J-matrix Analysis of Resonant States in the Shell Model: Charged Particles

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Motivation

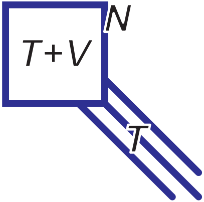
Main goal of this approach — extracting resonant characteristics from only $E_\nu(N, \hbar\Omega)$ in No Core Shell Model

Coulomb interaction is long-range $\Rightarrow$ applying "standard" J-matrix formalism to evaluate phase shift is impossible

J-matrix formalism: brief overview

Oscillator basis (with $\hbar\Omega$ parameter)

Potential energy matrix is truncated
to size $N \times N$ (matrix elements of short-range
potential quickly decrease in limit $n \to \infty$)



Kinetic energy matrix is full and infinity
(matrix elements lineary increase in limit $n \to \infty$)

$$\tan \delta_\ell(E) = -\frac{S_{N,\ell} - G_{N,N} T_{N,N+1,\ell} S_{N+1,\ell}}{C_{N,\ell} - G_{N,N} T_{N,N+1,\ell} C_{N+1,\ell}}$$

$S_{n,\ell}(E)$, $C_{n,\ell}(E)$ — free solutions in ocsillator basis

J-matrix formalism: brief overview

$$G_{N,N} = -\sum_{\nu=0}^{N} \frac{|\langle N|\nu\rangle|^2}{E_\nu - E},$$

E_ν — eigenvalue of truncated Hamiltonian
This sum is hard to evaluate!

In case of $E = E_\nu$

$$\tan\delta_\ell(E_\nu) = -\frac{S_{N+1,\ell}(E_\nu)}{C_{N+1,\ell}(E_\nu)}$$

J-matrix formalism in case of charged particle

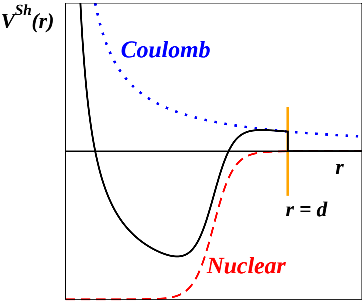
Auxiliary potential

$$V^{Sh} = \begin{cases} V^{Nucl} + V^{Coul}, & r \le d; \\ 0, & r > d; \end{cases} \quad d \ge R_{Nucl}$$

Optimal value of d is determined by classical turning point of basis state with highest oscillator quantum number N

$$d = 2r_0\sqrt{N + \ell/2 + 3/4}$$

$r_0 = \sqrt{\frac{\hbar}{\mu\Omega}}$ — oscillator radius



J-matrix formalism in case of charged particle

Calculating δ_ℓ^{Sh} through "standard" J-matrix
Matching with Coulomb asymptotics

$$\tan\delta_\ell = -\frac{W_d(j_\ell, F_\ell) + W_d(n_\ell, F_\ell)\tan\delta_\ell^{Sh}}{W_d(j_\ell, G_\ell) + W_d(n_\ell, G_\ell)\tan\delta_\ell^{Sh}}$$

Quasi-Wronskins

$$W_d(j_\ell, F_\ell) = \left(\frac{d}{dr}[j_\ell(kr)]F_\ell(\xi, kr) - j_\ell(kr)\frac{d}{dr}[F_\ell(\xi, kr)]\right)\Bigg|_{r=d}$$

j_ℓ, n_ℓ — spherical Bessel and Neumann functions,
F_ℓ, G_ℓ — Coulomb functions,
ξ — Sommerfeld parameter

J-matrix formalism in case of charged particle

Phase shift for V^{Sh} potential ($E = E_\nu$):

$$\tan\delta_\ell^{Sh}(E_\nu) = -\frac{S_{N+1,\ell}(E_\nu)}{C_{N+1,\ell}(E_\nu)}$$

E_ν — eigenvalue of Hamiltonian with truncated potential

$$\tan\delta_\ell(E_\nu) = -\frac{W_d(n_\ell, F_\ell)S_{N+1,\ell}(E_\nu) - W_d(j_\ell, F_\ell)C_{N+1,\ell}(E_\nu)}{W_d(n_\ell, G_\ell)S_{N+1,\ell}(E_\nu) - W_d(j_\ell, G_\ell)C_{N+1,\ell}(E_\nu)}$$

In vicinity of resonance

Phase shift in vicinity of resonance:

$$\delta_\ell(E) = -\arctan\frac{a\sqrt{E}}{E-b^2} + c\sqrt{E}$$

a, b describe resonant behavior

$$E_r = b^2 - \frac{a^2}{2}, \quad \frac{\Gamma}{2} = a\sqrt{b^2 - \frac{a^2}{4}}$$

c describes contribution of other S-matrix poles

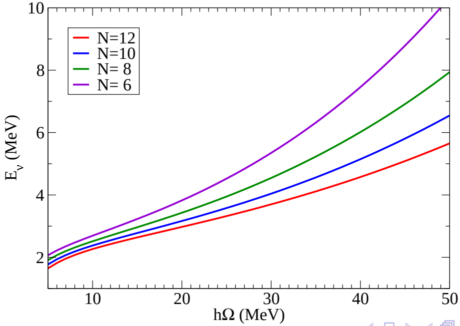
$E_\nu(N, \hbar\Omega)$ function

Equation for defining implicit function $E_\nu(N, \hbar\Omega)$ (with fixed a, b, c)

$$-\frac{W_d(n_\ell, F_\ell)S_{N+1,\ell}(E_\nu) - W_d(j_\ell, F_\ell)C_{N+1,\ell}(E_\nu)}{W_d(n_\ell, G_\ell)S_{N+1,\ell}(E_\nu) - W_d(j_\ell, G_\ell)C_{N+1,\ell}(E_\nu)} = $$

$$= -\arctan\frac{a\sqrt{E_\nu}}{E_\nu - b^2} + c\sqrt{E_\nu}$$

Typical dependence, note $d\delta_\ell/dE > 0$



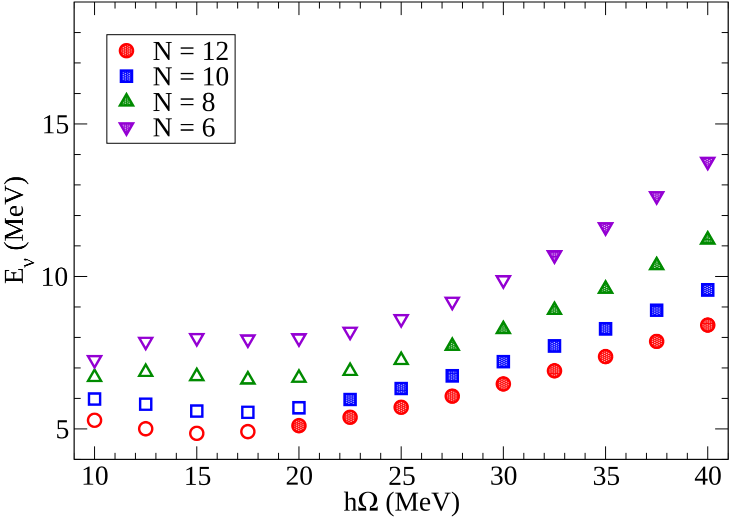
No Core Shell Model

- States $\frac{1}{2}^-$ and $\frac{3}{2}^-$ of ^{5}Li in NCSM
- NN potential — JISP16
- Energy from $p\alpha$ threshold

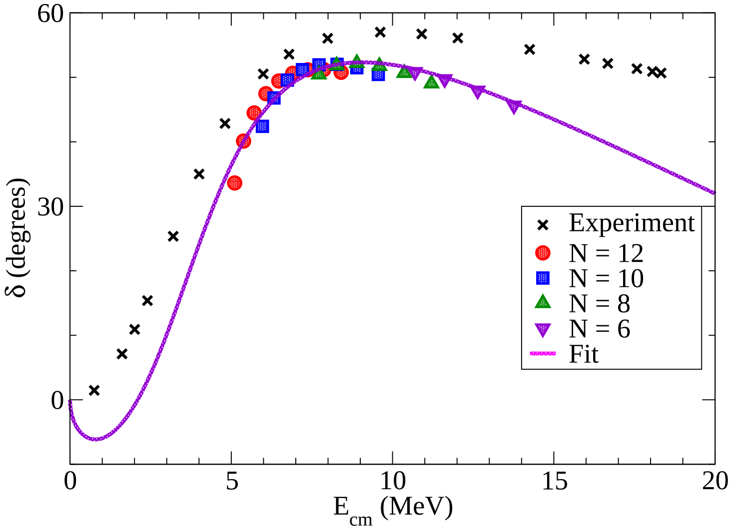
$$E_\nu(N,\hbar\Omega) = E_\nu^{^5\mathrm{Li}}(N,\hbar\Omega) - E_\nu^{^4\mathrm{He,\ gs}}(N,\hbar\Omega)$$

- a, b, c is fitted to NCSM results
- Selection criteria: $d\delta_\ell/dE > 0$

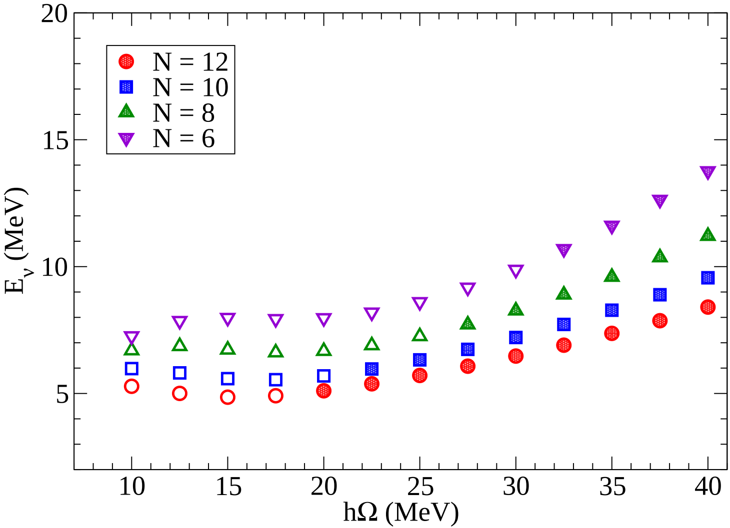
$p\alpha$ scattering, resonant state $1/2^-$



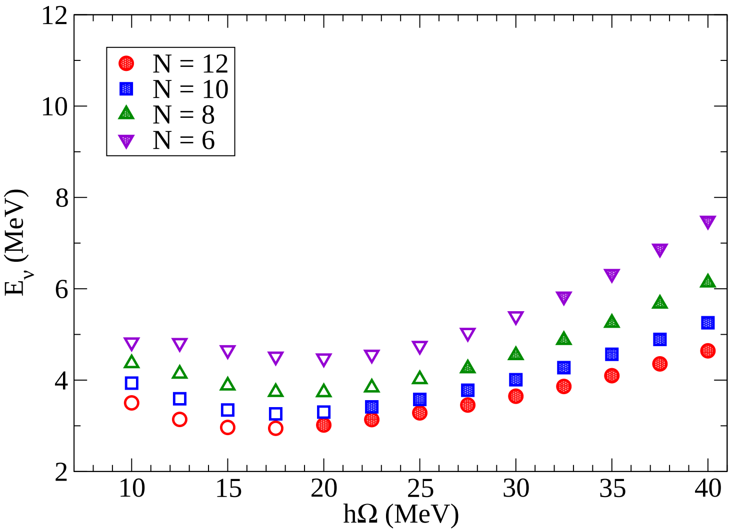
$p\alpha$ scattering, resonant state $1/2^-$



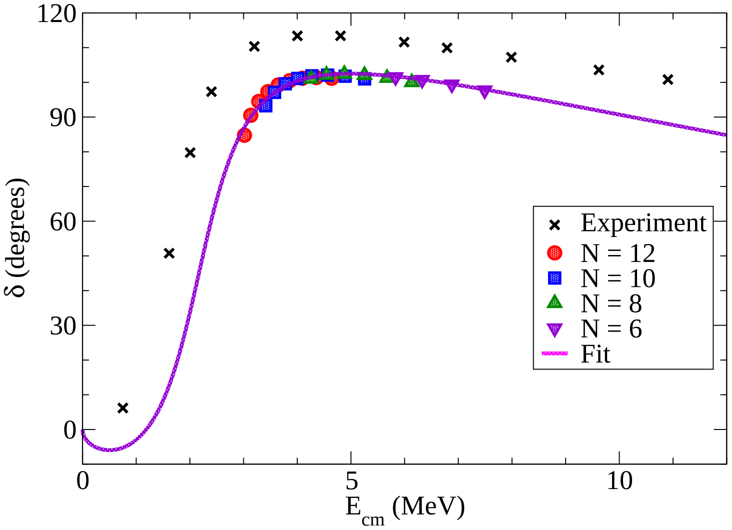
$p\alpha$ scattering, resonant state $1/2^-$



$p\alpha$ scattering, resonant state $3/2^-$



$p\alpha$ scattering, resonant state $3/2^-$



Results

Resonant state $\frac{1}{2}^-$

	Calculations	Experiment
E_r, MeV	3.478	2.85 ± 0.05
Γ, MeV	5.659	6.15 ± 0.10

Resonant state $\frac{3}{2}^-$

	Calculations	Experiment
E_r, MeV	2.157	1.658 ± 0.005
Γ, MeV	1.512	1.26 ± 0.01

Conclusions

- J-matrix approach can be extended to description of resonant states in systems with Coulomb interaction
- This approach was applied to describe $p\alpha$ scattering, convergence is obtained

Thank you for attention!