Origin and Properties of Strong Inter-Nucleon Interactions[1]

R. Machleidt

Department of Physics, University of Idaho, Moscow, Idaho 83844, USA

Abstract

I start with a historical review of the attempts to construct theories for the origin of nuclear forces, for which I also summaries the most important properties. The review then shifts to its main focus, which is the chiral effective field theory approach to nuclear forces. I summarize the current status of this approach and discuss the most important open issues: the proper renormalization of the chiral two-nucleon potential and sub-leading three-nucleon forces.

Keywords: *Nuclear forces; nucleon-nucleon scattering; low-energy QCD; effective field theory; renormalization; few-nucleon forces*

1 Introduction and overview

The nuclear force problem is as old as *James Vary*, namely seven decades. What a coincidence!

The development of a proper theory of nuclear forces has occupied the minds of some of the brightest physicists and has been one of the main topics of physics research in the 20th century. The original idea was that the force is caused by the exchange of lighter particles (than nucleons) known as mesons, and this idea gave rise to the birth of a new sub-field of modern physics, namely, (elementary) particle physics. The modern perception of the nuclear force is that it is a residual interaction (similar to the van der Waals force between neutral atoms) of the even stronger force between quarks, which is mediated by the exchange of gluons and holds the quarks together inside a nucleon.

1.1 Early history

After the discovery of the neutron in 1932, it was clear that the atomic nucleus is made up from protons and neutrons. In such a system, electromagnetic forces cannot be the reason why the constituents of the nucleus are sticking together. Therefore, the concept of a new strong nuclear interaction was introduced. In 1935, the first theory for this new force was developed by the Japanese physicist Yukawa [1], who suggested that the nucleons would exchange particles between each other and this mechanism would create the force. Yukawa constructed his theory in analogy to the theory of the electromagnetic interaction where the exchange of a (massless) photon is the cause of the force. However, in the case of the nuclear force, Yukawa assumed that the

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[1]Dedicated to James Vary on the occasion of his 70th birthday.

"force-makers" (which were eventually called "mesons") carry a mass of a fraction of the nucleon mass. This would limit the effect of the force to a finite range, since the uncertainty principal allows massive particles to travel only a finite distance. The meson predicted by Yukawa was finally found in 1947 in cosmic ray and in 1948 in the laboratory and called the pion. Yukawa was awarded the Nobel Prize in 1949. In the 1950's and 60's more mesons were found in accelerator experiments and the meson theory of nuclear forces was extended to include many mesons. These models became know as one-boson-exchange models, which is a reference to the fact that the different mesons are exchanged singly in this model. The one-boson-exchange model is very successful in explaining essentially all properties of the nucleon-nucleon interaction at low energies [2–6]. In the 1970's and 80's, also meson models were developed that went beyond the simple single-particle exchange mechanism. These models included, in particular, the explicit exchange of two pions with all its complications. Well-known representatives of the latter kind are the Paris [7] and the Bonn potential [8].

Since these meson models were quantitatively very successful, it appeared that they were the solution of the nuclear force problem. However, with the discovery (in the 1970's) that the fundamental theory of strong interactions is quantum chromodynamics (QCD) and not meson theory, all "meson theories" had to be viewed as models, and the attempts to derive a proper theory of the nuclear force had to start all over again.

1.2   QCD and the nuclear force

The problem with a derivation of nuclear forces from QCD is two-fold. First, each nucleon consists of three quarks such that the system of two nucleons is already a six-body problem. Second, the force between quarks, which is created by the exchange of gluons, has the feature of being very strong at the low energy-scale that is characteristic of nuclear physics. This extraordinary strength makes it difficult to find "converging" mathematical solutions. Therefore, during the first round of new attempts, QCD-inspired quark models became popular. The positive aspect of these models is that they try to explain nucleon structure (which consists of three quarks) and nucleon-nucleon interactions (six-quark systems) on an equal footing. Some of the gross features of the two-nucleon force, like the "hard core" are explained successfully in such models. However, from a critical point of view, it must be noted that these quark-based approaches are yet another set of models and not a theory. Alternatively, one may try to solve the six-quark problem with brute computing power, by putting the six-quark system on a four dimensional lattice of discrete points which represents the three dimensions of space and one dimension of time. This method has become known as lattice QCD and is making progress. However, such calculations are computationally very expensive and cannot be used as a standard nuclear physics tool.

1.3   Chiral effective field theory

Around 1990, a major breakthrough occurred when the nobel laureate Steven Weinberg applied the concept of an effective field theory (EFT) to low-energy QCD [9,10]. He simply wrote down the most general theory that is consistent with all the properties of low-energy QCD, since that would make this theory identical to low-energy QCD. A particularly important property is the so-called chiral symmetry, which is "spontaneously" broken. Massless particles observe chiral symmetry, which means that their spin and momentum are either parallel ("right-handed") or anti-parallel ("left-handed") and remain so forever. Since the quarks, which nucleons are made of ("up" and "down" quarks), are almost mass-less, approximate chiral symmetry is a given. Naively, this symmetry should have the consequence that one finds in nature mesons of the same mass, but with positive and negative parity. However, this is

not the case and such failure is termed a "spontaneous" breaking of the symmetry. According to a theorem first proven by Goldstone, the spontaneous breaking of a symmetry creates a particle, here, the pion. Thus, the pion becomes the main player in the production of the nuclear force. The interaction of pions with nucleons is weak as compared to the interaction of gluons with quarks. Therefore, pion-nucleon processes can be calculated without problem. Moreover, this effective field theory can be expanded in powers of momentum/scale, where "scale" denotes the "chiral symmetry breaking scale" which is about 1 GeV. This scheme is also known as chiral perturbation theory (ChPT) and allows to calculate the various terms that make up the nuclear force systematically power by power, or order by order. Another advantage of the chiral EFT approach is its ability to generate not only the force between two nucleons, but also many-nucleon forces, on the same footing [11]. In modern theoretical nuclear physics, the chiral EFT approach is becoming increasingly popular and is applied with great success [12, 13].

1.4   Main properties of the nuclear force and phenomenological potentials

Some properties of nuclear interactions can be deduced from the properties of nuclei. The property of saturation suggests that nuclear forces are of short range (a few fm) and strongly attractive at that range, which explains nuclear binding. But the nuclear force has also a very complex spin-dependence. First evidence came from the observation that the deuteron (proton-neutron bound state, smallest atomic nucleus) deviates slightly from a spherical shape. This suggests a force that depends on the orientation of the spins of the nucleons with regard to the line connecting the two nucleons (tensor force). In heavier nuclei, a shell structure has been observed which according to a suggestion by Mayer and Jensen can be explained by a strong force between the spin of the nucleon and its orbital motion (spin-orbit force). More clear-cut evidence for the spin-dependence is extracted from scattering experiments where one nucleon is scattered off another nucleon. In such experiments, the existence of the nuclear spin-orbit and tensor forces has clearly been established. Scattering experiments at higher energies (more than 200 MeV) show indications that the nucleon-nucleon interaction at very short distances (smaller than 0.5 fm) becomes repulsive ("hard core"). Besides the force between two nucleons (2NF), there are also three-nucleon forces (3NF), four-nucleon forces (4NF), etc. However, the 2NF is much stronger than the 3NF, which in turn is much stronger than the 4NF, etc. In exact calculations of the properties of light nuclei based upon the bare nuclear forces, it has been shown that 3NFs are important. Their contribution is small, but crucial. The need for 4NF for explaining nuclear properties has not (yet) been clearly established.

Phenomenological nucleon-nucleon (NN) potentials are constructed in close relationship to the empirical facts. In this regard, the most faithful method of construction is inverse scattering theory, which the so-called JISP-16 potentials are based upon [14].

In the following sections, I will elaborate more on the theory of nuclear forces with particular emphasis on the view according to which the forces between nucleons emerge from low-energy QCD via an effective field theory.

2   Effective field theory for low-energy QCD

Quantum chromodynamics (QCD) is the theory of strong interactions. It deals with quarks, gluons and their interactions and is a part of the Standard Model of Particle Physics. QCD is a non-Abelian gauge field theory with color SU(3) as the underlying gauge group. The non-Abelian nature of the theory has dramatic consequences. While the interaction between colored objects is weak at short distances or high momentum

transfer ("asymptotic freedom"); it is strong at long distances ($\gtrsim 1$ fm) or low energies, leading to the confinement of quarks into colorless objects, the hadrons. Consequently, QCD allows for a perturbative analysis at large energies, whereas it is highly non-perturbative in the low-energy regime. Nuclear physics resides at low energies and the force between nucleons is a residual color interaction similar to the van der Waals force between neutral molecules. Therefore, in terms of quarks and gluons, the nuclear force is a very complicated problem that, nevertheless, can be attacked with brute computing power on a discretized, Euclidean space-time lattice (known as lattice QCD). In a recent study [15], the neutron-proton scattering lengths in the singlet and triplet S-waves have been determined in fully dynamical lattice QCD. This result is then extrapolated to the physical pion mass with the help of chiral perturbation theory. The pion mass of 354 MeV is still too large to allow for reliable extrapolations, but the feasibility has been demonstrated and more progress can be expected for the near future. In a lattice calculation of a very different kind, the NN potential was studied [16]. The central part of the potential shows a repulsive core plus attraction of intermediate range. This is a very promising result, but it must be noted that also in this investigation still rather large pion masses are being used. In any case, advanced lattice QCD calculations are under way and continuously improved. However, since these calculations are very time-consuming and expensive, they can only be used to check a few representative key-issues. For everyday nuclear structure physics, a more efficient approach is needed.

The efficient approach is an effective field theory. For the development of an EFT, it is crucial to identify a separation of scales. In the hadron spectrum, a large gap between the masses of the pions and the masses of the vector mesons, like $\rho(770)$ and $\omega(782)$, can clearly be identified. Thus, it is natural to assume that the pion mass sets the soft scale, $Q \sim m_\pi$, and the rho mass is associated with the hard scale, $\Lambda_\chi \sim m_\rho$, also known as the chiral-symmetry breaking scale. This is suggestive of considering an expansion in terms of the soft scale over the hard scale, Q/Λ_χ. Concerning the relevant degrees of freedom, we noticed already that, for the ground state and the low-energy excitation spectrum of an atomic nucleus as well as for conventional nuclear reactions, quarks and gluons are ineffective degrees of freedom, while nucleons and pions are the appropriate ones. To make sure that this EFT is not just another phenomenology, it must have a firm link with QCD. The link is established by having the EFT to observe all relevant symmetries of the underlying theory. This requirement is based upon a 'folk theorem' by Weinberg [9]:

> If one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry principles.

In summary, the EFT program consists of the following steps:

1. Identify the soft and hard scales, and the degrees of freedom (DOF) appropriate for (low-energy) nuclear physics. Soft scale: $Q \sim m_\pi$, hard scale: $\Lambda_\chi \sim m_\rho \sim 1$ GeV; DOF: pions and nucleons.

2. Identify the relevant symmetries of low-energy QCD and investigate if and how they are broken: explicitly and spontaneously broken chiral symmetry (spontaneous symmetry breaking generates the pions as Goldstone bosons).

3. Construct the most general Lagrangian consistent with those symmetries and symmetry breakings, see Ref. [13].

4. Design an organizational scheme that can distinguish between more and less important contributions: a low-momentum expansion, $(Q/\Lambda_\chi)^\nu$, with ν determined by 'power counting'. For an irreducible diagram that involves A nucleons, we have:

$$\nu = -2 + 2A - 2C + 2L + \sum_i \Delta_i \,. \tag{1}$$

where

$$\Delta_i \equiv d_i + \frac{n_i}{2} - 2 \,, \tag{2}$$

with C being the number of separately connected pieces and L being the number of loops in the diagram; d_i is the number of derivatives or pion-mass insertions and n_i is the number of nucleon fields (nucleon legs) involved in vertex i; the sum runs over all vertices i contained in the diagram under consideration. Note that for an irreducible NN diagram ($A = 2$, $C = 1$), the power formula collapses to the very simple expression

$$\nu = 2L + \sum_i \Delta_i \,. \tag{3}$$

5. Guided by the expansion, calculate Feynman diagrams for the problem under consideration to the desired accuracy (see next Section).

3   The hierarchy of nuclear forces in chiral EFT

Chiral perturbation theory and power counting imply that nuclear forces emerge as a hierarchy controlled by the power ν, Fig. 1.
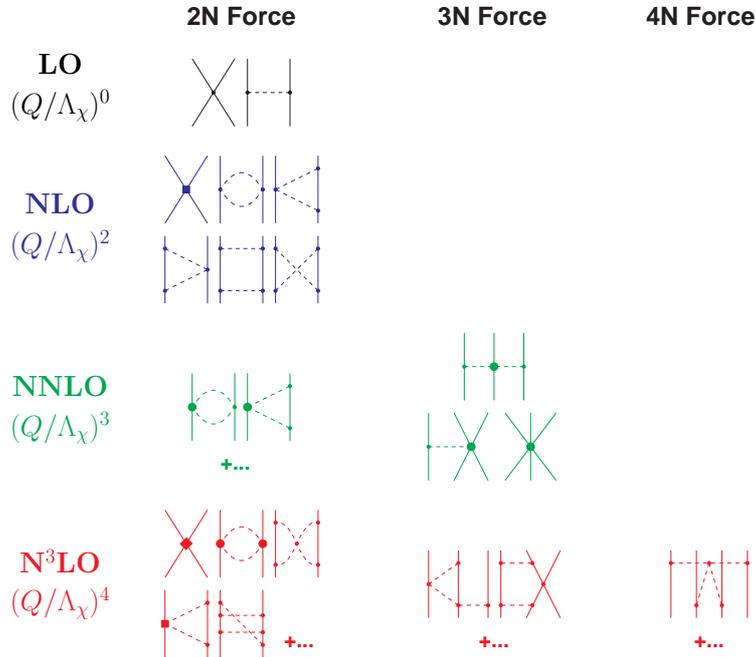

Figure 1: Hierarchy of nuclear forces in ChPT. Solid lines represent nucleons and dashed lines pions. Small dots, large solid dots, solid squares, and solid diamonds denote vertices of index $\Delta_i = 0$, 1, 2, and 4, respectively. Further explanations are given in the text.

In the lowest order, better known as leading order (LO, $\nu = 0$), the NN amplitude is made up by two momentum-independent contact terms ($\sim Q^0$), represented by the four-nucleon-leg graph with a small-dot vertex shown in the first row of Fig. 1, and static one-pion exchange (1PE), the second diagram in the first row of the figure. This is, of course, a rather rough approximation to the two-nucleon force (2NF), but it accounts already for some important features. The 1PE provides the tensor force, necessary to describe the deuteron, and it explains NN scattering in peripheral partial waves of very high orbital angular momentum. At this order, the two contacts which contribute only in S-waves provide the short- and intermediate-range interaction which is somewhat crude.

In the next order, $\nu = 1$, all contributions vanish due to parity and time-reversal invariance.

Therefore, the next-to-leading order (NLO) is $\nu = 2$. Two-pion exchange (2PE) occurs for the first time ("leading 2PE") and, thus, the creation of a more sophisticated description of the intermediate-range interaction is starting here. Since the loop involved in each pion-diagram implies already $\nu = 2$ [cf. Eq. (3)], the vertices must have $\Delta_i = 0$. Therefore, at this order, only the lowest order πNN and $\pi\pi NN$ vertices are allowed which is why the leading 2PE is rather weak. Furthermore, there are seven contact terms of $\mathcal{O}(Q^2)$, shown by the four-nucleon-leg graph with a solid square, which contribute in S and P waves. The operator structure of these contacts include a spin-orbit term besides central, spin-spin, and tensor terms. Thus, essentially all spin-isospin structures necessary to describe the two-nucleon force phenomenologically have been generated at this order. The main deficiency at this stage of development is an insufficient intermediate-range attraction.

This problem is finally fixed at order three ($\nu = 3$), next-to-next-to-leading order (NNLO). The 2PE involves now the two-derivative $\pi\pi NN$ seagull vertices (proportional to the c_i LECs) denoted by a large solid dot in Fig. 1. These vertices represent correlated 2PE as well as intermediate $\Delta(1232)$-isobar contributions. It is well-known from the meson phenomenology of nuclear forces [7, 8] that these two contributions are crucial for a realistic and quantitative 2PE model. Consequently, the 2PE now assumes a realistic size and describes the intermediate-range attraction of the nuclear force about right. Moreover, first relativistic corrections come into play at this order. There are no new contacts.

The reason why we talk of a hierarchy of nuclear forces is that two- and many-nucleon forces are created on an equal footing and emerge in increasing number as we go to higher and higher orders. At NNLO, the first set of nonvanishing three-nucleon forces (3NF) occur [17, 18], cf. column '3N Force' of Fig. 1. In fact, at the previous order, NLO, irreducible 3N graphs appear already, however, it has been shown by Weinberg [11] that these diagrams all cancel. Since nonvanishing 3NF contributions happen first at order $(Q/\Lambda_\chi)^3$, they are very weak as compared to 2NF which start at $(Q/\Lambda_\chi)^0$.

More 2PE is produced at $\nu = 4$, next-to-next-to-next-to-leading order (N^{3}LO), of which we show only a few symbolic diagrams in Fig. 1. Two-loop 2PE graphs show up for the first time and so does three-pion exchange (3PE) which necessarily involves two loops. 3PE was found to be negligible at this order [19, 20]. Most importantly, 15 new contact terms $\sim Q^4$ arise and are represented by the four-nucleon-leg graph with a solid diamond. They include a quadratic spin-orbit term and contribute up to D-waves. Mainly due to the increased number of contact terms, a quantitative description of the two-nucleon interaction up to about 300 MeV lab. energy is possible, at N^{3}LO (see red solid line in Fig. 2 and cf. Table 1). Besides further 3NF, four-nucleon forces (4NF) start at this order. Since the leading 4NF come into existence one order higher than the leading 3NF, 4NF are weaker than 3NF. Thus, ChPT provides a straightforward explanation for the empirically known fact that 2NF $\gg$ 3NF $\gg$ 4NF ... .
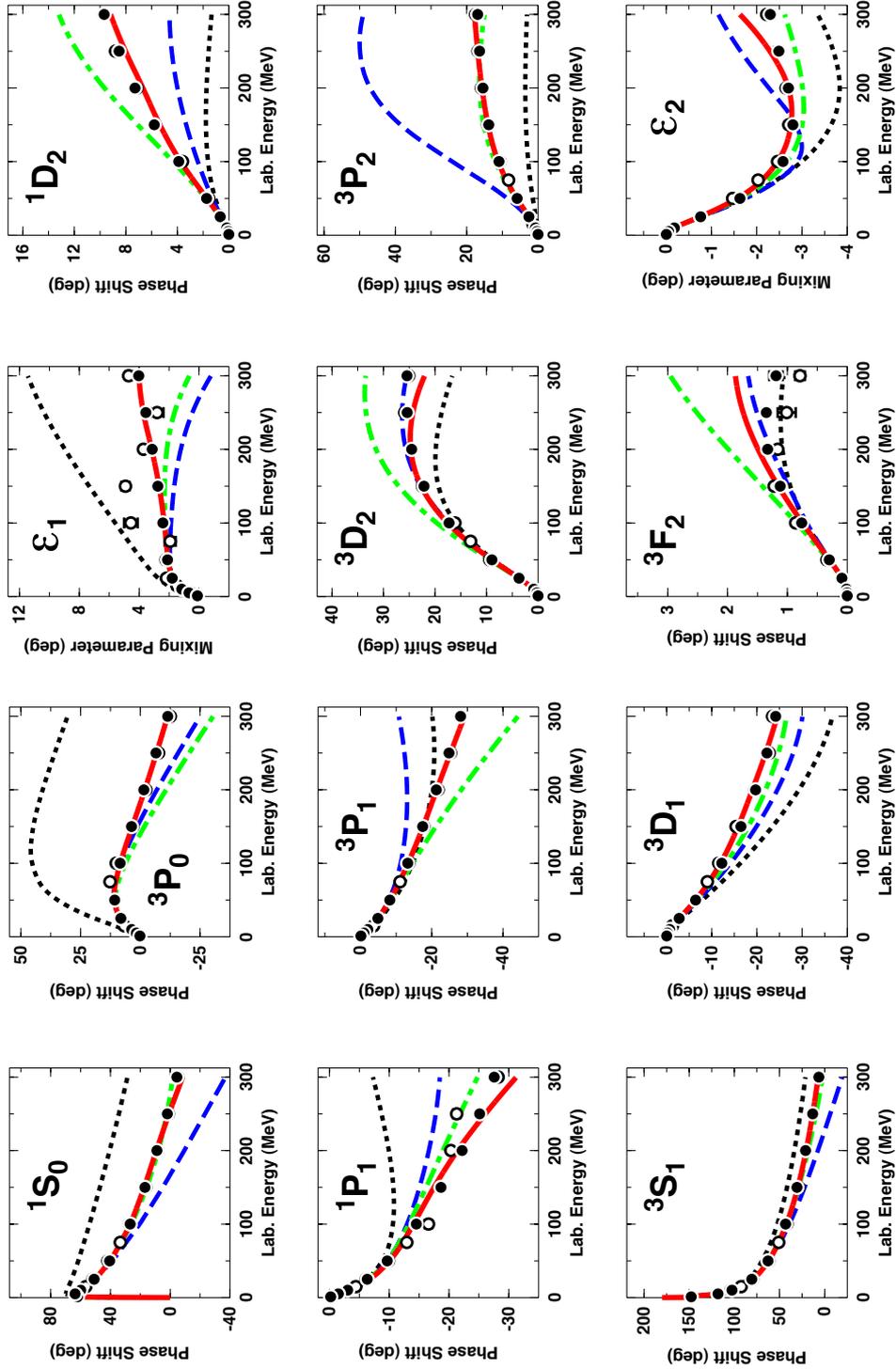
Figure 2: Phase shifts of np scattering as calculated from NN potentials at different orders of ChPT. The black dotted line is LO(500), the blue dashed is NLO(550/700) [21], the green dash-dotted NNLO(600/700) [21], and the red solid N^{3}LO(500) [22], where the numbers in parentheses denote the cutoffs in MeV. Partial waves with total angular momentum $J \leq 2$ are displayed. The solid dots and open circles are the results from the Nijmegen multi-energy np phase shift analysis [23] and the VPI/GWU single-energy np analysis SM99 [24], respectively.

Table 1: χ^2/datum for the reproduction of the 1999 np database [25] below 290 MeV by various np potentials. T_{lab} denotes the kinetic energy of the incident neutron in the laboratory system.

T_{lab} bin (MeV)	# of np data	N³LO [22]	NNLO [21]	NLO [21]	AV18 [26]
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

During the past decade or so, chiral two-nucleon forces have been used in many microscopic calculations of nuclear reactions and structure [27–33] and the combination of chiral two- and three-nucleon forces has been applied in few-nucleon reactions [18, 34–37], structure of light- and medium-mass nuclei [38–46], and nuclear and neutron matter [47–50] — with a great deal of success. The majority of nuclear structure calculations is nowadays based upon chiral forces.

However, in spite of this progress, we are not done. Due to the complexity of the nuclear force issue, there are still many subtle and not so subtle open problems. We will not list and discuss all of them, but instead just focus on the two open issues, which we perceive as the most important ones:

- The proper renormalization of chiral nuclear potentials and

- Subleading chiral few-nucleon forces.

4 Renormalization of chiral nuclear forces

4.1 The chiral NN potential

In mathematical terms, the various orders of the irreducible graphs in Fig. 1, which define the chiral NN potential, are given by:

$$V_{\text{LO}} = V_{\text{ct}}^{(0)} + V_{1\pi}^{(0)}, \tag{4}$$

$$V_{\text{NLO}} = V_{\text{LO}} + V_{\text{ct}}^{(2)} + V_{1\pi}^{(2)} + V_{2\pi}^{(2)}, \tag{5}$$

$$V_{\text{NNLO}} = V_{\text{NLO}} + V_{1\pi}^{(3)} + V_{2\pi}^{(3)}, \tag{6}$$

$$V_{\text{N}^3\text{LO}} = V_{\text{NNLO}} + V_{\text{ct}}^{(4)} + V_{1\pi}^{(4)} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)}, \tag{7}$$

where the superscript denotes the order ν of the low-momentum expansion. Contact potentials carry the subscript "ct" and pion-exchange potentials can be identified by an obvious subscript.

Multi-pion exchange, which starts at NLO and continues through all higher orders, involves divergent loop integrals that need to be regularized. An elegant way to do this is dimensional regularization which (besides the main nonpolynomial result) typically generates polynomial terms with coefficients that are, in part, infinite or scale dependent. One purpose of the contacts is to absorb all infinities and scale dependencies and make sure that the final result is finite and scale independent. This is the renormalization of the perturbatively calculated NN amplitude (which, by definition, is the "NN potential"). It is very similar to what is done in the ChPT calculations of $\pi\pi$ and πN scattering, namely, a renormalization order by order, which is the method of choice for any EFT. Thus, up to this point, the calculation fully meets the standards of an EFT and there are no problems. The perturbative NN amplitude can be used to make model independent predictions for peripheral partial waves.

4.2   Nonperturbative renormalization of the NN potential

For calculations of the structure of nuclear few and many-body systems, the lower partial waves are the most important ones. The fact that in S waves we have large scattering lengths and shallow (quasi) bound states indicates that these waves need to be treated nonperturbatively. Following Weinberg's prescription [10], this is accomplished by inserting the potential V into the Lippmann-Schwinger (LS) equation:

$$T(\vec{p}\,',\vec{p}) = V(\vec{p}\,',\vec{p}) + \int d^3 p'' \, V(\vec{p}\,',\vec{p}\,'') \, \frac{M_N}{p^2 - p''^2 + i\epsilon} \, T(\vec{p}\,'',\vec{p}), \qquad (8)$$

where M_N denotes the nucleon mass.

   In general, the integral in the LS equation is divergent and needs to be regularized. One way to do this is by multiplying V with a regulator function

$$V(\vec{p}\,',\vec{p}) \longmapsto V(\vec{p}\,',\vec{p}) \, e^{-(p'/\Lambda)^{2n}} \, e^{-(p/\Lambda)^{2n}} \,. \qquad (9)$$

Typical choices for the cutoff parameter Λ that appears in the regulator are $\Lambda \approx 0.5$ GeV $< \Lambda_\chi \approx 1$ GeV.

   It is pretty obvious that results for the T-matrix may depend sensitively on the regulator and its cutoff parameter. This is acceptable if one wishes to build models. For example, the meson models of the past [4] always depended sensitively on the choices for the cutoff parameters which, in fact, were important for the fit of the NN data. However, the EFT approach wishes to be fundamental in nature and not just another model.

   In field theories, divergent integrals are not uncommon and methods have been developed for how to deal with them. One regulates the integrals and then removes the dependence on the regularization parameters (scales, cutoffs) by renormalization. In the end, the theory and its predictions do not depend on cutoffs or renormalization scales. So-called renormalizable quantum field theories, like QED, have essentially one set of prescriptions that takes care of renormalization through all orders. In contrast, EFTs are renormalized order by order.

   Weinberg's implicit assumption [10, 51] was that the counterterms introduced to renormalize the perturbatively calculated potential, based upon naive dimensional analysis ("Weinberg counting"), are also sufficient to renormalize the nonperturbative resummation of the potential in the LS equation. In 1996, Kaplan, Savage, and Wise (KSW) [52] pointed out that there are problems with the Weinberg scheme if the LS equation is renormalized by minimally-subtracted dimensional regularization. This criticism resulted in a flurry of publications on the renormalization of the nonperturbative NN problem. The literature is too comprehensive to elaborate on all contributions. Therefore, we will restrict ourselves, here, to discussing just a few aspects that we perceive as particularly important. A more comprehensive consideration can be found in Ref. [13]

   Naively, the most perfect renormalization procedure is the one where the cutoff parameter Λ is carried to infinity while stable results are maintained. This was done successfully at LO in the work by Nogga *et al.* [53]. At NNLO, the infinite-cutoff renormalization procedure has been investigated in [54] for partial waves with total angular momentum $J \leq 1$ and in [55] for all partial waves with $J \leq 5$. At N³LO, the $^1S_0$ state was considered in Ref. [56], and all states up to $J = 6$ were investigated in Ref. [57]. From all of these works, it is evident that no counter term is effective in partial-waves with short-range repulsion and only a single counter term can effectively be used in partial-waves with short-range attraction. Thus, for the $\Lambda \to \infty$ renormalization prescription, even at N³LO, there exists either one or no counter term per partial-wave state. This is inconsistent with any reasonable power-counting scheme and prevents an order-by-order improvement of the predictions.

To summarize: In the infinite-cutoff renormalization scheme, the potential is admitted up to unlimited momenta. However, within the EFT, this potential is derived from has validity only for momenta smaller than the chiral symmetry breaking scale $\Lambda_\chi \approx 1$ GeV. The lack of order-by-order convergence and discrepancies in lower partial-waves demonstrate that the potential should not be used beyond the limits of the effective theory [57] (see Ref. [58] for a related discussion). The conclusion then is that cutoffs should be limited to $\Lambda \lesssim \Lambda_\chi$ (but see also Ref. [59]).

A possible solution of this problem was proposed already in [53] and reiterated in a paper by Long and van Kolck [60]. A calculation of the proposed kind has been performed by Valderrama [61] for the S, P, and D waves. The author renormalizes the LO interaction nonperturbatively and then uses the LO distorted wave to calculate the 2PE contributions at NLO and NNLO perturbatively. It turns out that perturbative renormalizability requires the introduction of about twice as many counter terms as compared to Weinberg counting, which reduces the predictive power. The order-by-order convergence of the NN phase shifts appears to be reasonable.

However, even if one considers the above method as successful for NN scattering, there is doubt if the interaction generated in this approach is of any use for applications in nuclear few- and many-body problems. In applications, one would first have to solve the many-body problem with the re-summed LO interaction, and then add higher order corrections in perturbation theory. It was shown in a recent paper [62] that the renormalized LO interaction is characterized by a very large tensor force from 1PE. This is no surprise since LO is renormalized with $\Lambda \to \infty$ implying that the 1PE, particulary its tensor force, is totally uncut. As a consequence of this, the wound integral in nuclear matter, κ, comes out to be about 40%. The hole-line and coupled cluster expansions are known to converge $\propto \kappa^{n-1}$ with n the number of hole-lines or particles per cluster. For conventional nuclear forces, the wound integral is typically between 5 and 10% and the inclusion of three-body clusters (or three hole-lines) are needed to obtain converged results in the many-body system. Thus, if the wound integral is 40%, probably, up to six hole-lines need to be included for approximate convergence. Such calculations are not feasible even with the most powerful computers of today and will not be feasible any time soon. Therefore, even if the renormalization procedure proposed in [60] will work for NN scattering, the interaction produced will be highly impractical (to say the least) in applications in few- and many-body problems because of convergence problems with the many-body energy and wave functions.

Crucial for an EFT are regulator independence (within the range of validity of the EFT) and a power counting scheme that allows for order-by-order improvement with decreasing truncation error. The purpose of renormalization is to achieve this regulator independence while maintaining a functional power counting scheme.

Thus, in the spirit of Lepage [63], the cutoff independence should be examined for cutoffs below the hard scale and not beyond. Ranges of cutoff independence within the theoretical error are to be identified using Lepage plots [63]. Recently, we have started a systematic investigation of this kind. In our work, we quantify the error of the predictions by calculating the χ^2/datum for the reproduction of the neutron-proton (np) elastic scattering data as a function of the cutoff parameter Λ of the regulator function Eq. (9). We have investigated the predictions by chiral np potentials at order NLO and NNLO applying Weinberg counting for the counter terms (NN contact terms). We show our results for the energy range 35–125 MeV in the upper frame of Fig. 3 and for 125–183 MeV in the lower frame. It is seen that the reproduction of the np data at these energies is generally poor at NLO, while at NNLO the χ^2/datum assumes acceptable values (a clear demonstration of order-by-order improvement). Moreover, at NNLO one observes "plateaus" of constant low χ^2 for cutoff parameters ranging from about 450 to 850 MeV. This may be perceived as cutoff independence (and, thus, successful renormalization) for the relevant range of
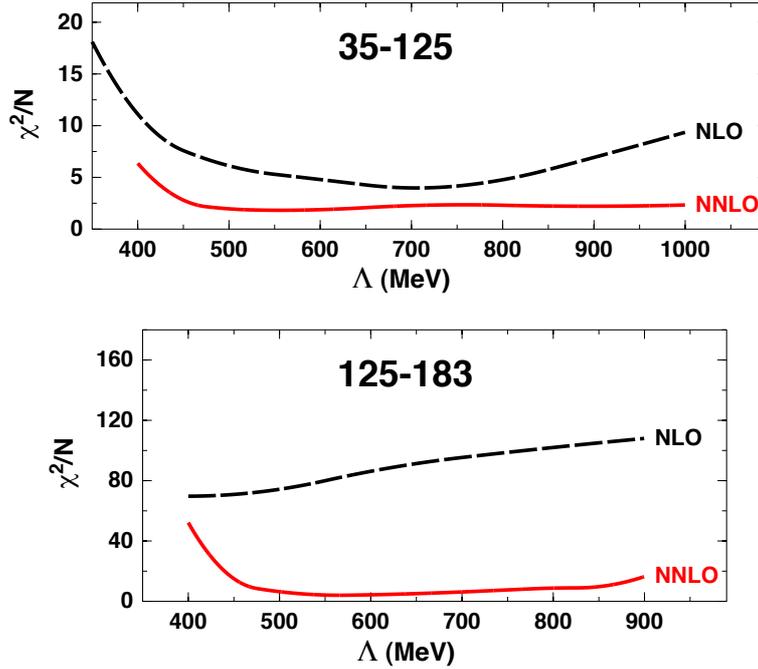
Figure 3: χ^2/datum for the reproduction of the np data in the energy range 35–125 MeV (upper frame) and 125–183 MeV (lower frame) as a function of the cutoff parameter Λ of the regulator function Eq. (9). The (black) dashed curves show the χ^2/datum achieved with np potentials constructed at order NLO and the (red) solid curves are for NNLO.

cutoff parameters.

5 Few-nucleon forces and what is missing

We will now discuss the other issue we perceive as unfinished and important, namely, subleading chiral few-nucleon forces.

Nuclear three-body forces in ChPT were initially discussed by Weinberg [11]. The 3NF at NNLO, was derived by van Kolck [17] and applied, for the first time, in nucleon-deuteron scattering by Epelbaum *et al.* [18]. The leading 4NF (at N³LO) was constructed by Epelbaum [64] and found to contribute in the order of 0.1 MeV to the ⁴He binding energy (total ⁴He binding energy: 28.3 MeV) in a preliminary calculation [65], confirming the traditional assumption that 4NF are essentially negligible. Therefore, the focus is on 3NFs.

For a 3NF, we have $A = 3$ and $C = 1$ and, thus, Eq. (1) implies

$$\nu = 2 + 2L + \sum_i \Delta_i \,. \tag{10}$$

We will use this equation to analyze 3NF contributions order by order. The first non-vanishing 3NF occurs at $\nu = 3$ (NNLO), which is obtained when there are no loops ($L = 0$) and $\sum_i \Delta_i = 1$, i.e., $\Delta_i = 1$ for one vertex while $\Delta_i = 0$ for all other vertices. There are three topologies which fulfill this condition, known as the two-pion exchange (2PE), one-pion exchange (1PE), and contact graphs (cf. Fig. 1).

The 3NF at NNLO has been applied in calculations of few-nucleon reactions [35], structure of light- and medium-mass nuclei [38–46], and nuclear and neutron matter [47–50] with a great deal of success. However, the famous 'A_y puzzle' of nucleon-
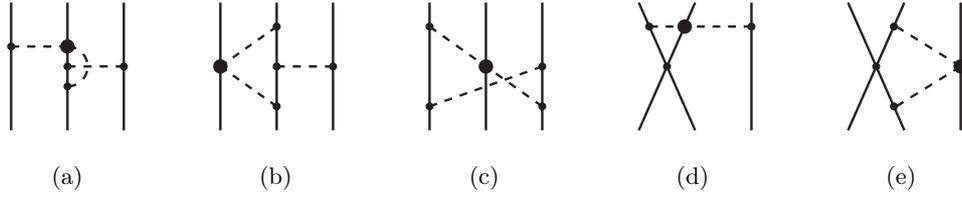
Figure 4: 3NF one-loop contributions at N^{4}LO ($\nu = 5$). We show one representative diagram for each of five topologies, which are: (a) 2PE, (b) 2PE-1PE, (c) ring, (d) 1PE-contact, and (e) 2PE-contact. Notation as in Fig. 1.

deuteron scattering [18] and the analogous problem with the analyzing power in p-^{3}He scattering [37] is not resolved. Furthermore, the spectra of light nuclei leave room for improvement [39]. Since we are dealing with a perturbation theory, it is natural to turn to the next order when looking for improvements.

The next order is N^{3}LO, where we have loop and tree diagrams. For the loops, we have $L = 1$ and, therefore, all Δ_i have to be zero to ensure $\nu = 4$. Thus, these one-loop 3NF diagrams can include only leading order vertices, the parameters of which are fixed from πN and NN analysis. One sub-group of these diagrams (the 2PE graphs) has been calculated by Ishikawa and Robilotta [66], and the other topologies have been evaluated by the Bochum–Bonn group [67,68]. The N^{3}LO 2PE 3NF has been applied in the calculation of nucleon-deuteron observables in Ref. [66] causing little impact. Very recently, the long-range part of the chiral N^{3}LO 3NF has been tested in the triton [69] and in three-nucleon scattering [70] yielding only moderate effects. The long- and short-range parts of this force have been used in neutron matter calculations (together with the N^{3}LO 4NF) producing relatively large contributions from the 3NF [71]. Thus, the ultimate assessment of the N^{3}LO 3NF is still outstanding and will require more few- and many-body applications.

In the meantime, it is of interest to take already a look at the next order of 3NFs, which is N^{4}LO or $\nu = 5$ (of the Δ-less theory to which the present discussion is restricted because of lack of space). The loop contributions that occur at this order are obtained by replacing one vertex in the N^{3}LO loops by a $\Delta_i = 1$ vertex (with LEC c_i), Fig. 4, which is why these loops may be more sizable than the N^{3}LO loops. The 2PE topology turns out to be of modest size [72]; moreover, it can be handled in a practical way by summing it up together with the 2PE topologies at NNLO and N^{3}LO [72]. The 2PE-1PE and ring topologies have also been derived [73]. Finally, there are also tree topologies at N^{4}LO (Fig. 5) which include a new set of 3N contact interactions (graph (c)). These 3N contacts have recently been derived by the Pisa group [74]. Contact terms are typically simple (as compared to loop diagrams) and their coefficients are unconstrained (except for naturalness). Therefore, it would be an attractive project to test some terms (in particular, the spin-orbit terms) of the N^{4}LO contact 3NF [74] in calculations of few-body reactions (specifically, the p-d and p-^{3}He A_y) and spectra of light nuclei.
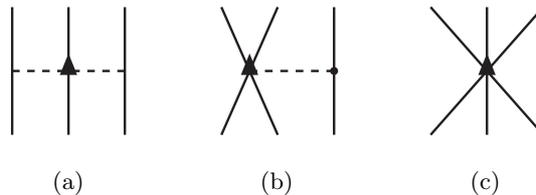

Figure 5: 3NF tree graphs at N^{4}LO ($\nu = 5$) denoted by: (a) 2PE, (b) 1PE-contact, and (c) contact. Solid triangles represent vertices of index $\Delta_i = 3$.

6   Conclusions and Outlook

The past 15 years have seen great progress in our understanding of nuclear forces in terms of low-energy QCD. A key to this development was the realization that the low-energy QCD is equivalent to an effective field theory which allows for a perturbative expansion that has become known as chiral perturbation theory. In this framework, two- and many-body forces emerge on an equal footing and the empirical fact that nuclear many-body forces are substantially weaker then the two-nucleon force is explained automatically.

In spite of the great progress and success of the past 15 years, there are still some unresolved issues. One problem is the proper renormalization of the chiral two- and many-nucleon potentials, where systematic investigations are already under way (cf. Sec. 4).

The other unfinished business is the few-nucleon forces beyond NNLO ("subleading few-nucleon forces") which are needed to hopefully resolve some important outstanding nuclear structure problems. At orders N^{3}LO and N^{4}LO very many new 3NF structures appear, some of which have already been tested. However, in view of the multitude of 3NF topologies it will take a while until we will have a proper overview of impact and convergence of these contributions.

If the open issues discussed in this paper will be resolved within the next few years, then, after 70 years of desperate struggle, we may finally claim that the nuclear force problem is essentially under control. The greatest beneficiaries of such progress will be the *ab initio* nuclear structure physicists, including *James Vary*. May this be a birthday present for him.

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