant center-of-mass (CM) coordinate.

In the case that C is the X-nucleon cluster, the WFs of the mother and the daughter nuclei $\Psi_M(R_M)$ and $\Psi_D(R_D)$ are superpositions of the oscillator WFs, the CM motions of the nuclei described by these WFs are zero oscillations the formula

$$F_{MDC} = \sum F_{MDC}^{nl} = \sum (-1)^n \left(\frac{A}{A-X} \right)^{n/2} X_{nl} F_{MD}^{R}(XN)$$

[Yu.F. Smirnov, Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977)] takes place. Here first two multipliers present the recoil factor and the multiplier

$$X_{nl} = < \Psi_{XN} | \phi_{nl}(\vec{R}_C) \Psi_C >$$

denotes cluster coefficient. Methods of calculation of this object for various cluster masses and nucleon configurations are developed in many papers.

As an example, a general expression for the cluster coefficients of light d, t, h, and α clusters takes the form:

$$X_{(n0)} \equiv \left< \prod_{i=1}^{X} n_i(n0):000 \,|\, \phi_{(n0)}(R_C)\Psi_C \right> =$$

$$X^{-n/2}\left(n!/\prod_{i=1}^{X} n_i! \right)^{1/2} \left(X!\prod_{j=1}^{k} \alpha_i! \right)^{1/2}.$$

[Ichimura et al. Nucl. Phys A 204, 225 (1973)]. The SU(3)-coupling of the one-nucleon WFs is implied here. The components of the symmetry $(n0); n = \sum n_i$ contribute to the expression only.

PHYSICS OF CLUSTERING

A long-term concepts was the measure of clustering – spectroscopic amplitude and the projection of the nuclear wave function onto the cluster channel i. e. FPC in the TISM (1) is one and the same [H.J. Mang Z. Phys. 148, 556 (1957); V.V. Balashov et al. JETP 37, 1385 (1959); a set of works by SINP MSU and VSU groups]:

$$C^{nl}_{MDC} = F^{nl}_{MDC}; \qquad F^{nl}_{MDC} = < \Psi_M \,|\, \hat{A}\{\Psi_D \phi_{nl}(\vec{\rho})\Psi_C\} >$$

and thus the the cluster form factor and spectroscopic factor can be expressed as:

$$\Phi_l(\rho) = < \Psi_M \,|\, \hat{A}\{\Psi_D \frac{1}{\rho^2}\delta(\rho - \rho')Y_{lm}(\Omega_{\rho'})\Psi_C\} >;$$

$$S_{MDC} \equiv \int \Phi(\rho)|^2 \rho^2 d\rho = \sum_n \left(C^{nl}_{MDC}\right)^2;$$

In the paper [T. Fliessbach and H.J. Mang, Nucl. Phys. A 263, 75 (1976)] this point of view was thrown doubt. The matter is that a certain matching procedure is required to deduce the amplitude and the width of a cluster channel. But the values of one and the same sense can solely be matched. I. e. the cluster form factor must be matched with the same projection of the cluster channel WF. Not:

$$\Phi_l(\rho) \not\longleftrightarrow f_l(\rho),$$

$f(\rho)$ – a solution of two-body problem, with the traditional norm but:

$$\Phi_l(\rho) \longleftrightarrow \Phi'_l(\rho)$$

where:

$$\Phi'_l(\rho) = \langle \Psi_{D+C} \,|\, \hat{A}\{\Psi_D \frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_{\rho'})\Psi_C\} \rangle$$

And the channel wave function:

$$\Psi_{D+C} = \hat{A}\{\Psi_D \varphi(\vec{\rho})\Psi_C\} -$$

microscopic solution of A-nucleon problem which may be RGM, OCM, etc. In the case that it is normalized as usual:

$$\left\langle \Psi_{D+C} \middle| \Psi_{D+C} \right\rangle = \begin{pmatrix} 1 \\ \delta(E-E'), \delta(k-k'), etc. \end{pmatrix}$$

the WF of the relative motion must be normalized as:

$$\left\langle \hat{N}_\rho^{1/2}\varphi(\vec{\rho}) \middle| \hat{N}_\rho^{1/2}\varphi(\vec{\rho}) \right\rangle = \begin{pmatrix} 1 \\ \delta(E-E'), \delta(k-k'), etc. \end{pmatrix}$$

where:

$$\hat{N}_\rho \varphi(\rho) \equiv \int N(\rho', \rho)\varphi(\rho')\rho'^2 d\rho'$$

$$N(\rho',\rho'') =$$

$$\left\langle \hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho')Y_{lm}(\Omega_\rho)\right\}\middle|\hat{A}\left\{\Psi_{A_1}\Psi_{A_2}\frac{1}{\rho^2}\delta(\rho-\rho'')Y_{lm}(\Omega_\rho)\right\}\right\rangle.$$

As a result:

$$\Phi'_l(\rho) = \hat{N}_\rho^{1/2}\varphi(\rho).$$

$$\Phi_l(\rho) \longleftrightarrow \hat{N}_\rho^{1/2}\varphi_l(\rho).$$

$$\hat{N}_\rho^{-1/2}\Phi_l(\rho) \longleftrightarrow \varphi_l(\rho)$$

$$S'_{MDC} \equiv \int |\hat{N}_\rho^{-1/2}\Phi(\rho)|^2 \rho^2 d\rho$$

R. Lovas et al. Phys. Rep. 294, 265 (1998).

In the case that the WFs Ψ_D, Ψ_C are presented in the form of superposition of the oscillator WFs the calculations of “new” characteristics can be carried out by the following way:

1. The eigenvalues ε_k and the eigenfunctions $f_{kl}(\rho)$ are found by diagonalization of the norm kernel matrix:

$$\| N_{nn'} \| = < \Psi_D \phi_{nl}(\rho) \Psi_C \,|\, \hat{A}^2 \,|\, \Psi_D \phi_{n'l}(\rho) \Psi_C > .$$

$$f_l^k(\rho) = \sum_n B_{nl}^k \phi_{nl}(\rho) .$$

$$\varepsilon_k = < \Psi_D f_l^k(\vec{\rho}) \Psi_C \,|\, \hat{A}^2 \,|\, \Psi_D f_l^k(\vec{\rho}) \Psi_C > .$$

2. The “new” cluster form factor $\Phi'_l(\rho)$ is expanded onto the eigenfunctions of the norm kernel :

$$\Phi'_l(\rho) = \sum_k \varepsilon_k^{-1/2} < \Phi'_l(\rho) \mid f_{kl}(\rho) f_{kl}(\rho) >=$$

$$\sum_k \varepsilon_k^{-1/2} \sum_n C_{MDC}^{nl} B_{nl}^k \, \phi_{nl}(\rho).$$

the “new” spectroscopic factor takes the form

$$S_{MDC}^{l'} = \sum_k \varepsilon_k^{-1} \sum_{nn'} C_{MDC}^{nl} C_{MDC}^{n'l} B_{nl}^k \, B_{n'l}^k .$$

Inserting the complete set of the resonance wave functions

$$1 = \sum_i |\Psi_{M_i}\rangle\langle\Psi_{M_i}|$$

it is easy to deduce the relationship:

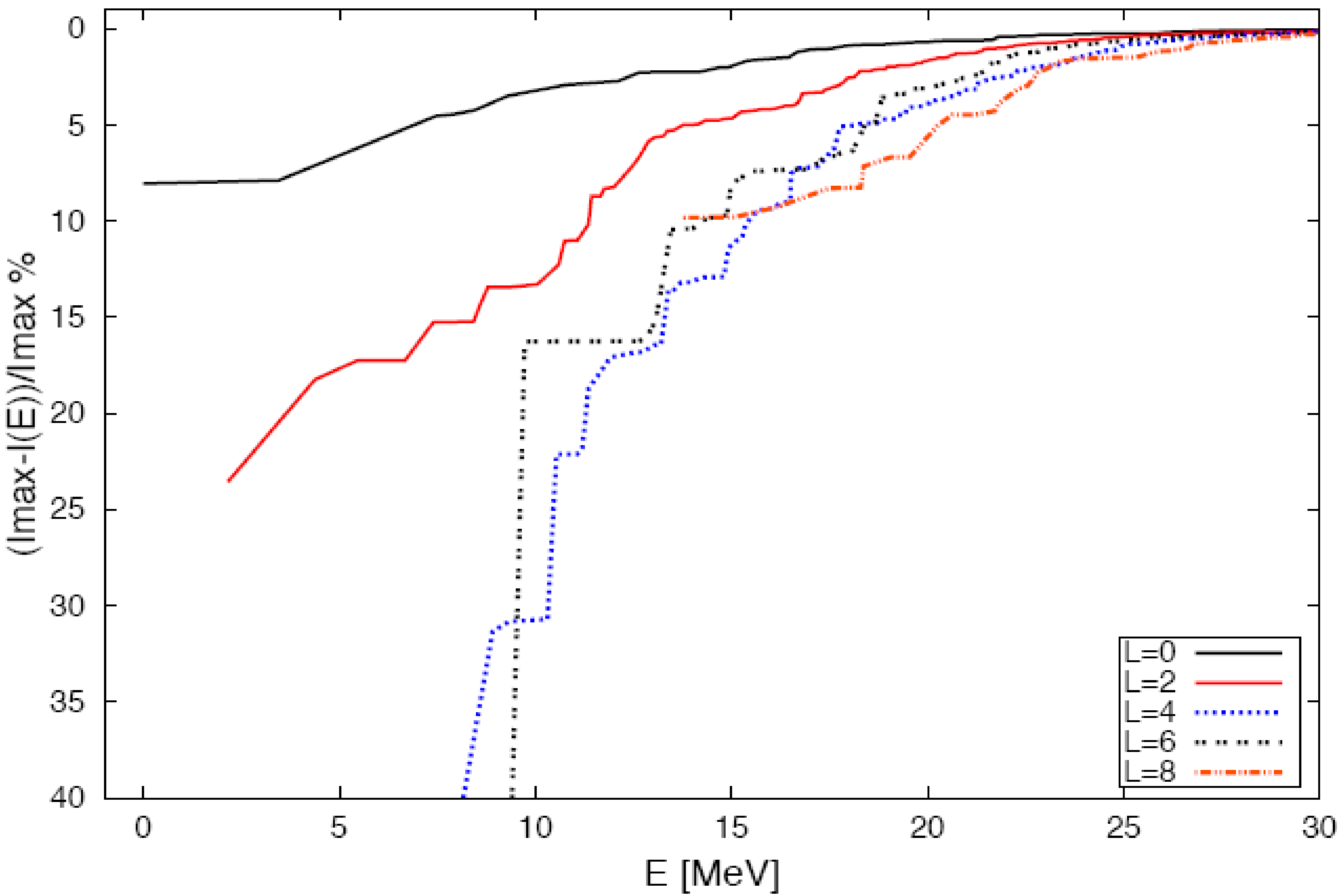
$$1 = \varepsilon_k^{-1} \sum_{inn'} C_{M_i DC}^{nl} C_{M_i DC}^{n'l} B_{nl}^{k} B_{n'l}^{k}$$

Performing summation over k one can obtain:

$$\sum_i S_{M_i DC}^{l'} = \dim \| k \|$$

The sum rule of the “new” spectroscopic factors corresponding to a fixed value of n (cluster strength in 2ħω domain turn out to be unity. Thus the statistical properties are described accurately. That is crutial for the dense spectra

α-CLUSTER STRENGTH IN 32S SPECTRUM



CLUSTERING IN ADVANCED SHELL MODEL APPROACHES

As usual the WFs of the modern versions of the shell model are:

a) presented in the form of a superposition of A-nucleon oscillator WFs,

b) fulfill the factorization condition:

$$\Psi_{M(D)}(R_{M(D)}) = \varphi_{000}(R_{M(D)})\Psi_{M(D)}.$$

Therefore they are convenient in operating in the just presented formalism.

As that is the case for approaches proposed earlier the lowest oscillator wave function of a cluster is used in the approach:

$$\Psi_{\alpha} = | X = 4N = 0[f] = [4](\lambda\mu)L = 0S = 0T = 0 >$$

where [f] is the symbol of the permutation symmetry (Young frame) and (λμ) – the SU(3) symmetry (Elliott symbol). The problem is concentrated on the calculation of the fractional parentage coefficient :

$$< \Psi_M(R_M) | \hat{A}\{\Psi_D(R_D)\Psi_{XN}\} >$$

To do that within the shell model approach normalized SU(3) states in are constructed by diagonalization of the SU(3) Casimir operator. In the explicit form these operators can be written as:

$$\hat{C} = (Q \cdot Q) - 3L^2$$

where the projection of the Hermitian conjugated quadrupole operator takes the form:

$$Q^m = \sqrt{4\pi / 5} \sum_{j=1}^{A} ((\rho_j^2 / \rho_{j0}^2) Y_{2m}(\vartheta j, \phi_{\rho j}) + (p_j^2 / p_{j0}^2) Y_{2m}(\vartheta_{pj}, \phi_{pj}))$$

L – operator of angular momentum.

From the technical point of view Casimir operator is conveniently expressed in the formalism of the fermion second quantization:

$$\hat{C} = \hat{B}^{\dagger}\hat{B}$$

$$\hat{B}^{\dagger} = \sum_{\{1,2,3,..X\}} b_{\{1,2,3,..,X\}} a_1^{\dagger} a_2^{\dagger} a_3^{\dagger} ... a_X^{\dagger}$$

$$\hat{B} = \sum_{\{1,2,3,..X\}} b^{*}_{\{1,2,3,..,X\}} a_X, a_{X-1}, ..., a_1$$

To determine the permutation symmetry in each state obtained by this way the operator:

$$F_{ij} = 1/2(1 + P_{ij}^{sp})$$

is used. Its mean values are different for different Young frames [f].

This approach as a whole was called Cluster-Nucleon Configuration Interaction Model (CNCIM) and presented first time in the paper [A. Volya, Yu.M. Tchuvil’sky. IASEN Conference Proceedings. World Scientific (2014)].

The results of calculations presented bellow are restricted by (sd)-shell.

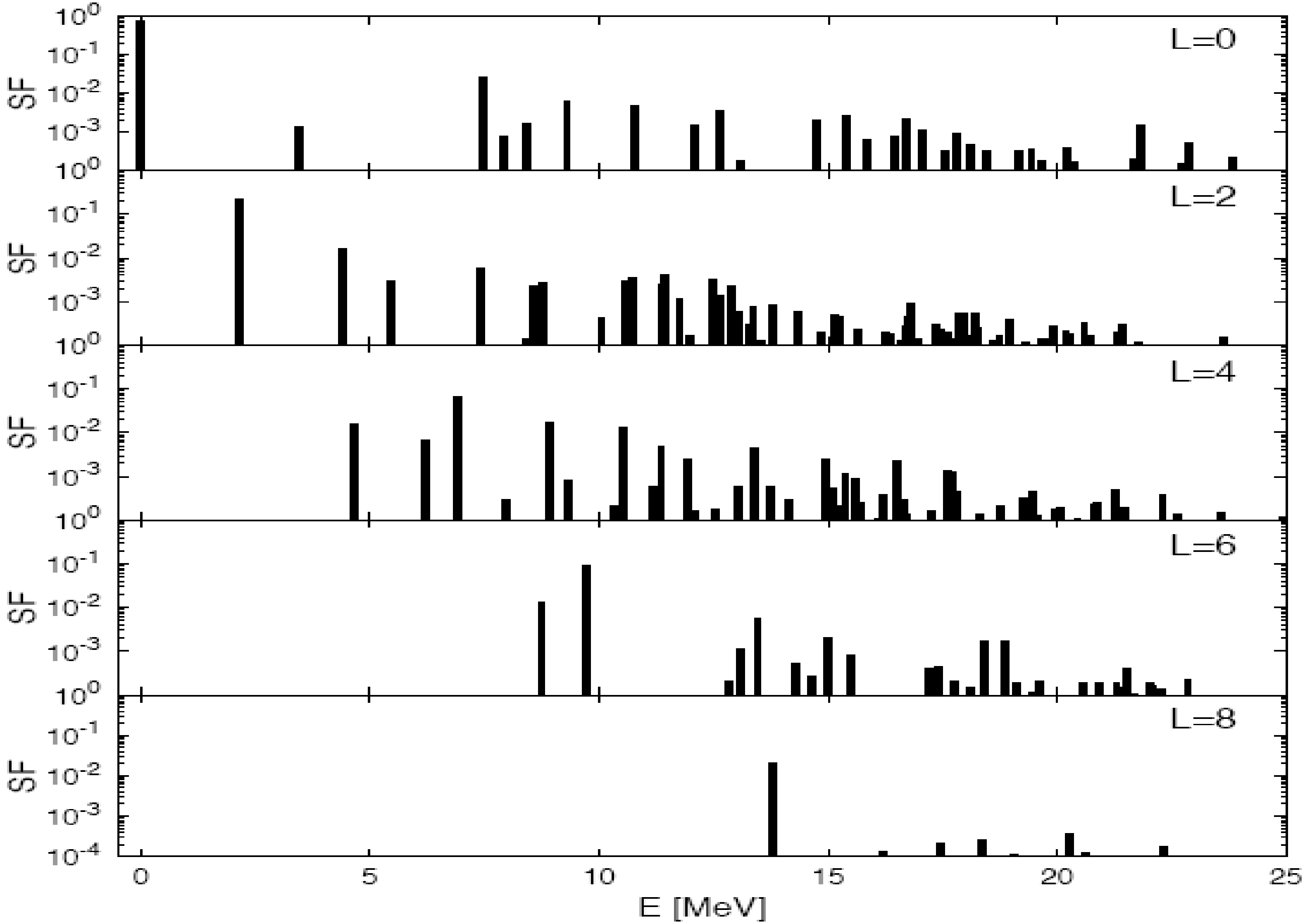
The Hamiltonian proposed in the paper [Y. Utsuno, S. Chiba, Phys. Rev. C 83, 021301 (2011) is used.

For 32S and 24Mg bellow the core is 16O and the size of the basis is about $10^4 \times 10^4$.

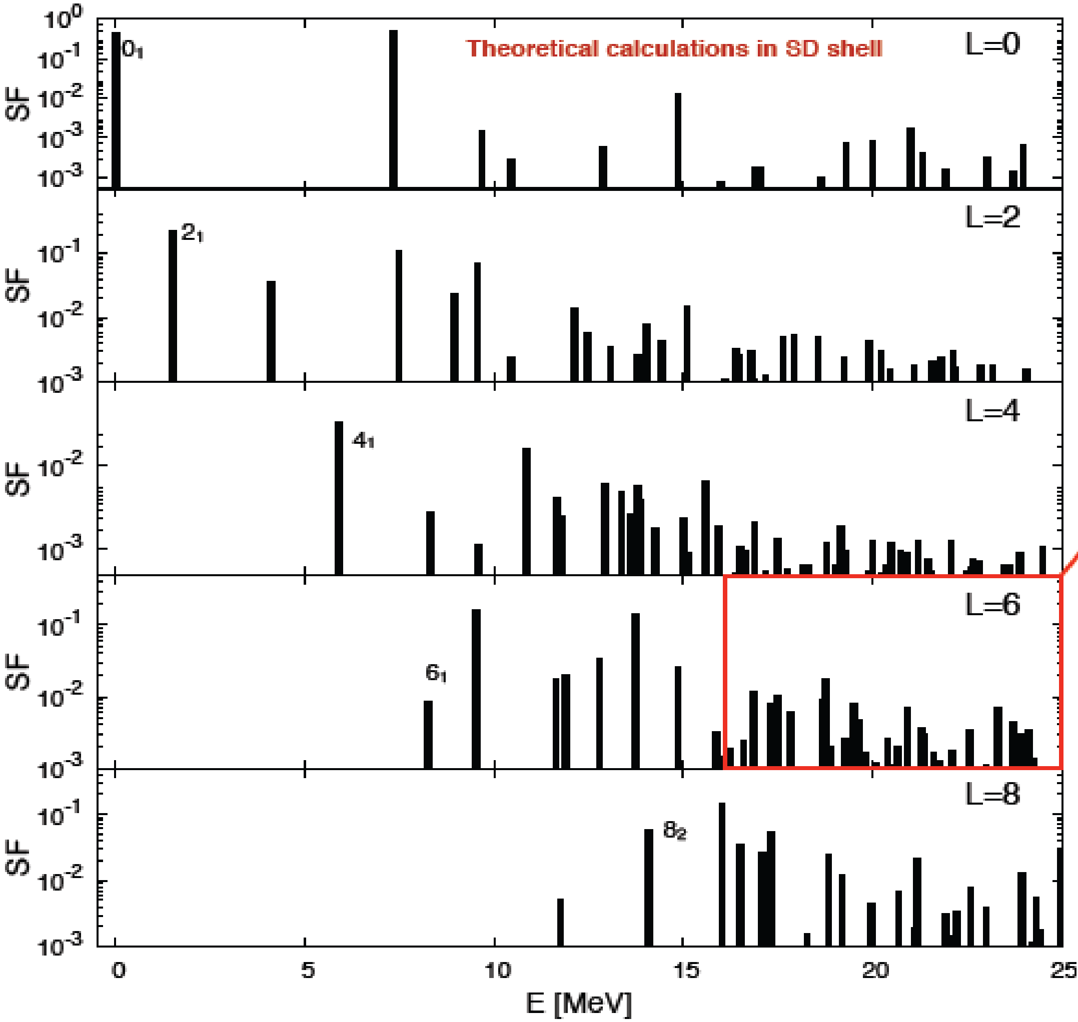
For 16O the core is 4He. The size of the basis is about $10^{7.5} \times 3 \cdot 10^{7.5}$.

EXAMPLES OF CALCULATIONS OF SPECTROSCOPIC FACTORS OF α-CLUSTERS

32S

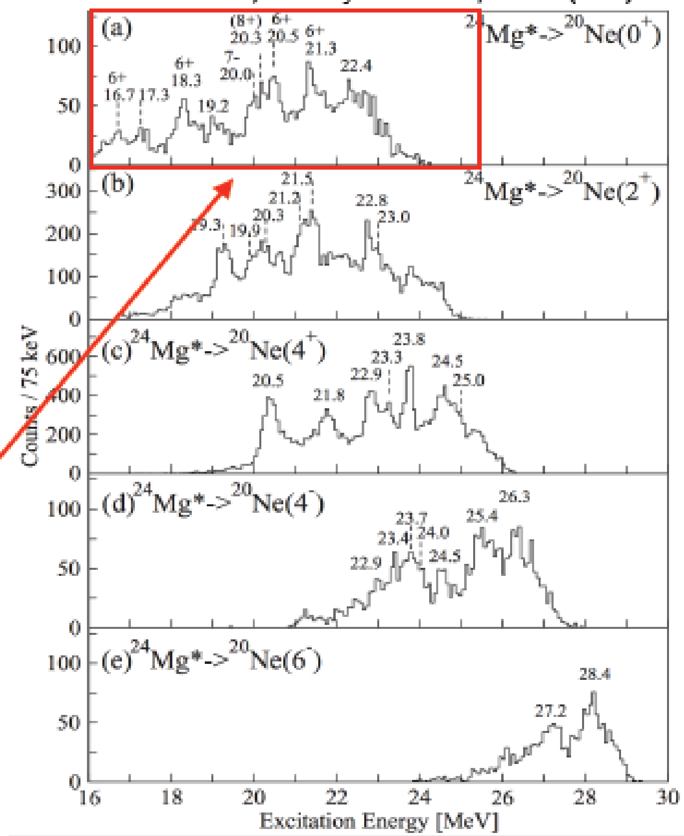


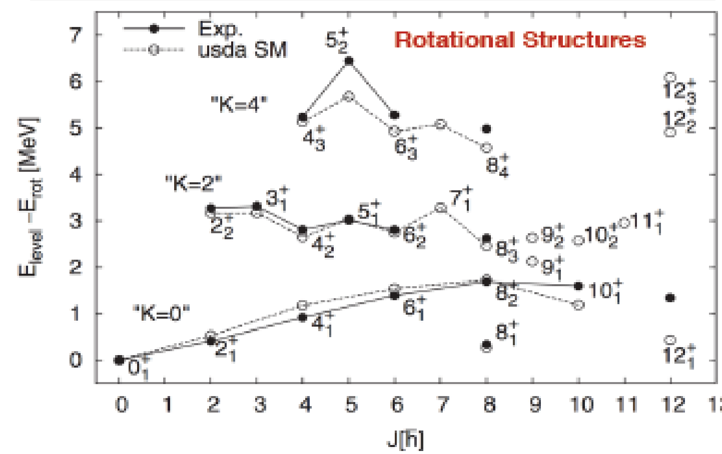
Alpha cluster spectroscopic factors in ^{24}Mg



Experimental results

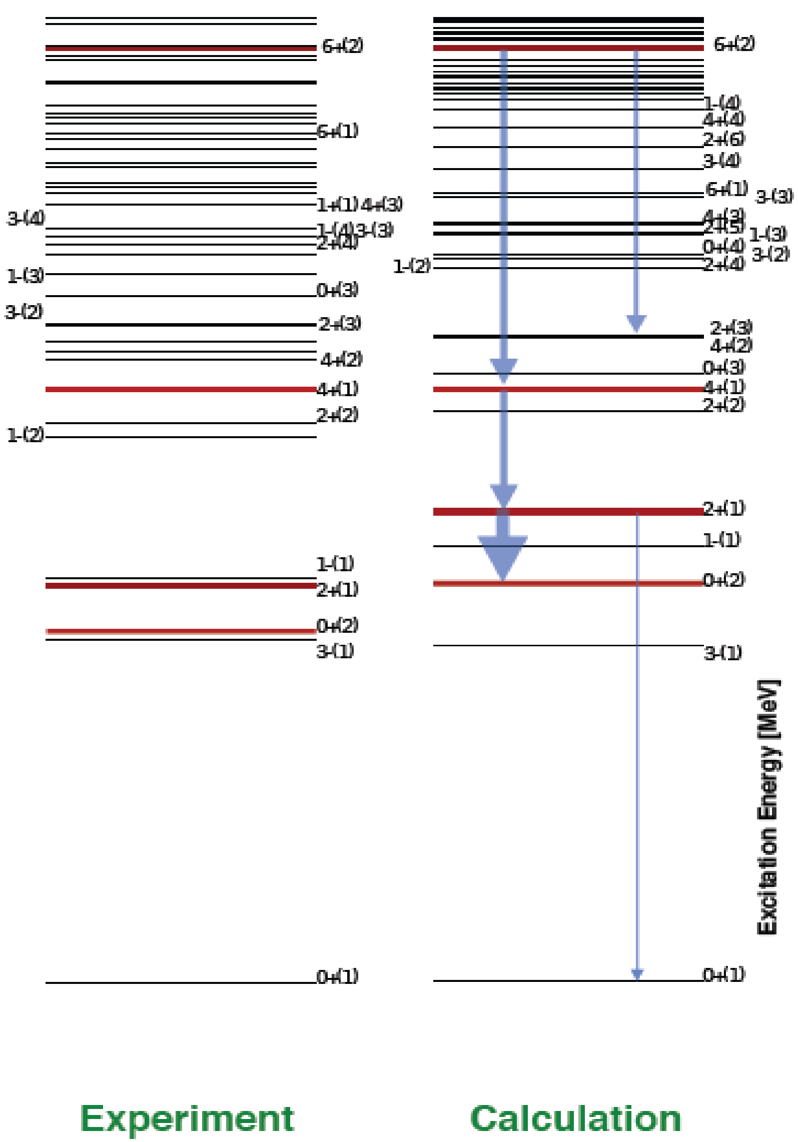
E. S. Diffenderfer, et.al Phys. Rev. C 85, 034311 (2012).

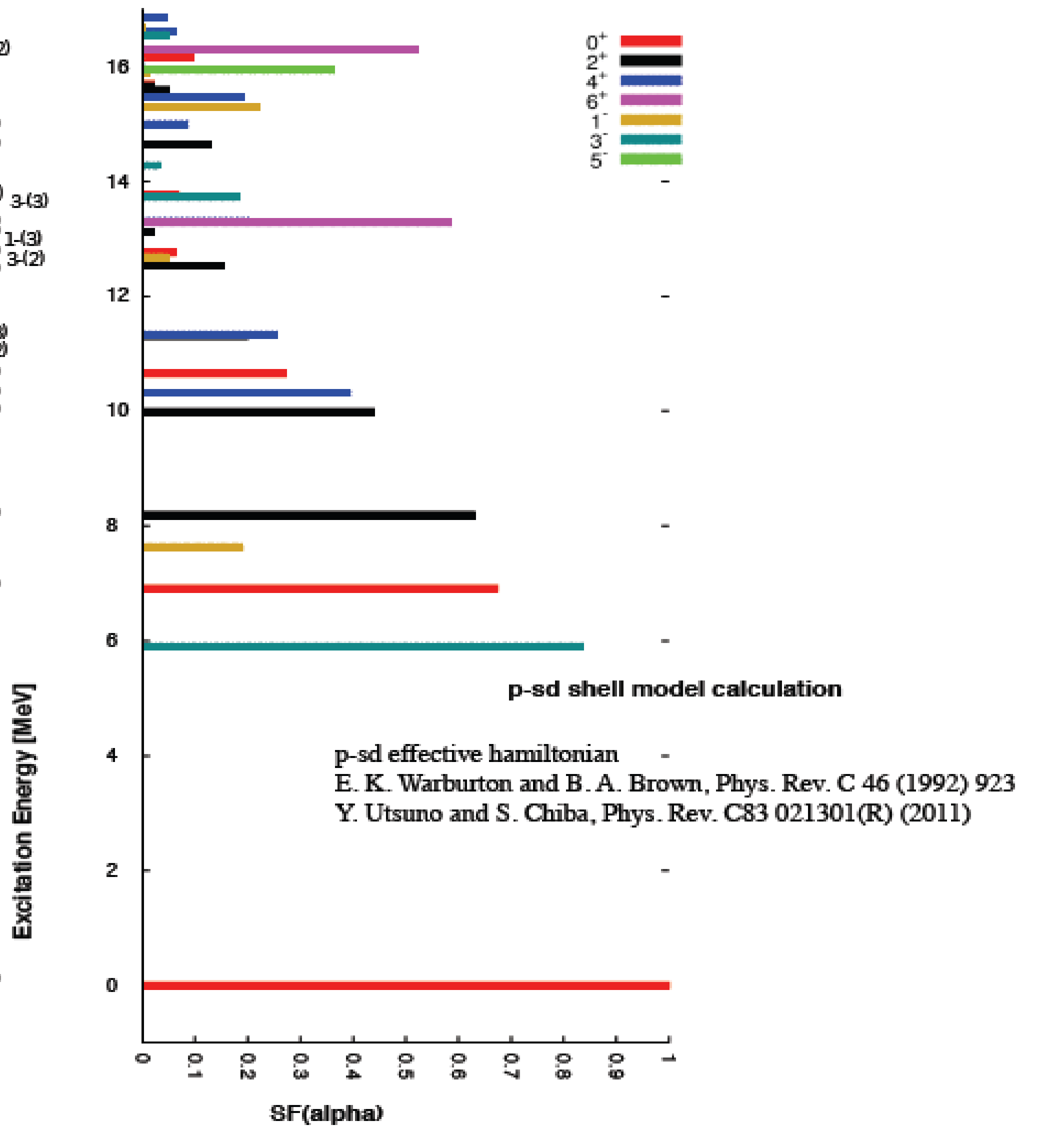




Alpha cluster rotational bands

^{16}O structure





CONCLUSIONS

1. A theoretical approach and mathematics making possible to calculate cluster spectroscopic amplitudes, form factors and spectroscopic factors in advanced versions of the shell model including no-core one is built.
2. It is proved that this the expedient allows one to describe accurately the statistical properties of dense cluster spectra.
3. Using this approach pioneering descriptions of the spectroscopic characteristics of dense spectra of highly excited states of nuclei are obtained.
4. The example demonstrating that the cluster observables may be a tool of the test on the quality of a dynamical model is found.

5. The approach already built looks promising for applications in various areas of the cluster physics.

6. We see ways of great improvement of the developed approach such as: involving of realistic cluster wave functions, description of heavy cluster channels, creation of hybrid models, etc.

THANK U 4 ATTENTION!