# Utilizing Symmetry Coupling Schemes in *Ab Initio* Nuclear Structure Calculations

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#### Abstract

We report on *ab initio* no-core shell model calculations in a symmetryadapted SU(3)-based coupling scheme that demonstrate that collective modes in *p*-shell nuclei emerge from first principles. The low-lying states of <sup>6</sup>Li, <sup>6</sup>He, <sup>8</sup>Be, <sup>8</sup>B, <sup>12</sup>C, and <sup>16</sup>O, are shown to exhibit orderly patterns that favor spatial configurations with strong quadrupole deformation and complementary low intrinsic spin values, a picture that is consistent with the nuclear symplectic model. The results also suggest a pragmatic path forward to accommodate deformation-driven collective features in *ab initio* analyses when they dominate the nuclear landscape.

Keywords: No-core shell model; SU(3) coupling scheme; p-shell nuclei

#### 1 Introduction

In the last few years, *ab initio* approaches to nuclear structure and reactions have considerably advanced our understanding and capability of achieving first-principle descriptions of *p*-shell nuclei [1–3]. These advances are driven by the major progress in the development of realistic nuclear potential models, such as *J*-matrix inverse scattering potentials [4] and two- and three-nucleon potentials derived from meson exchange theory [5] or by using chiral effective field theory [6], and, at the same time, by the utilization of massively parallel computing resources [7–9].

The predictive power that *ab initio* models hold [10, 11] makes them suitable for targeting short-lived nuclei that are inaccessible by experiment but essential to further modeling, for example, of the dynamics of X-ray bursts and the path of nucleosynthesis (see, e. g., Refs. [12, 13]). The main limitation of *ab initio* approaches is inherently coupled with the combinatorial growth in the size of the many-particle model space with increasing nucleon numbers and expansion in the number of single-particle levels in the model space as illustrated in Fig. 1. This points to the need of further major advances in many-body methods to access a wider range of nuclei and experimental observables, while retaining the *ab initio* predictive power.

These considerations motivate us to develop and investigate a novel model, the *ab initio* symmetry-adapted no-core shell model (SA-NCSM) [14], which by taking advantage of symmetries inherent to the nuclear dynamics [15,16] allows one to truncate a model space according to correlations indispensable for modeling important modes of nuclear collective dynamics, thereby overcoming the scale explosion bottleneck of *ab initio* nuclear structure computations.

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Figure 1: The dimensions of positive parity model spaces as functions of  $N_{\text{max}}$  for selected nuclei. Solid curves show the number of basis states with the projection of the total angular momentum M = 0. Dashed and dotted curves depict the number of basis states carrying selected values of the total angular momentum J.

## 2 Ab initio calculations in a SU(3)-based coupling scheme

The SA-NCSM joins a no-core shell model (NCSM) theory [2] with a multi-shell, SU(3)-based coupling scheme [15, 17]. Specifically, the many-nucleon basis states of the SA-NCSM are decomposed into spatial and intrinsic spin parts, where the spatial part is further classified according to the SU(3)  $\supset$  SO(3) group chain. The significance of the SU(3) group for a microscopic description of the nuclear collective dynamics can be seen from the fact that it is the symmetry group of the successful Elliott model [15], and a subgroup of the physically relevant Sp(3,  $\mathbb{R}$ ) symplectic model [16], which provides a comprehensive theoretical foundation for understanding the dominant symmetries of nuclear collective motion. The SA-NCSM basis states are labeled as

$$|\vec{\gamma}; N(\lambda \mu)\kappa L; (S_p S_n)S; JM\rangle,$$
 (1)

where N signifies the number of harmonic oscillator quanta with respect to the minimal number for a given nucleus. Quantum numbers  $S_p$ ,  $S_n$ , and S denote proton, neutron, and total intrinsic spins, respectively, and  $(\lambda \mu)$  represent a set of quantum numbers associated with SU(3) irreducible representations, irreps. The label  $\kappa$ distinguishes multiple occurrences of the same orbital momentum L in the parent irrep  $(\lambda \mu)$ . The L is coupled with S to the total angular momentum J and its projection M. The basis states bring forward important information about nuclear shapes and deformation according to an established mapping [18]. For example, (00),  $(\lambda 0)$ and  $(0\,\mu)$  describe spherical, prolate and oblate shapes, respectively. The symbol  $\vec{\gamma}$ schematically denotes the additional quantum numbers needed to specify a distribution of nucleon clusters over the major HO shells and their inter-shell coupling. Specifically, in each major HO shell  $\eta$  with degeneracy  $\Omega_{\eta}$ , clusters of protons and neutrons are arranged into antisymmetric  $U(\Omega_{\eta}) \times SU(2)_{S_{\eta}}$  irreps [19], with  $U(\Omega_{\eta})$  further reduced with respect to SU(3). The quantum numbers,  $[f_1, \ldots, f_{\Omega_n}] \alpha_\eta (\lambda_\eta \mu_\eta) S_\eta$ , along with  $SU(3) \times SU(2)_S$  labels of inter-shell coupling unambiguously determine SA-NCSM basis states (1). Note that a spatial symmetry associated with a Young

shape  $[f_1, \ldots, f_{\Omega_\eta}]$  is uniquely determined by the imposed antisymmetrization and the associated intrinsic spin  $S_\eta$ . A multiplicity index  $\alpha_\eta$  is required to distinguish multiple occurrences of SU(3) irrep  $(\lambda_\eta \mu_\eta)$  in a given U $(\Omega_\eta)$  irrep. It is important to note that any model space spanned by a complete set of equivalent SU $(3) \times SU(2)_S$ irreps, that is, a space spanned by all configurations carrying a fixed set of  $S_p S_n S$ and  $(\lambda \mu)$  quantum numbers, permits exact factorization of the center-of-mass motion.

The SA-NCSM implements fast methods for calculating matrix elements of arbitrary (currently up to two-body, but expandable to higher-rank) operators in the symmetry-adapted basis. This facilitates both the evaluation of the Hamiltonian matrix elements and the use of the resulting eigenvectors to evaluate other experimental observables. The underlying principle behind the SA-NCSM computational kernel is an SU(3)-type Wigner-Eckhart theorem, which factorizes interaction matrix elements into the product of SU(3) reduced matrix elements (*rme*) and the associated SU(3)coupling coefficient. The SA-NCSM configurations are constructed by the inter-shell coupling of a sequence of single-shell nucleon clusters arranged into  $U(\Omega) \times SU(2)_S$ , with  $U(\Omega) \supset SU(3)$ , irreps. Therefore, all the multi-shell *rme* are constructed from a set of single-shell *rme* computed in a configuration space of these irreps. This reduces the number of key pieces of information required to the single-shell *rme*, and these track with the number of  $U(\Omega) \times SU(2)_S$  irreps, with  $U(\Omega) \supset SU(3)$ , that represent building blocks of the SA-NCSM approach. It is therefore significant that their number grows slowly with the increasing nucleon number and  $N_{\rm max}$  cutoff as this allows these key pieces of information to be stored in CPU memory.

#### **3** Structure of nuclear wave functions

The expansion of calculated eigenstates in the physically relevant SU(3) basis unveils salient features that emerge from the complex dynamics of these strongly interacting many-particle systems. To explore the nature of the most important correlations, we analyze the probability distribution across Pauli-allowed  $(S_p S_n S)$  and  $(\lambda \mu)$  configurations of the four lowest-lying isospin-zero (T = 0) states of <sup>6</sup>Li  $(1_{gs}^+, 3_1^+, 2_1^+, \text{ and } 1_2^+)$ , the ground-state rotational bands of <sup>8</sup>Be, <sup>6</sup>He and <sup>12</sup>C, the lowest 1<sup>+</sup>, 3<sup>+</sup>, and 0<sup>+</sup> excited states of <sup>8</sup>B, and the ground state of <sup>16</sup>O. Results for the ground state of <sup>6</sup>Li and <sup>8</sup>Be, obtained with the JISP16 and chiral N<sup>3</sup>LO interactions, respectively, are shown in Figs. 2 and 3. These figures illustrates a feature common to all the low-energy solutions considered; namely, a highly structured and regular mix of intrinsic spins and SU(3) spatial quantum numbers that has heretofore gone unrecognized in other *ab initio* studies, and which, furthermore, does not seem to depend on the particular choice of realistic NN potential.

For a closer look at these results, first consider the spin content. We found that the calculated eigenstates project at a 99% level onto a comparatively small subset of intrinsic spin combinations. For instance, the lowest-lying eigenstates in <sup>6</sup>Li are almost entirely realized in terms of configurations characterized by the following intrinsic spin  $(S_p S_n S)$  triplets:  $(\frac{3}{2} \frac{3}{2} 3)$ ,  $(\frac{1}{2} \frac{3}{2} 2)$ ,  $(\frac{3}{2} \frac{1}{2} 2)$ , and  $(\frac{1}{2} \frac{1}{2} 1)$ , with the last one carrying over 90% of each eigenstate. Likewise, the same spin components as in the case of <sup>6</sup>Li are found to dominate the ground state and the lowest 1<sup>+</sup>, 3<sup>+</sup>, and 0<sup>+</sup> excited states of <sup>8</sup>B (Table 1). The ground state bands of <sup>8</sup>Be, <sup>6</sup>He, <sup>12</sup>C, and <sup>16</sup>O are found to be dominated by many-particle configurations carrying total intrinsic spin of the protons and neutrons equal to zero and one, with the largest contributions due to  $(S_p S_n S) = (000)$  and (112) configurations.

Second, consider the spatial degrees of freedom. Our results show that the mixing of  $(\lambda \mu)$  quantum numbers, induced by the SU(3) symmetry breaking terms of realistic interactions, exhibits a remarkably simple pattern. One of its key features is the preponderance of a single  $0\hbar\Omega$  SU(3) irrep. This so-called leading irrep, according to the established geometrical interpretation of SU(3) labels  $(\lambda \mu)$  [18], is characterized



Figure 2: Probability distributions for proton, neutron, and total intrinsic spin components  $(S_p S_n S)$  across the Pauli-allowed  $(\lambda \mu)$  values (horizontal axis) for the calculated 1<sup>+</sup> ground state of <sup>6</sup>Li obtained for  $N_{\text{max}} = 10$  and  $\hbar \Omega = 20$  MeV with the JISP16 interaction. The total probability for each  $N\hbar\Omega$  subspace is given in the upper left-hand corner of each histogram. Adapted from Ref. [14].

by the largest value of the intrinsic quadrupole deformation. For instance, the lowlying states of <sup>6</sup>Li project at a 40%–70% level onto the prolate  $0\hbar\Omega$  SU(3) irrep (20), as illustrated in Figs. 2 and 3 for the ground state. For the considered states of

Table 1: Probability amplitude of the dominant  $(S_p S_n S)$  spin configuration and the dominant nuclear shapes according to Eq. (2) for the ground state of *p*-shell nuclei.

Nucleus	$(S_p S_n S)$	Prob. [%]	$(\lambda_0\mu_0)$	Prob. [%]
<sup>6</sup> Li	$\left(\frac{1}{2},\frac{1}{2},1\right)$	93.26	(20)	98.13
$^{8}\mathrm{B}$	$(\frac{1}{2}, \frac{1}{2}, 1)$	85.17	(21)	87.94
<sup>8</sup> Be	(000)	85.25	(40)	90.03
$^{12}\mathrm{C}$	(000)	55.19	(04)	48.44
$^{16}O$	(000)	83.60	(00)	89.51



Figure 3: Probability distributions for proton, neutron, and total intrinsic spin components  $(S_p S_n S)$  across the Pauli-allowed  $(\lambda \mu)$  values (horizontal axis) for the calculated 0<sup>+</sup> ground state of <sup>8</sup>Be obtained for  $N_{\text{max}} = 8$  and  $\hbar\Omega = 25$  MeV with the chiral N<sup>3</sup>LO interaction. The total probability for each  $N\hbar\Omega$  subspace is given in the upper left-hand corner of each histogram. Adapted from Ref. [14].

<sup>8</sup>B, <sup>8</sup>Be, <sup>12</sup>C, and <sup>16</sup>O, qualitatively similar dominance of the leading  $0\hbar\Omega$  SU(3) irreps is observed — (2 1), (4 0), (0 4), and (0 0) irreps, associated with triaxial, prolate, oblate, and spherical shapes, respectively. The clear dominance of the most deformed  $0\hbar\Omega$  configuration within low-lying states of light *p*-shell nuclei indicates that the quadrupole-quadrupole interaction of the Elliott SU(3) model of nuclear rotations [15] is realized naturally within an *ab initio* framework.

The analysis also reveals that the dominant SU(3) basis states at each  $N\hbar\Omega$  subspace (N = 0, 2, 4, ...) are typically those with ( $\lambda \mu$ ) quantum numbers given by

$$\lambda + 2\mu = \lambda_0 + 2\mu_0 + N \tag{2}$$

where  $\lambda_0$  and  $\mu_0$  denote labels of the leading SU(3) irrep in the  $0\hbar\Omega$  (N = 0) subspace. We conjecture that this regular pattern of SU(3) quantum numbers reflects the presence of an underlying symplectic Sp(3,  $\mathbb{R}$ ) symmetry of microscopic nuclear collective motion [16] that governs the low-energy structure of both even-even and odd-odd *p*-shell nuclei. This can be seen from the fact that ( $\lambda \mu$ ) configurations that satisfy condition (2) can be determined from the leading SU(3) irrep ( $\lambda_0 \mu_0$ ) through a successive application of a specific subset of the Sp(3,  $\mathbb{R}$ ) symplectic  $2\hbar\Omega$  raising operators. This subset is composed of the three operators,  $\hat{A}_{zz}$ ,  $\hat{A}_{zx}$ , and  $\hat{A}_{xx}$ , that distribute two oscillator quanta in *z* and *x* directions, but none in *y* direction, thereby inducing SU(3) configurations with ever-increasing intrinsic quadrupole deformation. These three operators are the generators of the Sp $(2, \mathbb{R}) \subset$  Sp $(3, \mathbb{R})$  subgroup [20], and give rise to deformed shapes that are energetically favored by an attractive quadrupole-quadrupole interaction [21]. Note that this is consistent with our earlier findings of a clear symplectic Sp $(3, \mathbb{R})$  structure with the same pattern (2) in *ab initio* eigensolutions for <sup>12</sup>C and <sup>16</sup>O [22].

Furthermore, there is an apparent hierarchy among states that fulfill condition (2). In particular, the  $N\hbar\Omega$  configurations with  $(\lambda_0+N \mu_0)$ , the so-called stretched states, carry a noticeably higher probability than the others. For instance, the (2+N 0) stretched states contribute at the 85% level to the ground state of <sup>6</sup>Li, as can be readily seen in Figs. 2 and 3. Moreover, the dominance of the stretched states is rapidly increasing with the increasing many-body basis cutoff  $N_{\text{max}}$  as illustrated in Fig. 4. The sequence of the stretched states is formed by consecutive applications of the  $\hat{A}_{zz}$  operator, the generator of  $\text{Sp}(1,\mathbb{R}) \subset \text{Sp}(2,\mathbb{R}) \subset \text{Sp}(3,\mathbb{R})$  subgroup, over the leading SU(3) irrep. This translates into distributing N oscillator quanta along the direction of the *z*-axis only and hence rendering the largest possible deformation. The important role of the stretched configurations for the description of the rotational bands in N = Z even-even nuclei was recognized heretofore using a simple microscopic Hamiltonian [23]. In the present study, for the first time, this structure is clearly and simply unveiled within the context of a fully microscopic framework starting from first principles.

#### 4 Efficacy of the SU(3) basis

The observed patterns of intrinsic spin and deformation mixing supports a symmetryguided basis selection philosophy referenced above. Specifically, one can take advantage of dominant symmetries to refine the definition of the NCSM model space, which is based solely on the  $N_{\max}$  cutoff. A SA-NCSM model space, which we denote as  $\langle N_{\max}^{\perp} \rangle N_{\max}^{\top}$ , can be constructed using a symmetry-guided selection that includes the complete basis up through some  $N_{\max}^{\perp} \leq N_{\max}$  along with configurations carrying a restricted set of  $(\lambda \mu)$  and  $(S_p S_n S)$  quantum numbers in the  $N_{\max}^{\perp}$  to  $N_{\max}^{\top}$  space. Ultimately, we aim to achieve  $N_{\max}^{\top} \geq N_{\max}$ , where  $N_{\max}$  is the largest value for which complete-space results can be currently calculated. This concept focuses on retaining the most important configurations that support the strong many-nucleon correlations of a nuclear system using the underlying  $\mathrm{Sp}(1,\mathbb{R}) \subset \mathrm{Sp}(2,\mathbb{R}) \subset \mathrm{Sp}(3,\mathbb{R})$ symmetry considerations. Within this context, it is important to note that for model spaces truncated according to  $(\lambda \mu)$  irreps and intrinsic spins  $(S_p S_n S)$ , the spurious center-of-mass motion can be factored out exactly, which represents an important advantage of this scheme.

The efficacy of the symmetry-guided concept is illustrated for SA-NCSM results obtained in a model space, which is expanded beyond the complete  $N_{\text{max}}^{\perp} = 6$  (or 8) space by relatively few dominant intrinsic spin components and quadrupole deformations that satisfy condition (2). We use selected spaces up through  $N_{\text{max}}^{\top} = 12$ , which allows a comparison to available results obtained in the complete  $N_{\text{max}} = 12$ space and hence, probes the efficacy of the SA-NCSM symmetry-guided model space selection concept. For this analysis, a Coulomb plus bare JISP16 NN interaction for  $\hbar\Omega$  values ranging from 17.5 up to 25 MeV is used. SA-NCSM eigenstates are used to determine spectroscopic properties of low-lying T = 0 states of <sup>6</sup>Li for a  $\langle 6 \rangle 12$ model space and of the ground-state band of <sup>6</sup>He for  $\langle 8 \rangle 12$ . We utilize a complete space of  $N_{\text{max}}^{\perp} = 6$  for <sup>6</sup>Li and of  $N_{\text{max}}^{\perp} = 8$  for <sup>6</sup>He, as these spaces seem sufficient to accommodate essential mixing of low-energy HO excitations.

The results indicate that the observables obtained in the symmetry-guided truncated spaces under consideration are excellent approximations to the corresponding complete-space counterparts. In particular, the ground-state binding energies repre-



Figure 4: Probabilities of the most important  $(\lambda \mu)$   $(S_p S_n S)$  components in <sup>6</sup>Li at  $4\hbar\Omega$  subspace (a),  $6\hbar\Omega$  subspace (b),  $8\hbar\Omega$  subspace (c), and  $10\hbar\Omega$  subspace as a function of the model space cutoff  $N_{\text{max}}$ .



Figure 5: Experimental and theoretical excitation energies: (a) T = 0 states of <sup>6</sup>Li, and (b) the two lowest-lying states of the halo <sup>6</sup>He nucleus. Experimental results [24] are given in the first column. The theoretical results shown are for JISP16 and  $\hbar\Omega = 20$  MeV in the complete  $N_{\text{max}} = 12$  space (second column), symmetryguided truncated model space (third column) and the complete  $N_{\text{max}} = 6$  or 8 spaces (last column). Note the relatively large change in the calculated excitation spectrum of <sup>6</sup>Li when  $N_{\text{max}}$  is increased from 6 to 12, and that the  $\langle 6 \rangle 12$  SA-NCSM results (third column) track the latter closely.

sent from 98% up to 98.7% of the complete-space binding energy in the case of <sup>6</sup>Li, and reach over 99% for <sup>6</sup>He. Furthermore, the excitation energies differ only by 11 keV to a few hundred keV from the corresponding complete-space results, see Fig. 5, and the agreement with known experimental data is reasonable over a broad range of  $\hbar\Omega$  values.

As illustrated in Table 2, the magnetic dipole moments for <sup>6</sup>Li agree to within 0.3% for odd-J values, and 5% for  $\mu(2_1^+)$ . Qualitatively similar agreement is achieved for  $\mu(2_1^+)$  of <sup>6</sup>He, as shown in Table 3. The results suggest that it may suffice to include all low-lying  $\hbar\Omega$  states up to a fixed limit, e. g.,  $N_{\max}^{\perp} = 6$  for <sup>6</sup>Li and  $N_{\max}^{\perp} = 8$  for <sup>6</sup>He, to account for the most important correlations that contribute to the magnetic dipole moment.

To explore how close one comes to reproducing the important long-range correlations of the complete  $N_{\text{max}} = 12$  space in terms of nuclear collective excitations within

Table 2: Magnetic dipole moments  $\mu$  [ $\mu_N$ ] and point-particle rms matter radii  $r_m$  [fm] of T = 0 states of <sup>6</sup>Li calculated in the complete  $N_{\text{max}} = 12$  space and the  $\langle 6 \rangle 12$  subspace for JISP16 and  $\hbar\Omega = 20$  MeV. The experimental value for the 1<sup>+</sup> ground state is known to be  $\mu = +0.822 \ \mu_N$  [24].

	$1^{+}_{1}0$	$3^{+}_{1}0$	$2^{+}_{1}0$	$1^{+}_{2}0$
	μ		-	
Full $N_{\rm max} = 12$	0.838	1.866	0.960	0.336
$\langle 6 \rangle 12$	0.840	1.866	1.015	0.337
	rms			
Full $N_{\rm max} = 12$	2.146	2.092	2.257	2.373
$\langle 6 \rangle 12$	2.139	2.079	2.236	2.355

	$N_{\rm max} = 12$	$\langle 8 \rangle 12$
$B(E2; 2_1^+ \to 0_1^+) \ [e^2 \text{fm}^4]$	0.181	0.184
$Q(2_1^+) \ [e \cdot {\rm fm}^2]$	-0.690	-0.711
$\mu(2_1^+) \ [\mu_N]$	-0.873	-0.817
$r_m (2_1^+)$ [fm]	2.153	2.141
$r_m (0^+_1)$ [fm]	2.113	2.110

Table 3: Selected observables for the two lowest-lying states of <sup>6</sup>He obtained in the complete  $N_{\text{max}} = 12$  space and (8)12 model subspace for JISP16 and  $\hbar\Omega = 20$  MeV.

the symmetry-truncated spaces under consideration, we compared observables that are sensitive to the tails of the wavefunctions; specifically, the point-particle rms matter radii, the electric quadrupole moments and the reduced electromagnetic B(E2)transition strengths that, in addition, could hint at rotational features [25]. As Table 3 clearly shows, the complete-space results for these observables are remarkably well reproduced by the SA-NCSM for <sup>6</sup>He in the restricted  $\langle 8 \rangle 12$  space. Similarly, the  $\langle 6 \rangle 12$ eigensolutions for <sup>6</sup>Li yield results for B(E2) strengths and quadrupole moments that track very closely with their complete  $N_{\rm max} = 12$  space counterparts for all values of  $\hbar\Omega$  (Fig. 6). The B(E2) strengths almost double upon increasing the model space from  $N_{\rm max} = 6$  to  $N_{\rm max} = 12$ . This result suggests that further expansion of the model space will be needed to reach convergence [26]. The close correlation between the  $N_{\rm max} = 12$  and  $\langle 6 \rangle 12$  results is nevertheless impressive. In addition to being in agreement, the results reproduce the challenging sign and magnitude of the groundstate quadrupole moment that is measured to be  $Q(1^+) = -0.0818(17) \ e \cdot \text{fm}^2$  [24].



Figure 6: Electric quadrupole transition probabilities in units of  $e^2 \text{fm}^4$  [(a) and (b), as shown], and quadrupole moments in units of  $e \cdot \text{fm}^2$  (c) as a function of  $\hbar\Omega$ for T = 0 states of <sup>6</sup>Li calculated using JISP16 in the complete  $N_{\text{max}} = 12$  space (dashed black line), the complete  $N_{\text{max}} = 6$  space (solid blue line), and symmetrytruncated  $\langle 6 \rangle 12$  (solid red line) model spaces. Note that while the  $N_{\text{max}} = 6$ results differ considerably from their  $N_{\text{max}} = 12$  counterparts, in all cases the latter are nearly indistinguishable from the truncated  $\langle 6 \rangle 12$  results. Experimentally,  $B(E2; 1_1^+ \rightarrow 3_1^+) = 25.6(20) e^2 \text{fm}^4$  [24].

Finally, the results for the rms matter radii of <sup>6</sup>Li, listed in Table 2, agree to within 1%.

The differences between truncated-space and complete-space results are found to be essentially insensitive to the choice of  $\hbar\Omega$  and appear sufficiently small as to be inconsequential relative to the residual dependences on  $\hbar\Omega$  and on  $N_{\rm max}$  (see Fig. 6). Since the NN interaction dominates contributions from three-nucleon forces (3NFs) in light nuclei, except for selected cases [27–29], we expect our results to be robust and carry forward to planned applications that will include 3NFs.

### 5 Conclusion

We have developed a novel approach that capitalizes on advances being made in *ab initio* methods while exploiting exact and partial symmetries of nuclear many-body system. Using this approach we have demonstrated that the low-lying eigenstates of <sup>6</sup>Li, <sup>8</sup>Be, <sup>12</sup>C, and <sup>16</sup>O, which were obtained using the JISP16 and N<sup>3</sup>LO *NN* interaction, exhibit a strong dominance of few intrinsic spin components and carry an intriguingly simple pattern of dominant deformations. The results very clearly underscore the significance of the SU(3) scheme, *LS*-coupling, and underlying symplectic symmetry in enabling an extension, through symmetry-guided model space reductions, of *ab initio* methods to heavier nuclei beyond <sup>16</sup>O.

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