Scattering
in
Time-dependent Basis Light-Front Quantization

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Time-dependent Basis Light-front Quantization

- BLFQ: for quantum field eigenspectrum
- tBLFQ: for quantum field evolution

BLFQ

$$P^- |\psi\rangle = P^-_\psi |\psi\rangle$$

tBLFQ

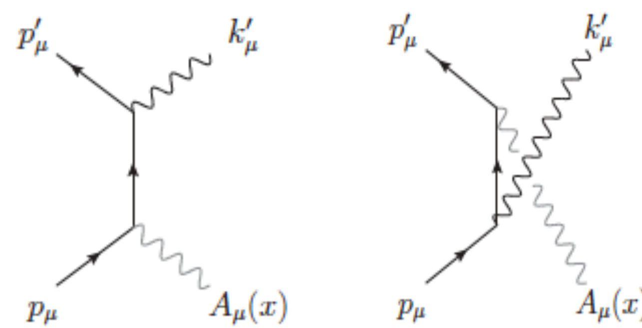
$$i \frac{\partial}{\partial x^+} |\psi(x^+)\rangle = \frac{1}{2} P^- |\psi(x^+)\rangle$$

- Real-time framework: BLFQ ⇨ tBLFQ
- tBLFQ is designed for:
  - time-dependence in dynamical processes
  - in strong/time-dependent background field

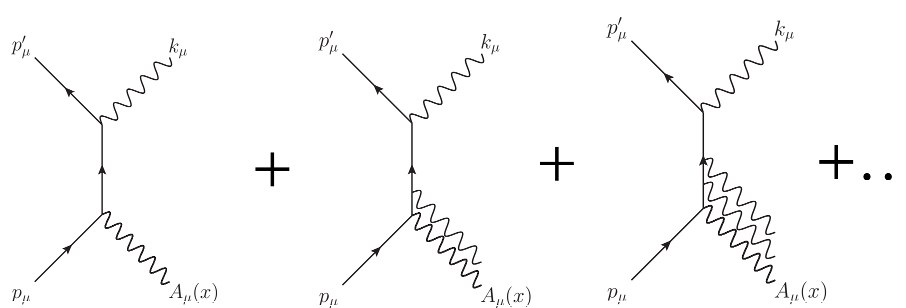
Application to Strong QED: Nonlinear Compton Scattering

- $e + n\gamma(laser) \to e' + \gamma'$
- 10^{20} photons in a laser: model as background field

- Perturbation theory:
- $\sigma \propto \text{Klein-Nishina} \times \tilde{A}^2$

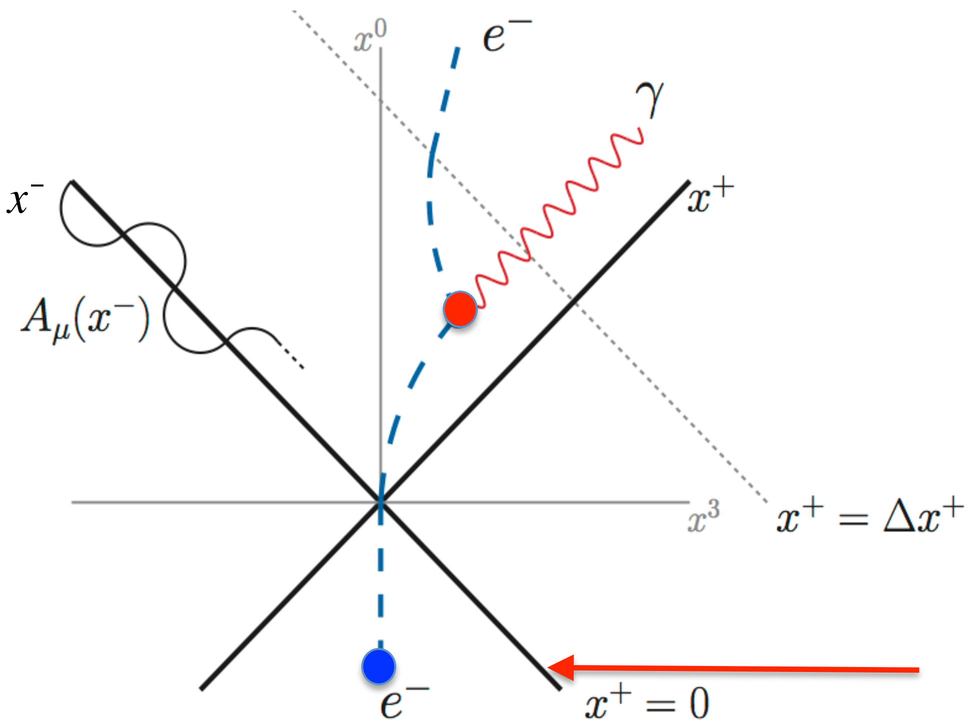


- At high intensity: nonperturbative treatment needed



Setup for Nonlinear Compton Scattering

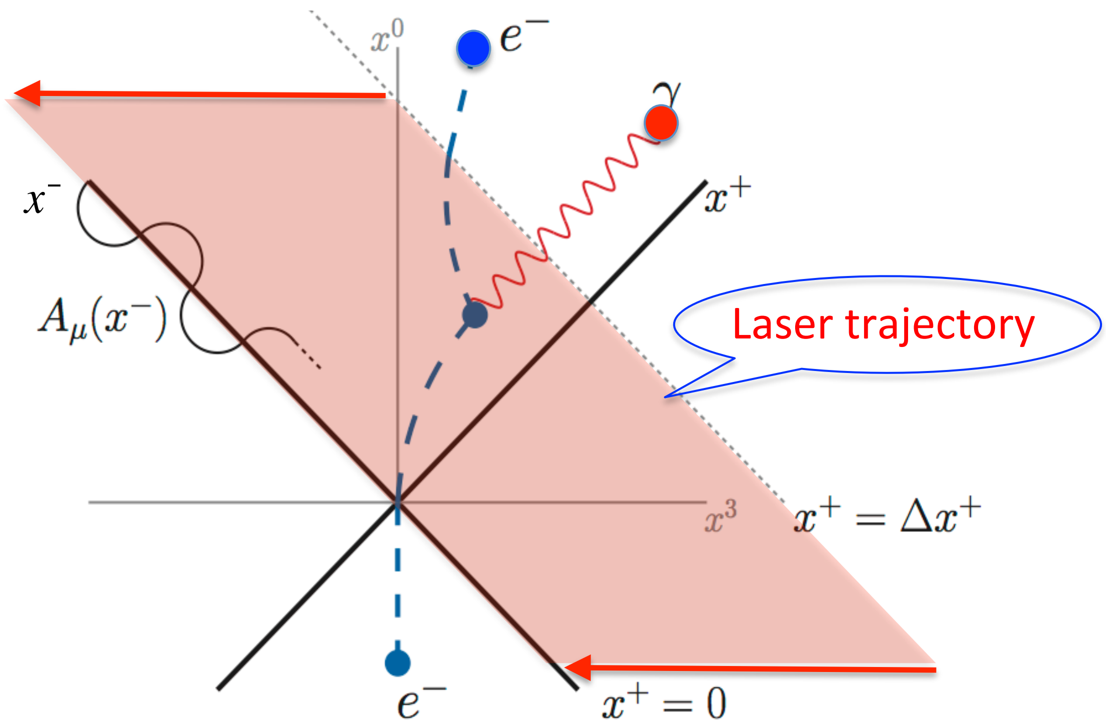
- Space-time structure



- Two effects: acceleration and radiation

Setup for Nonlinear Compton Scattering

- Space-time structure



- Two effects: acceleration and radiation

A Simple Laser Field Profile

$$eA^{-}(x^{-}) = m_e a_0 \cos\left(l_- x^{-}\right)$$

- Key properties:
  - $\mathcal{A}$ is treated classically w/ $\mathcal{A}^{+} = 0$
  - $\mathcal{A}^{-}$ only; uniform in $x^{1,2}$ and light-front time x^{+}
  - $\mathcal{A}^{-}$ depends only on x^{-} ⟹ $\mathcal{F}^{+-} = E^{-} \neq 0$

    ⟹ electric field in longitudinal direction
  - a_0: the field strength
  - l_-: the laser field's spatial frequency in x^{-}

    (longitudinal momentum)

General Procedure for tBLFQ

1. Derive Lightfront-Hamiltonian from Lagrangian
2. Switch to the interaction picture
3. Prepare the initial (‘in’) state
4. Evolve the initial state until the background field subsides
5. Project the scattering final state onto ‘out’ states (constructed out of QED eigenstates) and obtain S-matrix element

$$S = {}_I\langle out|\mathcal{T}_+ \exp\left(-\frac{i}{2}\int_0^{x_f^+} V_I\right)|in\rangle_I$$

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Derive Lightfront QED Hamiltonian

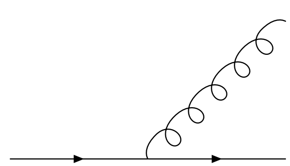
- QED Lagrangian $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$
- Lightfront Hamiltonian from Legendre transform

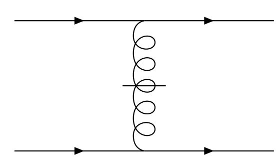
$$\left(A^+ = 0\right)$$

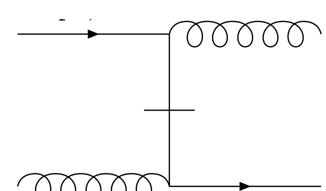
$$P^-_{QED} = \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \; F^{\mu+}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

$$= \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \; \underbrace{\frac{1}{2}\bar{\Psi}\gamma^+ \frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2} A^j (i\partial^\perp)^2 A^j}_{\text{kinetic energy terms}}$$

$$\underbrace{+\, e j^\mu A_\mu}_{\text{vertex interaction}} + \underbrace{\frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+}_{\text{instantaneous photon interaction}} + \underbrace{\frac{e^2}{2} \bar{\Psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu \Psi}_{\text{instantaneous fermion interaction}}$$







QED in background EM field

- Replace $A \rightarrow \mathcal{A} + A$ in P^-_{QED} and obtain P^-

- $P^- = P^-_{QED} + \int d^2 x^\perp dx^- e j^\mu \mathcal{A}_\mu$

$$+ \frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi + \frac{e^2}{2}\bar{\Psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi + \frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu \Psi$$

$$= P^-_{QED} + V$$

A Simple Laser Field Profile

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- Key properties:
  - $\mathcal{A}$ is treated classically w/ $\mathcal{A}^{+} = 0$
  - $\mathcal{A}^{-}$ is uniform in $x^{1,2}$ and light-front time x^{+}
  - $\mathcal{A}^{-}$ depends only on x^{-} ⇨ $\mathcal{F}^{+-} = E^{-} \neq 0$

    ⇨ electric field in longitudinal direction
  - a_0 describes the field strength
  - l_- describes the laser field's spatial frequency in x^{-} (longitudinal momentum)

QED in background EM field

- Replace $A \to \mathcal{A} + A$ in P^-_{QED} and obtain P^-

- $P^- = P^-_{QED} + \int d^2x^\perp dx^- e j^\mu \mathcal{A}_\mu$

$$\cancel{+\frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi + \frac{e^2}{2}\bar{\Psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi + \frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu \Psi}$$

$$= P^-_{QED} + V$$

these terms vanish
if $A^\perp = 0$

General Procedure for tBLFQ

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3. Prepare the initial ('in') state
4. Evolve the initial state until the background field subsides
5. Project the scattering final state onto 'out' states (constructed out of QED eigenstates) and obtain S-matrix element

$$S = {}_I\langle\, out \,|\mathcal{T}_+ \exp\left(-\frac{i}{2}\int_0^{x_f^+} V_I\right)|\, in \,\rangle_I$$

QED in background EM field

- Time dependence in external field only

$$P^-(x^+) = P^-_{QED} + \int d^2x^\perp dx^- e j^\mu \mathcal{A}_\mu(x^+)$$

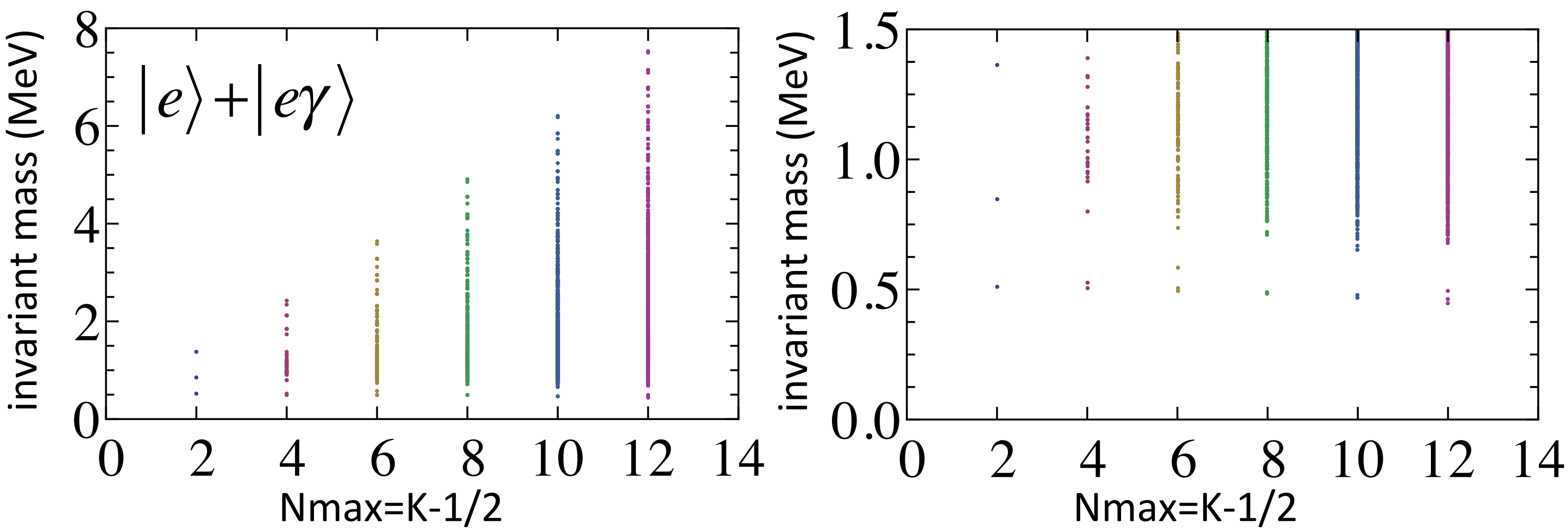
$$= P^-_{QED} + V(x^+)$$

- In interaction picture, only external field induces evolution

$$i\frac{\partial}{\partial x^+}\left|\psi(x^+)\right\rangle = \frac{1}{2}P^-\left|\psi(x^+)\right\rangle \quad \Rightarrow \quad i\frac{\partial}{\partial x^+}\left|\psi(x^+)\right\rangle_I = \frac{1}{2}V_I(x^+)\left|\psi(x^+)\right\rangle_I$$

- Need to work in the eigenstate basis of P^-_{QED}, obtained by solving $P^-_{QED}\left|\beta\right\rangle = P^-_\beta\left|\beta\right\rangle$ in BLFQ

Eigenspectrum of QED (N_f=1)



- Single electron(bound state) + eγ scattering states (continuum)
- Larger basis covers wider QED spectrum
- Eigenstates of P^{-}_{QED} serve as basis states in tBLFQ

General Procedure for tBLFQ

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$$S = {}_I\langle out|\mathcal{T}_+ \exp\left(-\frac{i}{2}\int_0^{x_f^+} V_I\right)|in\rangle_I$$

General Procedure for tBLFQ

1. Derive Lightfront-Hamiltonian from Lagrangian
2. Switch to the interaction picture
3. Prepare the initial ('in') state: a single physical electron
4. Evolve the initial state until the background field subsides
5. Project the scattering final state onto 'out' states (constructed out of QED eigenstates) and obtain S-matrix element

$$S = {}_I\langle out| \mathcal{T}_+ \exp\left(-\frac{i}{2} \int_0^{x_f^+} V_I \right) |in\rangle_I$$

General Procedure for tBLFQ

1. Derive Lightfront-Hamiltonian from Lagrangian
2. Switch to the interaction picture
3. Prepare the initial (‘in’) state
4. Evolve the initial state until the background field subsides
5. Project the scattering final state onto ‘out’ states (constructed out of QED eigenstates) and obtain S-matrix element

$$S = {}_I\langle out | \mathcal{T}_+ \exp\left(-\frac{i}{2} \int_0^{x_f^+} V_I \right) | in \rangle_I$$

$$= {}_I\langle out | \left(1 - \tfrac{i}{2} V_I\,(x^+)\delta x^+\right) \cdots \left(1 - \tfrac{i}{2} V_I\,(x_2^+)\delta x^+\right)\left(1 - \tfrac{i}{2} V_I\,(x_1^+)\delta x^+\right) | in \rangle_I$$

General Procedure for tBLFQ

1. Derive Lightfront-Hamiltonian from Lagrangian
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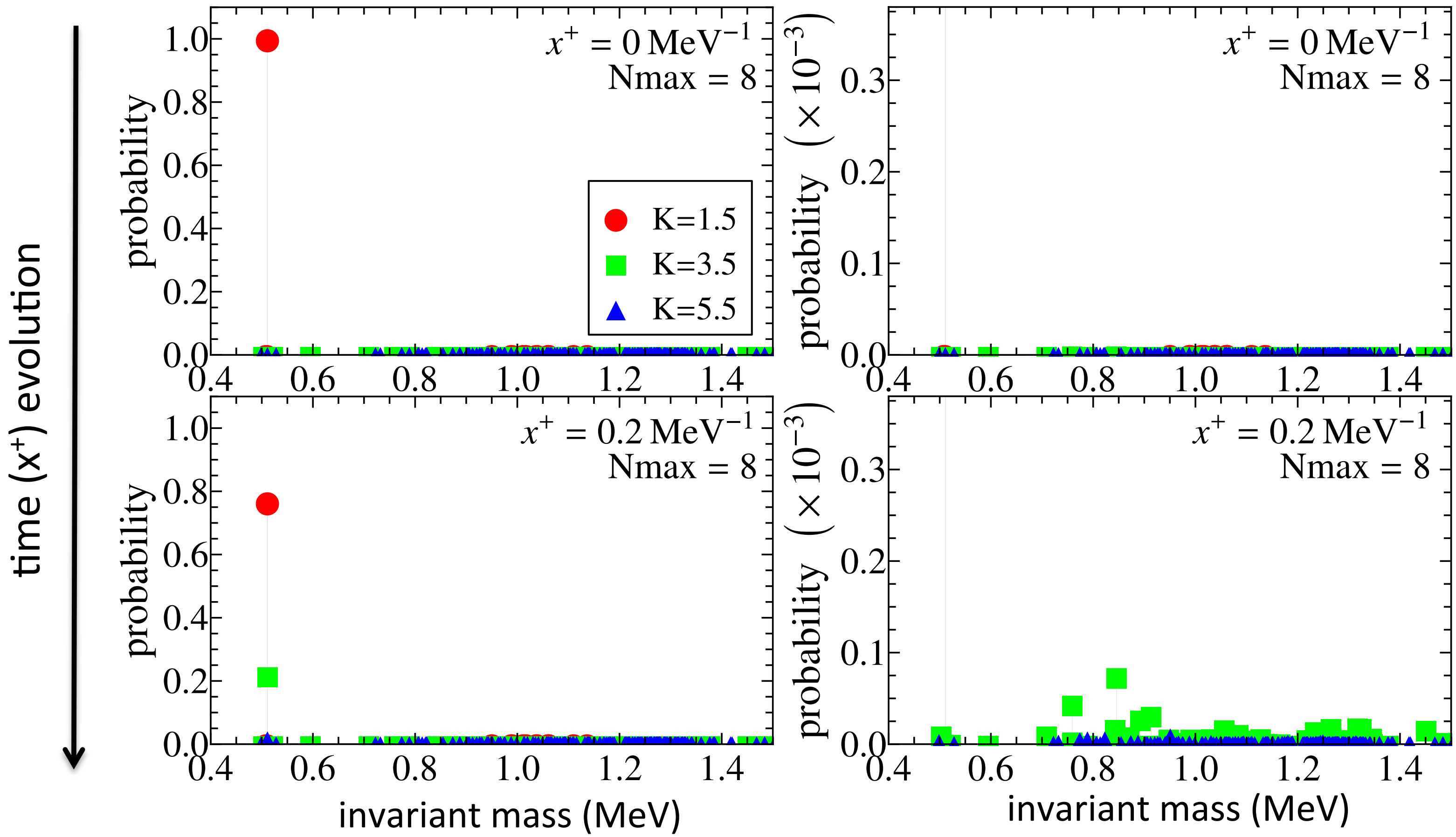
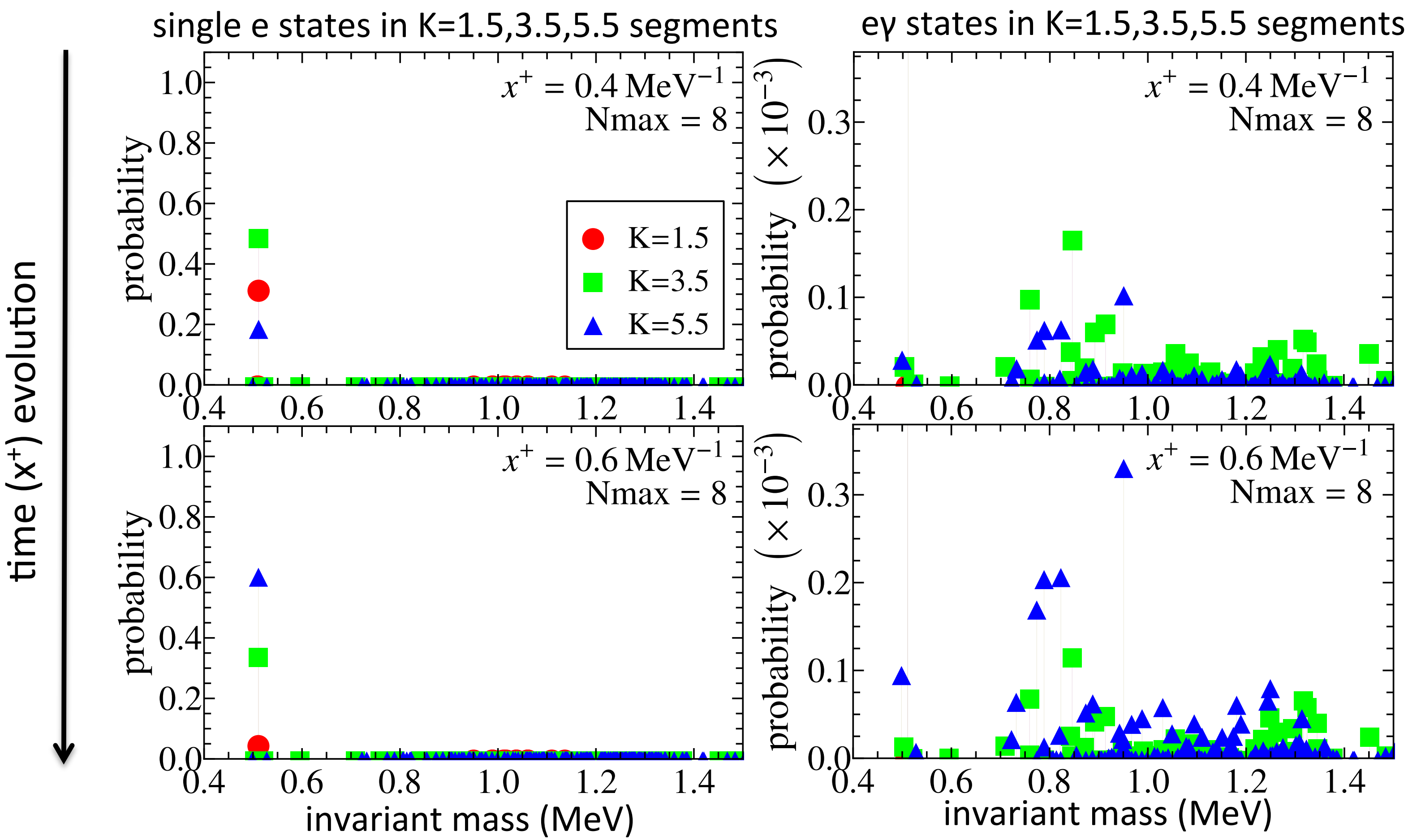
Results: Nonlinear Compton Scattering

- Laser profile: $e\mathcal{A}^-(x^-) = m_e a_0 \cos\left(l_- x^-\right)$ with $a_0 = 10, \quad l_- \sim 2MeV$
- Basis space: $|e\rangle + |e\gamma\rangle$, Nmax=8, K: 1.5+3.5+5.5 (three segments)
- Initial state (x^+=0): a single physical electron in K=1.5 segment



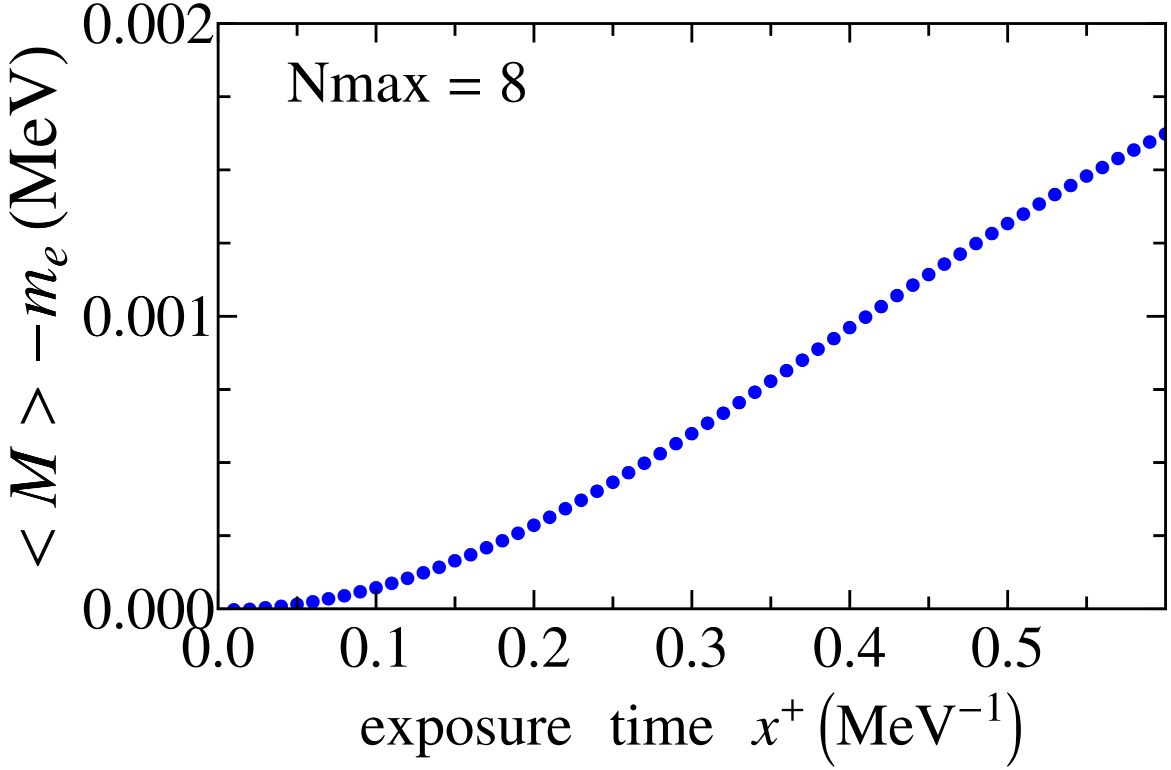
Results: Nonlinear Compton Scattering



[Zhao, Ilderton, Maris, Vary arXiv: 1303:3273]

- Acceleration and radiation are treated in the same Hilbert space
- Entire process is nonperturbative (initial state changes significantly)

Invariant Mass Increases Over Time



- Invariant mass increases due to photon emission, reflecting energy injection by the background laser field
- Experimentally accessible by measuring p_μ^e and p_μ^γ

Conclusion and Outlook

- Time-dependent Basis Light-Front Quantization (tBLFQ) approach
  - straightforward extension of BLFQ into time-dependent regime
  - solve for time-evolution of quantum field configurations
  - wavefunction ("snapshot") of system accessible at intermediate times
  - initial application to nonlinear Compton scattering simulates the electron acceleration and radiation coherently in nonperturbative regime

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- Strong field QED
- Hadronization in vacuum / jet evolution in hot medium
- Ultimate goal: nuclear/hadron scattering
- Evolution of quantum ensembles?

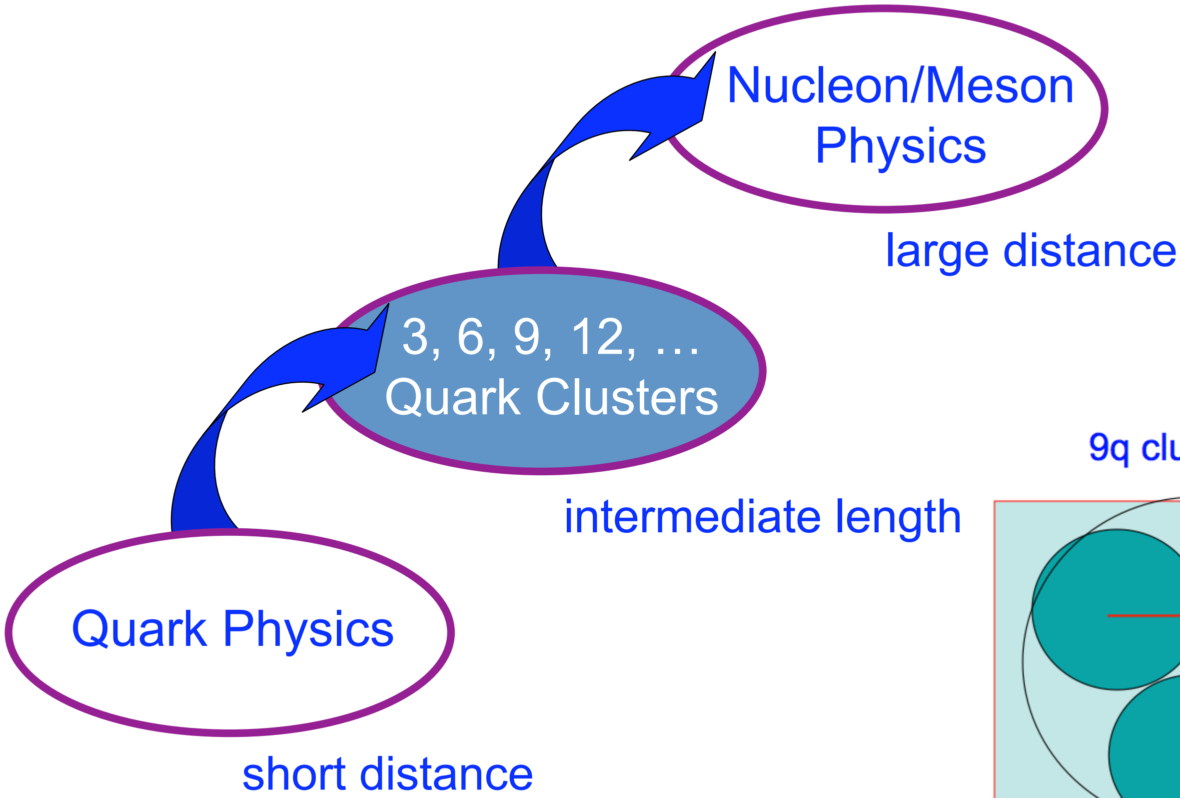
Thank you!

based on: arXiv: 1303:3273

Backup Slides

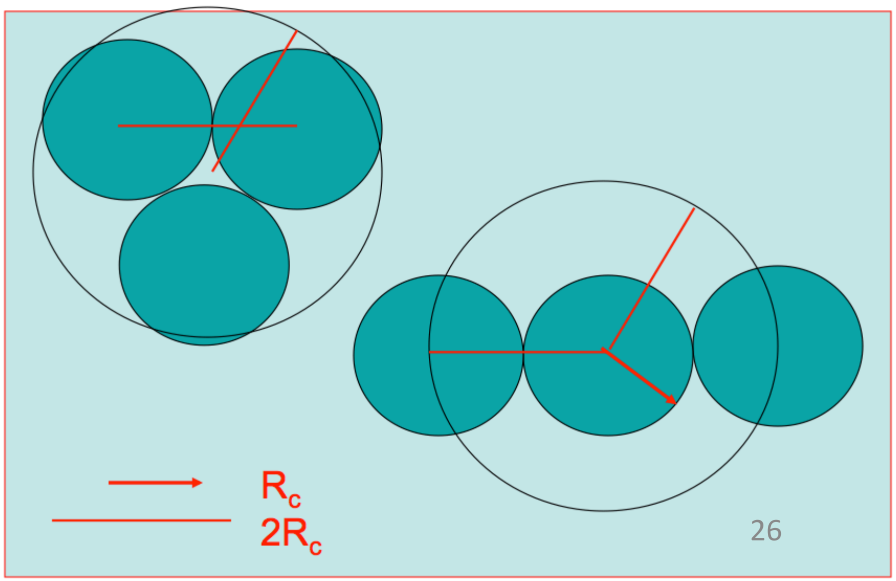
Under what conditions do we require a quark-based description on nuclear structure?

“Quark Percolation in Cold and Hot Nuclei”



Spin content of the proton
Nuclear form factors
DIS on nuclei – Bjorken x > 1
Nuclear Equation of State
Probes with Q > 1 GeV/c

9q cluster at geometrical limits of formation



H.J. Pirner and J.P. Vary,
Phys. Rev. C. **84**, 015201(2011);
arXiv: nucl-th/1008.4962

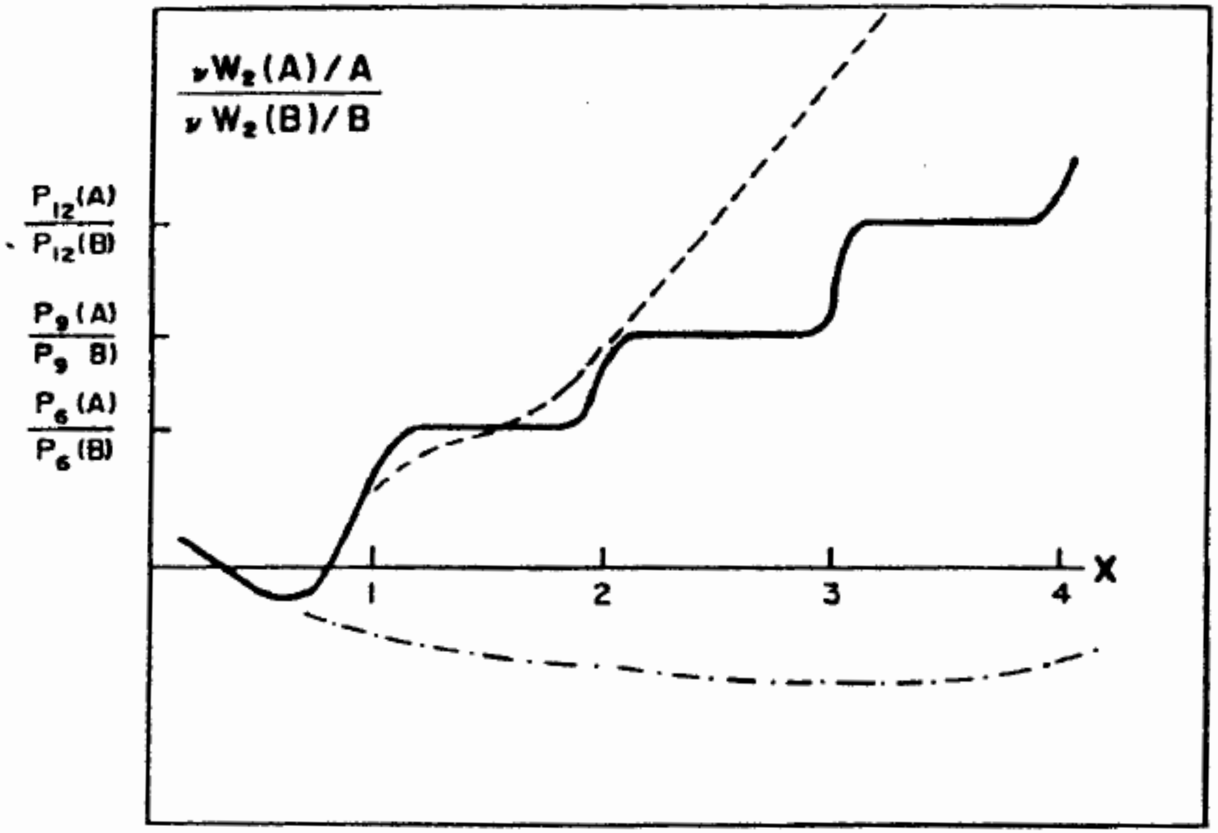
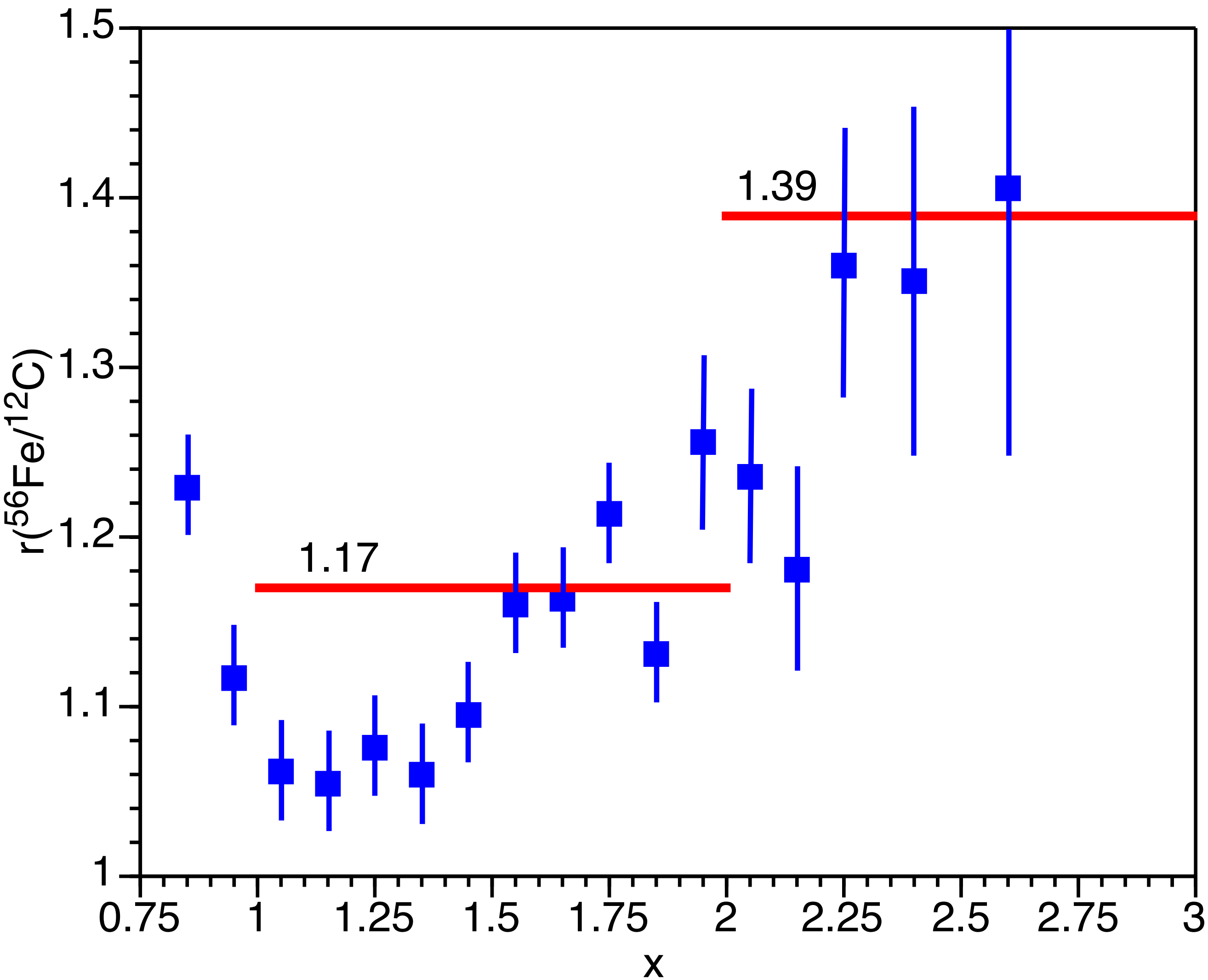


Fig. 2. Characteristic behaviour of the ratio of nuclear structure functions per nucleon for different models over a wide kinematic range of x. The QCM gives the solid curve. The dashed curve is due to the model of reference 22. The dashed-dot curve approximates the predictions of references 23 and 24.

J.P. Vary, Proc. VII Int'l Seminar on High Energy Physics Problems, "Quark Cluster Model of Nuclei and Lepton Scattering Results," Multiquark Interactions and Quantum Chromodynamics, V.V. Burov, Ed., Dubna #D-1, 2-84-599 (1984) 186 [staircase function for x > 1]

See also: Proceedings of HUGS at CEBAF1992, & many conf. proceedings

Comparison between Quark-Cluster Model and JLAB data



Data: K.S. Egiyan, et al., Phys. Rev. Lett. **96**, 082501 (2006)
Theory: H.J. Pirner and J.P. Vary, Phys. Rev. Lett. **46**, 1376 (1981)
and Phys. Rev. C **84**, 015201 (2011); nucl-th/1008.4962;
M. Sato, S.A. Coon, H.J. Pirner and J.P. Vary, Phys. Rev. C **33**, 1062 (1986)

Steps to implement BLFQ

- Enumerate Fock-space basis subject to symmetry constraints
- Evaluate/renormalize/store H in that basis
- Diagonalize (Lanczos)
- Iterate previous two steps for sector-dep. renormalization
- Evaluate observables using eigenvectors (LF amplitudes)
- Repeat previous 4 steps for new regulator(s)
- Extrapolate to infinite matrix limit – remove all regulators
- Compare with experiment or predict new experimental results

Above achieved for QED test case – electron in a trap
H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky,
Phys. Rev. Lett. 106, 061603 (2011)

Improvements: trap independence, (m,e) renormalization, . . .
X. Zhao, H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, in prep'n

New Measurements of High-Momentum Nucleons and Short-Range Structures in Nuclei

N. Fomin,[1,2,3] J. Arrington,[4] R. Asaturyan,[5,*] F. Benmokhtar,[6] W. Boeglin,[7] P. Bosted,[8] A. Bruell,[8] M. H. S. Bukhari,[9] M. E. Christy,[8] E. Chudakov,[8] B. Clasie,[10] S. H. Connell,[11] M. M. Dalton,[3] A. Daniel,[9] D. B. Day,[3] D. Dutta,[12,13] R. Ent,[8] L. El Fassi,[4] H. Fenker,[8] B. W. Filippone,[14] K. Garrow,[15] D. Gaskell,[8] C. Hill,[3] R. J. Holt,[4] T. Horn,[6,8,16] M. K. Jones,[8] J. Jourdan,[17] N. Kalantarians,[9] C. E. Keppel,[8,18] D. Kiselev,[17] M. Kotulla,[17] R. Lindgren,[3] A. F. Lung,[8] S. Malace,[18] P. Markowitz,[7] P. McKee,[3] D. G. Meekins,[8] H. Mkrtchyan,[5] T. Navasardyan,[5] G. Niculescu,[19] A. K. Opper,[20] C. Perdrisat,[21] D. H. Potterveld,[4] V. Punjabi,[22] X. Qian,[13] P. E. Reimer,[4] J. Roche,[20,8] V. M. Rodriguez,[9] O. Rondon,[3] E. Schulte,[4] J. Seely,[10] E. Segbefia,[18] K. Slifer,[3] G. R. Smith,[8] P. Solvignon,[8] V. Tadevosyan,[5] S. Tajima,[3] L. Tang,[8,18] G. Testa,[17] R. Trojer,[17] V. Tvaskis,[18] W. F. Vulcan,[8] C. Wasko,[3] F. R. Wesselmann,[22] S. A. Wood,[8] J. Wright,[3] and X. Zheng[3,4]

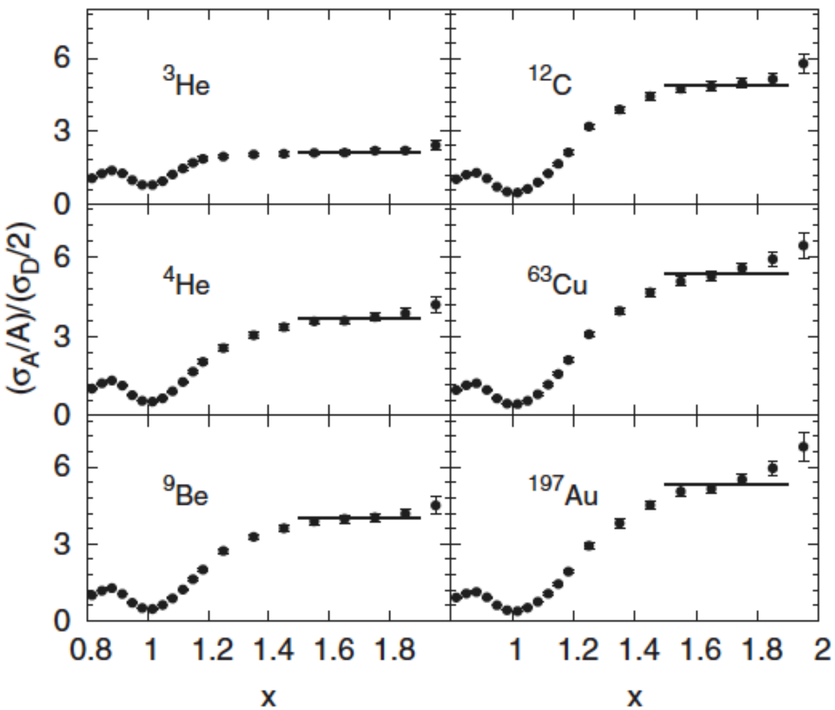


FIG. 2. Pernucleon cross section ratios vs x at $\theta_e = 18°$.

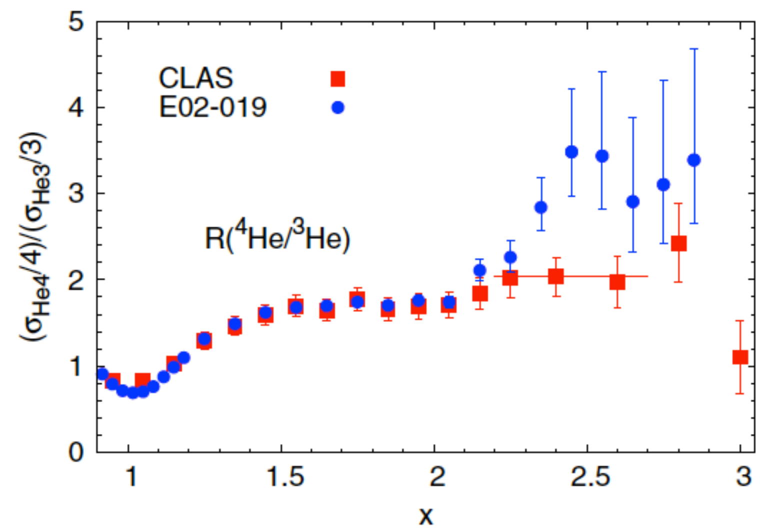


FIG. 3 (color online). The ^{4}He/^{3}He ratios from E02-019 ($Q^2 \approx 2.9$ GeV^2) and CLAS ($\langle Q^2 \rangle \approx 1.6$ GeV^2); errors are combined statistical and systematic uncertainties. For $x > 2.2$, the uncertainties in the ^{3}He cross section are large enough that a one-sigma variation of these results yields an asymmetric error band in the ratio. The error bars shown for this region represent the central 68% confidence level region.

Discretized Light Cone Quantization (c1985)

$$\Downarrow$$

Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_{\alpha}(\vec{x}) a_{\alpha}^{+} + f_{\alpha}^{*}(\vec{x}) a_{\alpha} \right]$$

where $\{a_{\alpha}\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_{\alpha}(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_{\alpha}(\vec{x}) f_{\alpha'}^{*}(\vec{x}) d^3x = \delta_{\alpha\alpha'}$

Complete: $\sum_{\alpha} f_{\alpha}(\vec{x}) f_{\alpha}^{*}(\vec{x}') = \delta^3(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_{a}(\vec{x})$ and our initial choice is

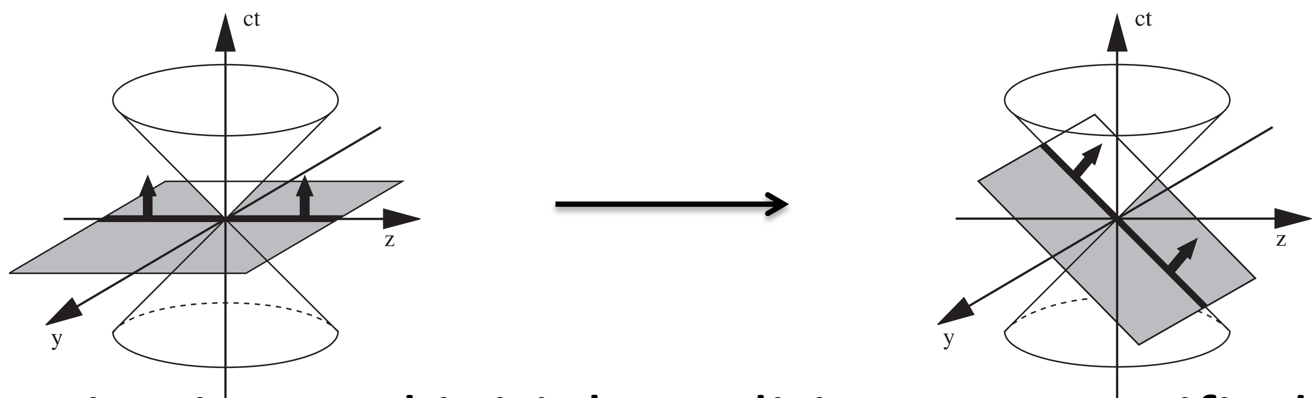
$$f_{\alpha}(\vec{x}) = Ne^{ik^{+}x^{-}} \Psi_{n,m}(\rho,\varphi) = Ne^{ik^{+}x^{-}} f_{n,m}(\rho) \chi_{m}(\varphi)$$

*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Light-front Dynamics

[Dirac 1949]

- Time is redefined $t = x^0 \rightarrow \underline{x^+ \equiv x^0 + x^3}$ Light front
- Hamiltonian is thus $H = P^0 \rightarrow P^- \equiv P^0 - P^3$



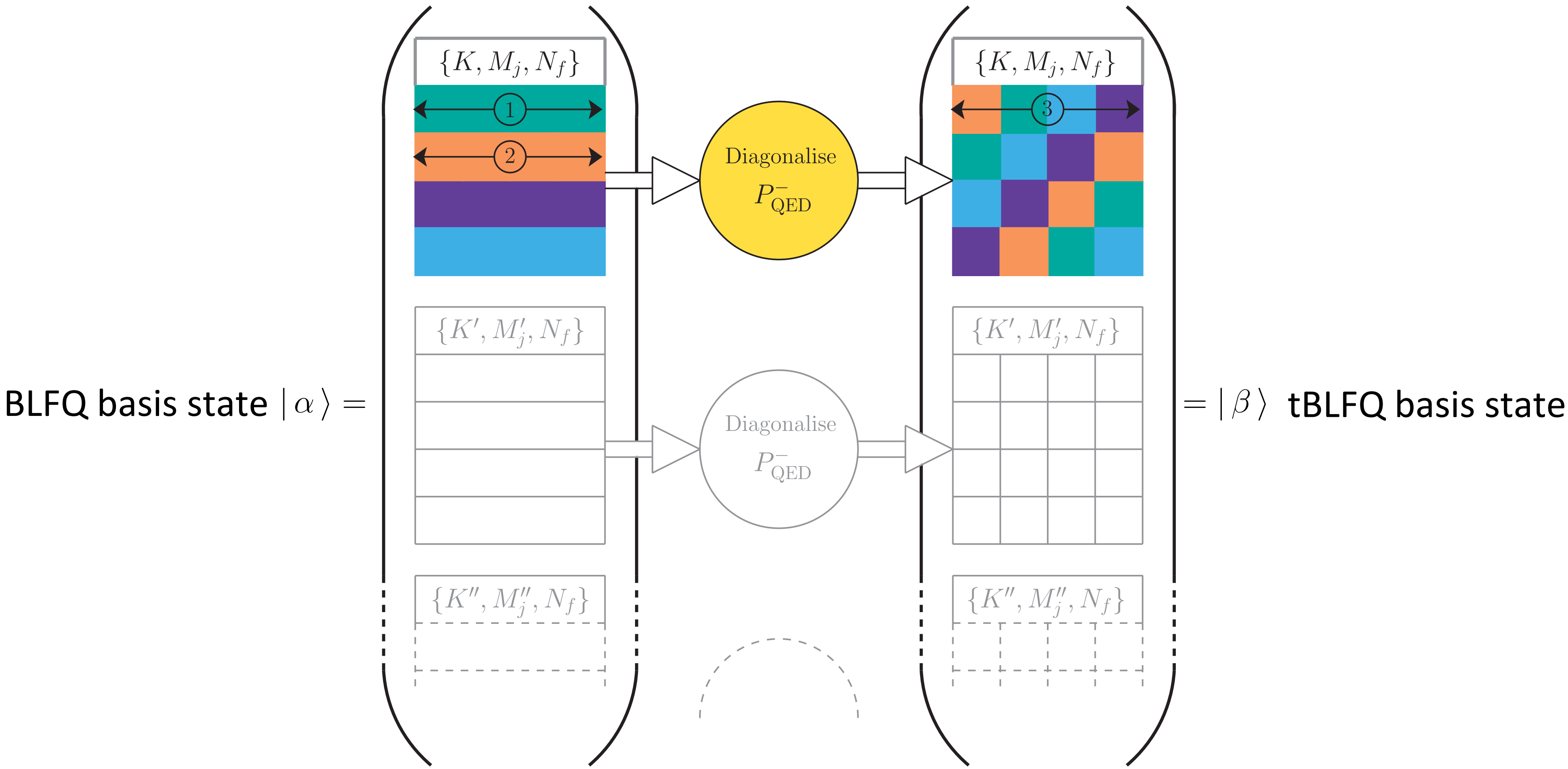
- Quantization and initial condition are specified on the equal light-front time plane
- State evolution is governed by LF-Schrodinger Eq.

$$i\frac{\partial}{\partial x^+}\left|\psi(x^+)\right\rangle = \frac{1}{2}P^-\left|\psi(x^+)\right\rangle$$

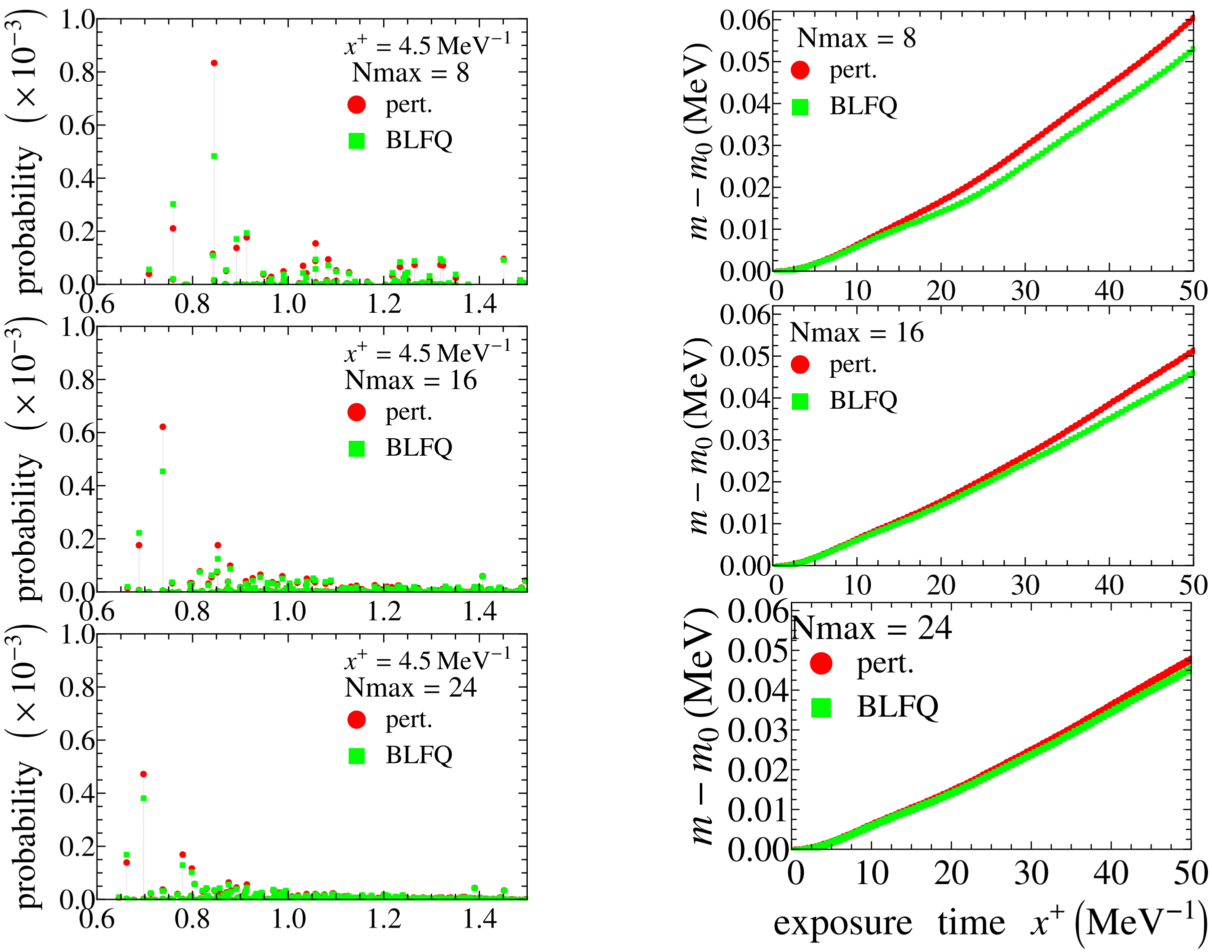
Challenges and Solution

- Challenges
  - Covariant perturbation theory calculates S-matrix between in- and out-states with infinite evolution time in-between
  - Nontrivial transform between results in BLFQ basis and momentum basis (often used in perturbative calculation):
    1. Integration over HO wave function needed
    2. Different normalization for basis states,
       Kronecker delta (BLFQ basis) vs. Dirac delta (momentuem basis)
    3. Nmax truncation exclusive for BLFQ basis
- Solution -- Lightfront (LF) perturbation theory in BLFQ basis
  - Able to calculate transition amplitude per unit time
  - Allows for comparison with nonpert. calculation on the level of transition matrix element of the laser field $\langle\Psi'|V^L|\Psi\rangle$, where $|\Psi\rangle$ and $|\Psi'\rangle$ are eigenstates of P_+^{QED} (adopt the interaction picture)

tBLFQ Basis Construction



Comparison with Perturbation Theory

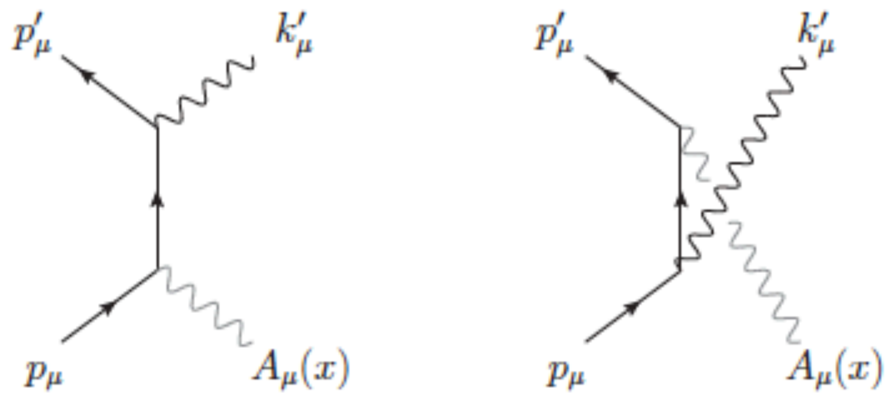


invariant mass (MeV)

'Nonlinear Compton scattering'

- Simplest laser-particle scattering process.
- $e^- \to e^- + \gamma$ within a laser field. Reiss, Nikishov, Ritus, Kibble...
- 10^{20} photons in a laser: model as a background field.

- Perturbation theory:
- Looks like ordinary Compton
- $\sigma \propto$ Klein-Nishina $\times \tilde{A}^2$



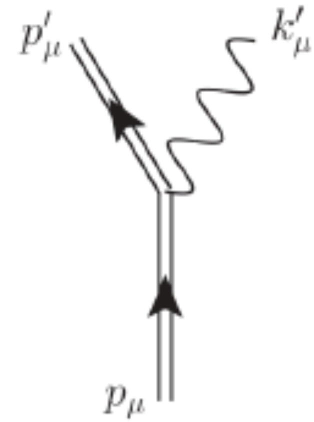
- But! High intensity $\Longrightarrow$ background should be treated nonperturbatively.

At high intensity

- Fermions become 'dressed' by the background.
-  = exact propagator in background.
- Most analytic progress for plane waves. Volkov, 1935

  Harvey, Heinzl, Ilderton PRA 79 (2009) 063407

  Heinzl, Seipt, Kämpfer, PRA 81 (2010) 022125



$\ddot{\frown}$ Few analytic results for realistic background fields.
(Need solution of Dirac equation in background.)

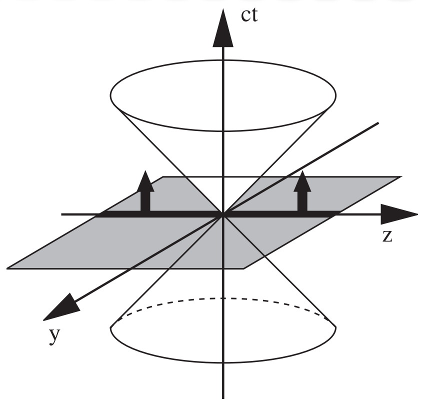
$\ddot{\frown}$ Few results for $1 \to 3$, $2 \to 2 \ldots$ scattering. Ilderton, PRL 106 (2011) 020404
(Even plane wave calculations become very complex.)

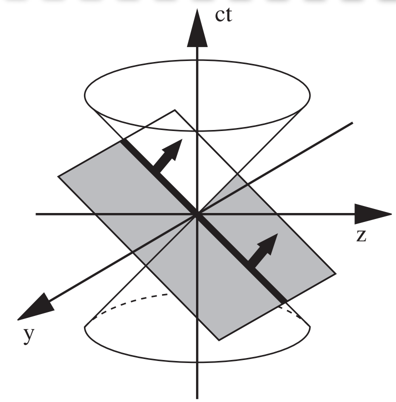
? A different approach needed ?

Light-front vs Equal-time Quantization

[Dirac 1949]

equal time dynamics — VS — light front dynamics





$$i\frac{\partial}{\partial t}|\varphi(t)\rangle = H|\varphi(t)\rangle$$

$$i\frac{\partial}{\partial x^+}|\varphi(x^+)\rangle = \frac{1}{2}P^-|\varphi(x^+)\rangle$$

$$H|\beta\rangle = E_\beta|\beta\rangle$$

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

Basis Functions for Single Particle States

[Vary et al '10, Honkanen et al '11]

- Optimal basis is chosen to speed up numerical calculation
- $\left|e;\, p^{+}, p^{\perp}\right\rangle = \left|e;\, p^{+}\right\rangle \otimes \left|e;\, p^{\perp}\right\rangle$
- Plane wave basis for longitudinal direction: $\left|e;\, p^{+}\right\rangle \sim \exp(ip^{+}x^{-})$
- Harmonic oscillator basis for transverse direction: $\left|e;\, p^{\perp}\right\rangle \sim \left|e;\, n, m\right\rangle$
  - $\left|n, m\right\rangle$: eigenstates of 2D-harmonic oscillator (HO) of frequency ω
- $\left|e;\, p_e^{+}, p_e^{\perp}\right\rangle \rightarrow \left|e;\, p_e^{+}, n_e, m_e\right\rangle$

  $\left|e\gamma;\, p_e^{+}, p_e^{\perp}, p_\gamma^{+}, p_\gamma^{\perp}\right\rangle \rightarrow \left|e\gamma;\, p_e^{+}, n_e, m_e, p_\gamma^{+}, n_\gamma, m_\gamma\right\rangle$

  $\vdots$

- In each Fock sector we truncate states with total HO quantum number beyond Nmax $= \sum_i (2n_i + |m_i| + 1)$
  - Larger Nmax -> larger basis -> more realistic results

    -> numerically more expensive

Outline for BLFQ approach

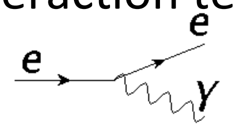
- Set up Hilbert space by Fock space expansion:

$$|e_{\text{physical}}\rangle = a|e\rangle + b|e\gamma\rangle + c|e\gamma\gamma\rangle + d|e\gamma e\bar{e}\rangle + \ldots$$

- Calculate the Hamiltonian matrix in the Fock Space: $\langle i|H|j\rangle$

$$H = T + V$$

  - T: Kinetic energy term for each particle in each Fock sector
  - V: Interaction term coupling different states (and different sectors)

  E.g., 

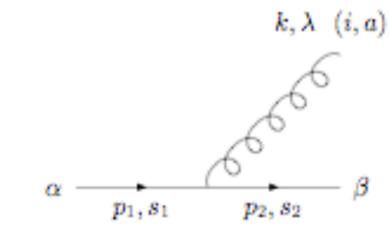
  , See Young Li's talk for more details

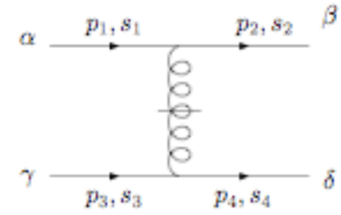
- Diagonalize H and obtain eigenvalues and eigenstates
- Extract observables from the eigenstates: $|e_{physical}\rangle$

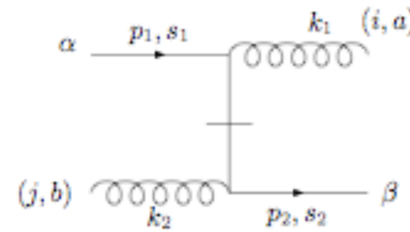
$$O \equiv \langle e_{physical}|\hat{O}|e_{physical}\rangle$$

$\hat{O}$ is the quantum operator for O

Elementary vertices in LF gauge

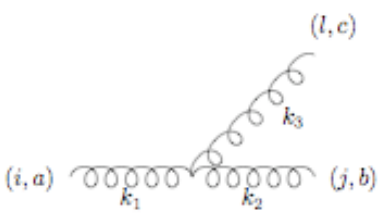


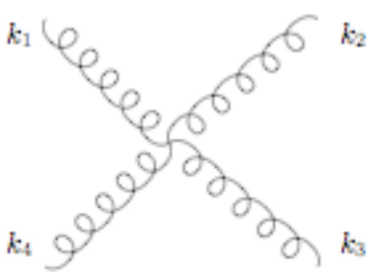




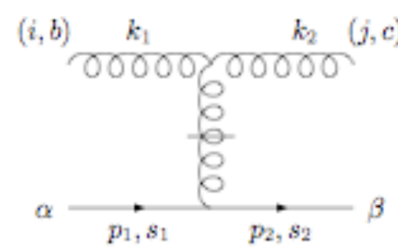
H_{qqqg1}

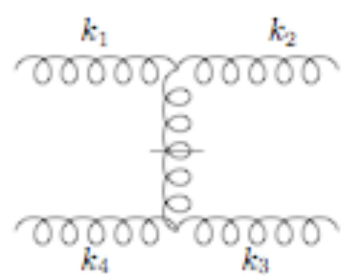
QED & QCD





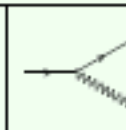
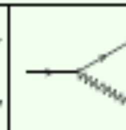
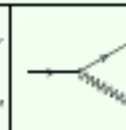
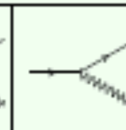
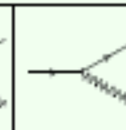
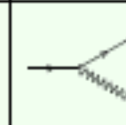
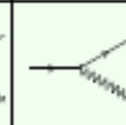
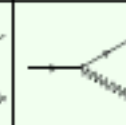
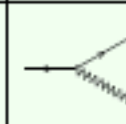
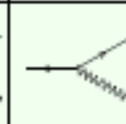
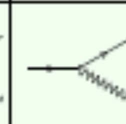
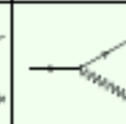
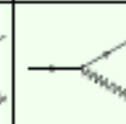
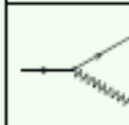
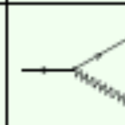
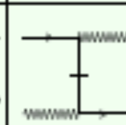
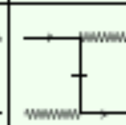
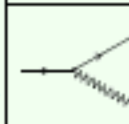
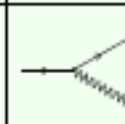
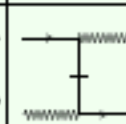
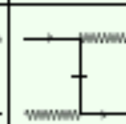
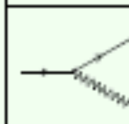
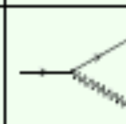
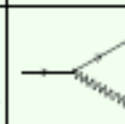
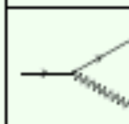
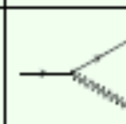
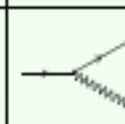
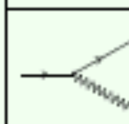
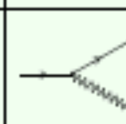
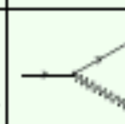
H_{qqgg2}





QCD

Matrix Example: $N_{max} = 3, K = 2$

e: ↑, n=m=0	0	0	0					
e: ↓, n=0, m=1	0	0	0	0	0			
e: ↑, n=1, m=0	0	0	0					
e: ↑, n=0, m=-1 γ: ↑, n=m=0		0				0	0	0
e: ↑, n=m=0 γ: ↑, n=0, m=-1		0				0	0	0
e: ↓, n=m=0 γ: ↑, n=m=0				0	0	0	0	0
e: ↑, n=m=0 γ: ↓, n=0, m=1				0	0	0	0	0
e: ↑, n=0, m=1 γ: ↓, n=m=0				0	0	0	0	0

$$
\begin{aligned}
H = &\frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A_a^i(\mathrm{i}\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi}\frac{1}{(\mathrm{i}\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \\
&- g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+\left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
&+ g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
&+ g\int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
&+ 2g\int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \widetilde{A}^\nu\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

Time-dependent Basis Light-front Quantization

- A numerical non-perturbative approach for time-dependent problems in quantum field theory
- Solves the generalized wave-equation for time-evolution of quantum field configurations $|\varphi(x^+)\rangle$

Generalized wave-eq.

$$\boxed{i\frac{\partial}{\partial x^+}|\varphi(x^+)\rangle = P_+|\varphi(x^+)\rangle}$$

BLFQ

$$\boxed{P_+^0|\Phi_i\rangle = \tilde{P}_+^{0,i}|\Phi_i\rangle}$$

tBLFQ

$$\boxed{i\frac{\partial}{\partial x^+}|\varphi(x^+)\rangle_I = V_I|\varphi(x^+)\rangle_I}$$

Provide basis

- Works in the interaction picture: $P_+(x^+) = P_+^0 + V(x^+)$
- Typical applications: strong field laser physics, heavy-ion physics…

Solving Nonlinear Compton Scattering in tBLFQ

1. Write down the Hamiltonian P_+:

$$P_+(x^+) = P_+^{QED} + V^{LAS}(x^+)$$

2. Solve $P_+^{QED}|\Phi_i\rangle = \tilde{P}_+^i|\Phi_i\rangle$ for the tBLFQ basis $|\Phi_i\rangle$
3. Prepare initial state $|\varphi(0)\rangle$
   - physical electron: the ground state of P_+^{QED} with n_f=1
4. Calculate matrix elements for V^{LAS}

$$\langle\Phi_j|V^{LAS}(x^+)|\Phi_i\rangle_I = e^{i(P_+^j - P_+^i)x^+}\langle\Phi_j|V^{LAS}(x^+)|\Phi_i\rangle$$

5 . Solve for the generalized wave-equation numerically

$$i\frac{\partial}{\partial x^+}\langle\Phi_i|\varphi(x^+)\rangle_I = \sum_j \langle\Phi_i|V^{LAS}|\Phi_j\rangle_I \langle\Phi_j|\varphi(x^+)\rangle_I$$

$$|\Psi(x^+)\rangle_I = U(x^+,0)|\Psi(0)\rangle_I = T\exp\left(-i\int_0^{x^+} V_I^L(x^{+\prime})dx^{+\prime}\right)|\Psi(0)\rangle_I$$

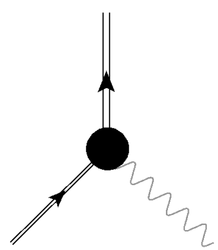
$$\rightarrow (1 - iV_I^L(x^+)\Delta x^+)\cdots(1 - iV_I^L(x_2^+)\Delta x^+)(1 - iV_I^L(x_1^+)\Delta x^+)|\Psi(0)\rangle_I$$

Nonpert. Vs Pert. Laser Matrix Element $\langle\Psi'|V^L|\Psi\rangle$

Nonperturbative evaluation

1. Diagonalize P_+^{QED} for $|\Psi'\rangle$ $|\Psi\rangle$ in BLFQ basis
2. Compute V^L in BLFQ basis
3. Sandwich V^L with $|\Psi'\rangle$ $|\Psi\rangle$ and obtain

$$\langle\Psi'|V^L|\Psi\rangle\Big|_{nonpert} =$$



Perturbative evaluation

1. Diagonalize $P_+^{kinetic}$ for $|\Psi_0'\rangle$ $|\Psi_0\rangle$ in BLFQ basis
2. Convert $|\Psi_0'\rangle$ $|\Psi_0\rangle$ to momentum basis
3. Evaluate $|\Psi'\rangle$ $|\Psi\rangle$ from $|\Psi_0'\rangle$ $|\Psi_0\rangle$ using LF perturbation theory

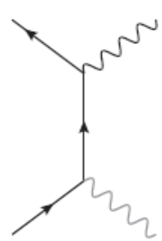
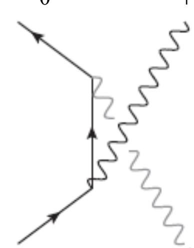
$$|\Psi'\rangle = \left(1 + \frac{1}{\varepsilon' - P_0^{kinetic} + i0_+} V^Q\right)|\Psi_0'\rangle$$

$$|\Psi\rangle = \left(1 + \frac{1}{\varepsilon - P_0^{kinetic} + i0_+} V^Q\right)|\Psi_0\rangle$$

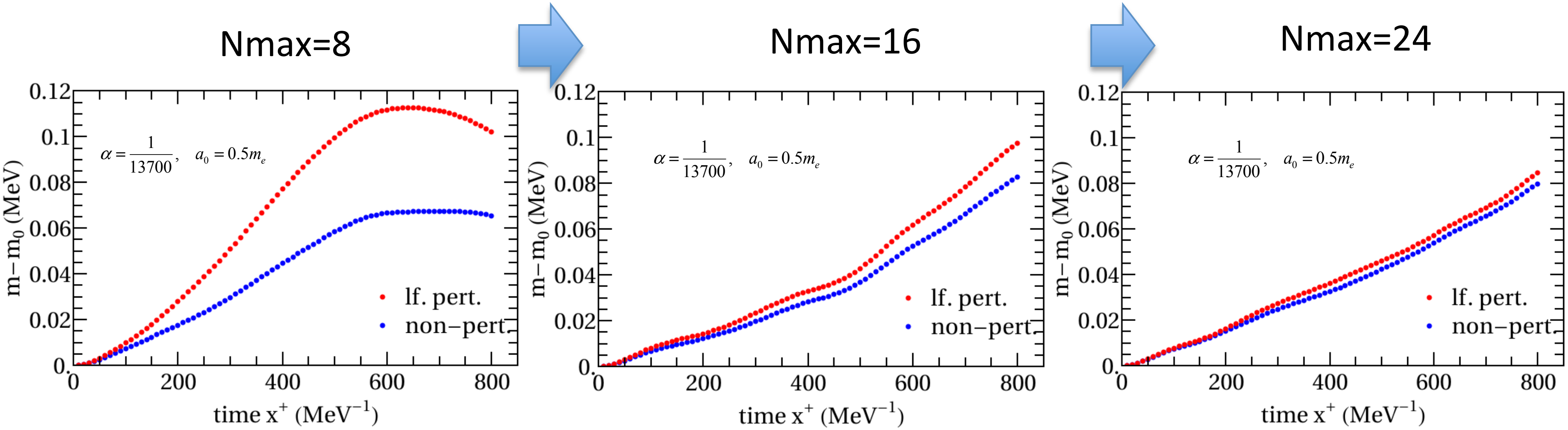
These calculations are actually done in momentum basis (no basis truncation).

4. Sandwich V^L with $|\Psi'\rangle$ $|\Psi\rangle$ and keep terms of leading order in V^L, V^Q:

$$\langle\Psi'|V^L|\Psi\rangle\Big|_{pert} = \langle\Psi_0'|V^Q \frac{1}{\varepsilon' - P_0^{kinetic} + i0_+} V^L|\Psi_0\rangle + \langle\Psi_0'|V^{L*} \frac{1}{\varepsilon - P_0^{kinetic} + i0_+} V^Q|\Psi_0\rangle$$

$=$  $+$ 

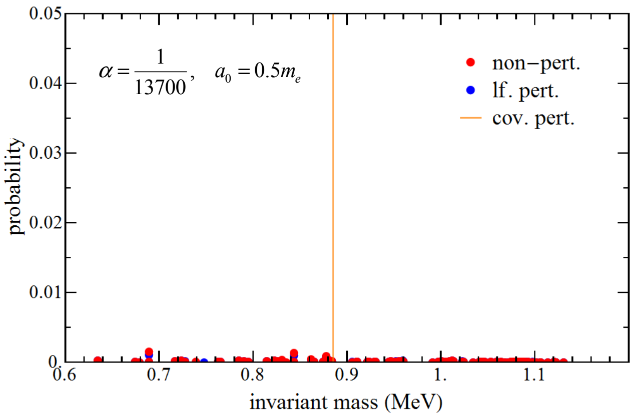
Evolution of Invariant Mass of the System



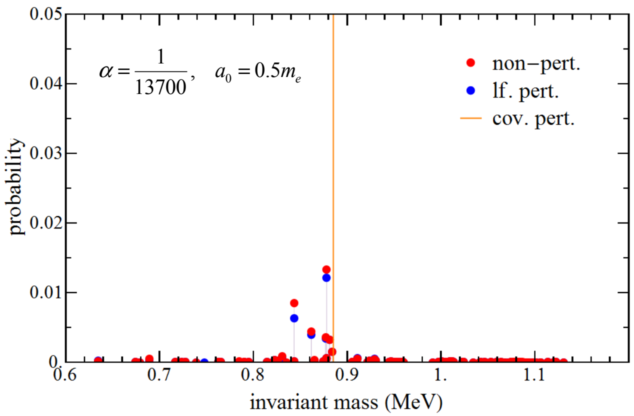
- Invariant mass increases with time as laser field "pumps" energy in
- As Nmax increases better agreements are achieved between calculations based on laser matrix elements from LF. pert. and nonpert. methods, intermediate truncation effects are removed gradually in the nonperturbative case
- Quasi-linear dependence on x^+ is expected in the perturbative regime

Evolution of Excited States for Nmax=24

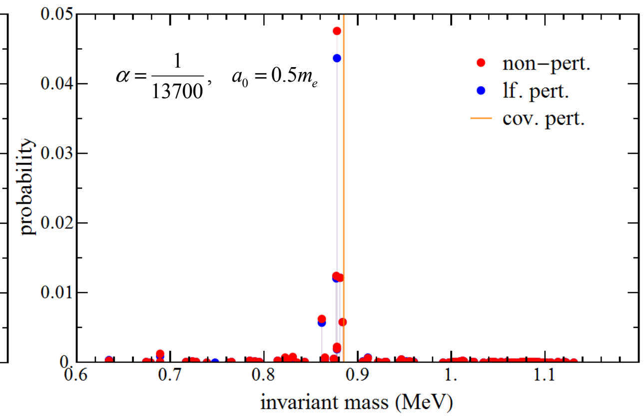
x^{+}=10MeV^{-1}



x^{+}=40MeV^{-1}



x^{+}=80MeV^{-1}



- Evolution of all excited states in the basis are tracked
- Excited states are being populated as time increases
  - Decent agreement between nonpert. and lf. pert. laser matrix elements
- Peak structure emerges for transitions conserving energy
  - Only transitions conserving (light-front) energy keep increasing with time
  - Transitions not conserving (light-front) energy oscillate with time
  - Peak location agrees with covariant perturbation theory
  - Peak width consistent with energy-time uncertainty principle

Application to Scattering Process

- S-matrix

$$S = {}_I\langle\, out \,|\mathcal{T}_+ \exp\left(-\frac{i}{2}\int_0^{x_f^+} V_I \right)|\, in \,\rangle_I$$

1. Construct "in" state out of QED eigenstates and use as initial state $\left|\psi(0)\right\rangle_I = \left|in\right\rangle_I$
2. Evolve $\left|\psi(0)\right\rangle$ until the background field subsides at x_f^+ and obtain the scattering final state $\left|\psi(x_f^+)\right\rangle$
3. Project $\left|\psi(x_f^+)\right\rangle$ onto "out" states (constructed out of QED eigenstates) and obtain S-matrix element

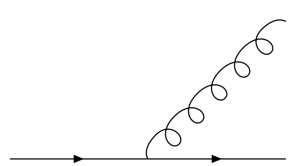
Example: Obtain LF QED Hamiltonian

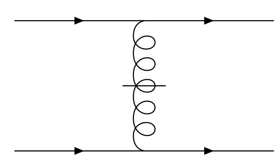
- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$
- Derived Light-front Hamiltonian

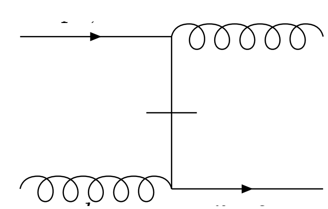
$$P^- = \int \mathrm{d}^2x^\perp \mathrm{d}x^- \;\; F^{\mu+}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

$$= \int \mathrm{d}^2x^\perp \mathrm{d}x^- \;\; \underbrace{\frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^\perp)^2 A^j}_{\text{kinetic energy terms}}$$

$$+ \underbrace{ej^\mu A_\mu}_{\text{vertex interaction}} + \underbrace{\frac{e^2}{2}j^+\frac{1}{(i\partial^+)^2}j^+}_{\text{instantaneous photon interaction}} + \underbrace{\frac{e^2}{2}\bar{\Psi}\gamma^\mu A_\mu\frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu\Psi}_{\text{instantaneous fermion interaction}}$$







Switch to the Interaction Picture

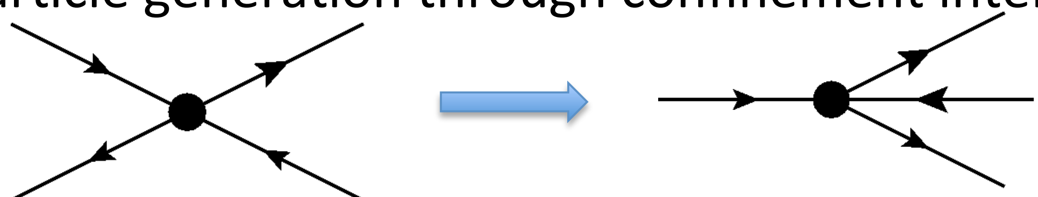
- Time-dependent term treated as “interaction” part

$$P^- = P^-_{QED} + \int \mathrm{d}^2 x^\perp \mathrm{d}x^-\ ej^\mu \mathcal{A}_\mu(x^+)$$
$$= P^-_{QED} + V(x^+)$$

- $i\dfrac{\partial}{\partial x^+}\left|\psi(x^+)\right\rangle = \dfrac{1}{2}P^-\left|\psi(x^+)\right\rangle \to i\dfrac{\partial}{\partial x^+}\left|\psi(x^+)\right\rangle_I = \dfrac{1}{2}V_I\left|\psi(x^+)\right\rangle_I$
- Different splitting from traditional perturb. theory
- $\left\langle \beta' \middle| V_I(x^+) \middle| \beta \right\rangle$ is needed, where QED eigenstates $\left|\beta\right\rangle$ are found by BLFQ

Hadronization / Jet Evolution

- Hadronization in vacuum: a nontrivial dynamic process in QCD
  1. evolve the initial jet qq-bar pair state with tBLFQ
  2. particle generation through confinement interaction



  3. project the evolved wavefunction to desired final states

- Jet evolution in medium created by heavy-ion collision – similar to the evolution in vacuum, but
  – replace the confining interaction by a time-dependent background field modeling the interaction with the medium

Common Variables in Light-front Dynamics

- LF-time $x^+ = x^0 + x^3$
- LF-Hamiltonian $P^- = P^0 - P^3$
- Longitudinal coordinate $x^- = x^0 - x^3$
- Longitudinal momentum $P^+ = P^0 + P^3$
- Transverse coordinate $x^\perp = x^{1,2}$
- Transverse momentum $P^\perp = P^{1,2}$
- Equal-time dispersion relation $P^0 = \sqrt{m^2 + \vec{P}^2}$
- LF dispersion relation $P^- = \dfrac{m^2 + P_\perp^2}{P^+}$

Comparison with Lattice Gauge Approach

- BLFQ: complimentary to Lattice Gauge Approach with the following advantages:
  - (boost invariant) lightfront-wavefunction is directly accessible
  - convenience in calculating observables such as parton distribution function
  - time evolution of quantum field configurations can be straightforwardly calculated

QED in background EM field

$$P^- = P^-_{QED} + \int \mathrm{d}^2 x^\perp \mathrm{d}x^- \; e j^\mu \mathcal{A}_\mu$$

$$\cancel{+ \frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi + \frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi + \frac{e^2}{2}\bar{\Psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \Psi}$$

$$= P^-_{QED} + V$$

A Simple Laser Field Profile

$$eA^-(x^-) = m_e a_0 \cos\left(l_- x^-\right)$$

- Key properties:
  - $\mathcal{A}^-$ is treated classically
  - $\mathcal{A}^-$ is in lightcone gauge, $\mathcal{A}^+ = 0$
  - $\mathcal{A}^-$ is uniform in $x^{1,2}$ and light=front time x^+
  - $\mathcal{A}^-$ depends on $x^- \longrightarrow \mathcal{F}^{+-}$: electric field in longitudinal (x^-) direction
  - a_0 describes the field strength
  - l_- describes the laser field's spatial frequency in x^-

Outline

- Basis Light-Front Quantization (BLFQ) [Vary et al '10, Honkanen et al '11]
  - For spectrum and wavefunction of eigenstates of quantum field theory (ultimate goal: hadrons & nuclear structure)

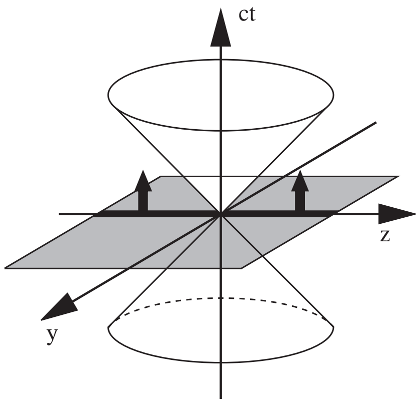
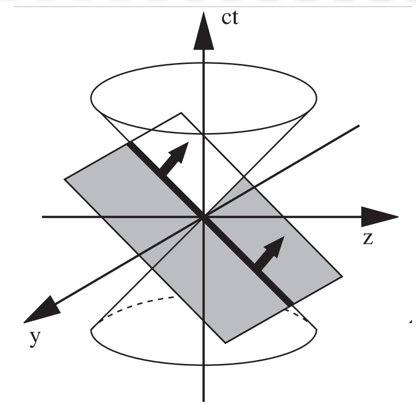
- Time-dependent Basis Light-Front Quantization (tBLFQ)
  - For time evolution of quantum field configurations (ultimate goal: hadrons & nuclear dynamics) [Zhao et al '13]

Basis Light-front Quantization

- BLFQ: approach for solving quantum field theory
  - nonperturbative
    - for systems with strong interaction / strong background field
  - first-principles
    - accept first-principles / effective Hamiltonian as input
  - Hamiltonian formalism
    - direct access to wavefunction of bound states
  - light-front dynamics
    - boost invariant light-front wavefunction
    - lightfront vacuum = vacuum of free Hamiltonian
    - parton interpretation of wavefunction
    - dispersion relation similar to nonrelativistic case

Light-front vs Equal-time Quantization

[Dirac 1949]

equal-time dynamics	$\longrightarrow$ light-front dynamics
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
$i\dfrac{\partial}{\partial t}\left|\varphi(t)\right\rangle = H\left|\varphi(t)\right\rangle$	$i\dfrac{\partial}{\partial x^+}\left|\varphi(x^+)\right\rangle = \dfrac{1}{2}P^-\left|\varphi(x^+)\right\rangle$
$H = P^0$	$P^- = P^0 - P^3$

Common Variables in Light-front Dynamics

- LF-time $x^{+} = x^{0} + x^{3}$
- LF-Hamiltonian $P^{-} = P^{0} - P^{3}$
- Longitudinal coordinate $x^{-} = x^{0} - x^{3}$
- Longitudinal momentum $P^{+} = P^{0} + P^{3}$
- Transverse coordinate $x^{\perp} = x^{1,2}$
- Transverse momentum $P^{\perp} = P^{1,2}$
- Equal-time dispersion relation $P^{0} = \sqrt{m^{2} + \vec{P}^{2}}$
- LF dispersion relation $P^{-} = \dfrac{m^{2} + P_{\perp}^{2}}{P^{+}}$

General Procedure for BLFQ

1. Derive LF-Hamiltonian from Lagrangian
2. Construct basis states $|\alpha\rangle$
3. Calculate Hamiltonian matrix elements $\langle\alpha'|P^-|\alpha\rangle$
4. Diagonalize P^- (solve $P^-|\beta\rangle = P_\beta^-|\beta\rangle$) and obtain its eigenspectrum
5. Evaluate observables $O \equiv \langle\beta|\hat{O}|\beta\rangle$

Discretized Light Cone Quantization (c1985)

$\Downarrow$

Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_{\alpha}(\vec{x}) a_{\alpha}^{+} + f_{\alpha}^{*}(\vec{x}) a_{\alpha} \right]$$

where $\{a_{\alpha}\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_{\alpha}(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_{\alpha}(\vec{x}) f_{\alpha'}^{*}(\vec{x}) d^{3}x = \delta_{\alpha\alpha'}$

Complete: $\sum_{\alpha} f_{\alpha}(\vec{x}) f_{\alpha}^{*}(\vec{x}') = \delta^{3}(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_{a}(\vec{x})$ and our initial choice is

$$f_{\alpha}(\vec{x}) = N e^{ik^{+}x^{-}} \Psi_{n,m}(\rho,\varphi) = N e^{ik^{+}x^{-}} f_{n,m}(\rho) \chi_{m}(\varphi)$$

*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

General Procedure for BLFQ

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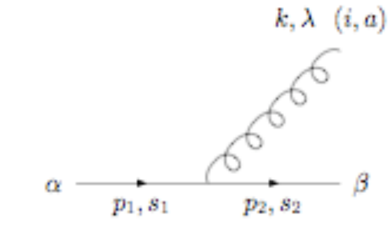
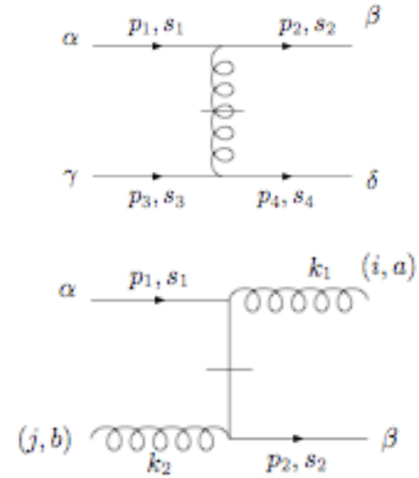
5. Evaluate observables $O \equiv \langle\beta|\hat{O}|\beta\rangle$

LF QCD Hamiltonian

$$
\begin{aligned}
H = {} & \frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A_a^i(\mathrm{i}\partial^\perp)^2 A_{ia} \\
& - \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
& + \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \frac{1}{(\mathrm{i}\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi} \\
& - g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
& + g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
& + \frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
& + g \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
& + 2g \int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \widetilde{A}^\nu\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

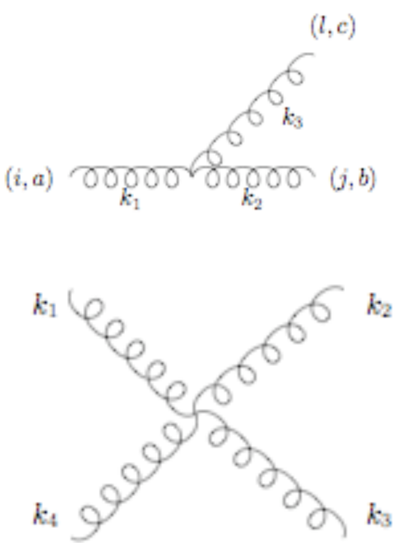
Elementary Vertices in LF Gauge



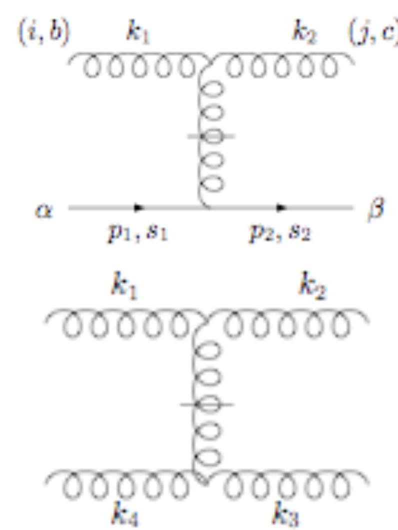


QED & QCD

H_{qqgg1}



H_{qqgg2}



QCD

General Procedure for BLFQ

1. Derive LF-Hamiltonian from Lagrangian
2. Construct basis states $|\alpha\rangle$
3. Calculate Hamiltonian matrix elements $\langle\alpha'|P^-|\alpha\rangle$
4. Diagonalize P^- (solve $P^-|\beta\rangle = P^-_\beta|\beta\rangle$) and obtain its eigenspectrum
5. Evaluate observables $O \equiv \langle\beta|\hat{O}|\beta\rangle$

Basis Construction

- Basis is chosen to reflect the symmetries of P^-
- Procedure:

1. Fock-space expansion

   e.g. $|e_{phys}\rangle = a|e\rangle + b|e\gamma\rangle + c|e\gamma\gamma\rangle + d|ee\bar{e}\rangle + \dots .$

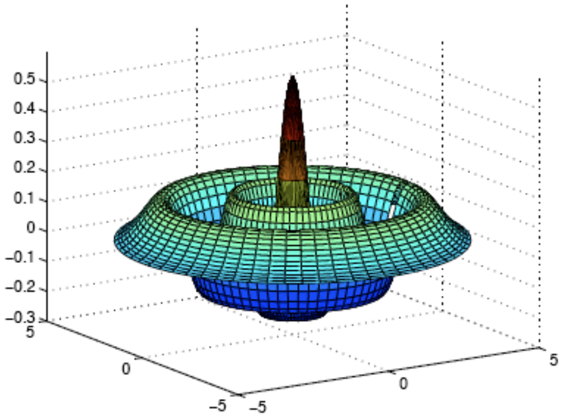
2. For each Fock particle:
   - transverse ($x^{\perp} = x^{1,2}$) 2D-HO basis, labeled by n,m quantum number (HO basis parameter $b = \sqrt{M\Omega}$)
   - longitudinal ($x^{-} = x^{0} - x^{3}$) plane-wave basis, labeled by k

   e.g. $|e\gamma\rangle = |e\rangle \otimes |\gamma\rangle$

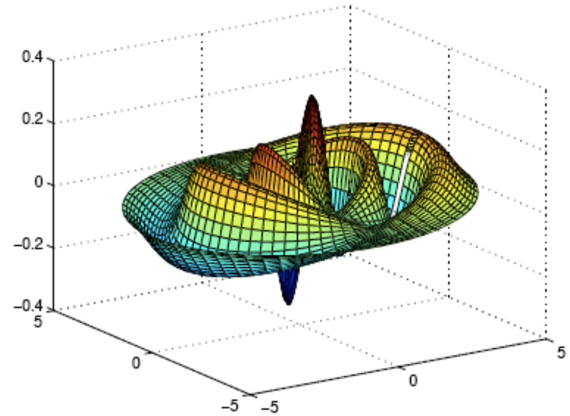
   with $e = \{n^e, m^e, k^e, \lambda^e\}$ and $\gamma = \{n^\gamma, m^\gamma, k^\gamma, \lambda^\gamma\}$

Set of Transverse 2D HO Modes for n=4

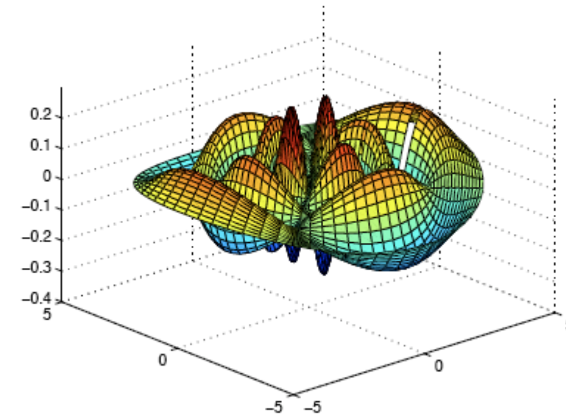
m=0



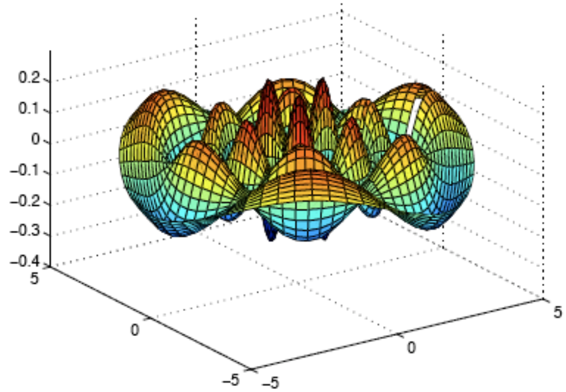
m=1



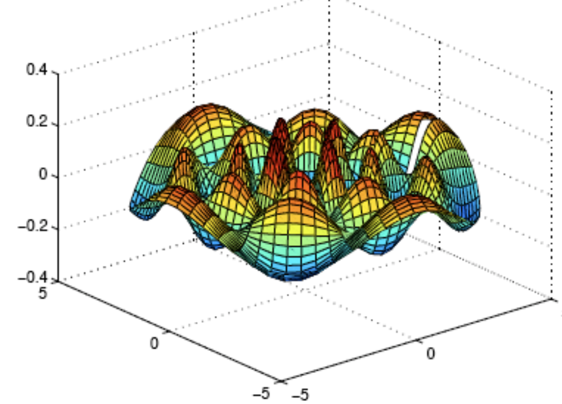
m=2



m=3



m=4



J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath,
G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010).
ArXiv:0905:1411

Basis Reduction

- "Pruning" based on symmetry of Hamiltonian:

  - net fermion number: $\sum_i n_i^f = N^f$

  - total longitudinal angular momentum: $\sum_i (m_i + s_i) = J_z$

  - longitudinal momentum: $\sum_i k_i = K$

  - global color singlets (QCD)

- Truncation:

  - Fock sector truncation

  - longitudinal periodic boundary condition (integer or half integer k_i) $\sum_i k_i = K$

  - "N_{max}" truncation in the transverse directions $\sum_i [2n_i + |m_i| + 1] \leq N_{max}$

General Procedure for BLFQ

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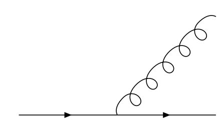
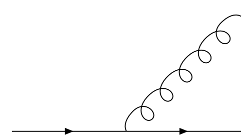
QED Hamiltonian in BLFQ Basis

- QED LF- Hamiltonian in a small basis

$\langle \alpha' \| P^{-}_{QED} \| \alpha \rangle$ (MeV)	$\| e \rangle$	$\| e\gamma \rangle$
$\| e \rangle$	0.3482 (kinetic energy)	-0.0119 
$\| e\gamma \rangle$	-0.0119 	0.9139 (kinetic energy)

Basis parameters: Nmax=2, K=1.5, b=m_e, N_f=1, $|e\rangle + |e\gamma\rangle$ sectors

General Procedure for BLFQ

1. Derive LF-Hamiltonian from Lagrangian
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QED Eigenstates in BLFQ Basis: Solution

- Eigenspectrum of P^-_{QED} (N_f=1) in the small basis :

	P^-_β (MeV)	M_β (MeV)	$\langle e \vert \beta \rangle$	$\langle e\gamma \vert \beta \rangle$
$\vert e \rangle_{phys}$	0.348	0.511	-0.9998	-0.021
$\vert e\gamma \rangle_{scat}$	0.914	1.054	-0.021	-0.9998

- Bound states and scattering states are obtained

Remove Center-of-Mass Motion

- Center-of-mass (cm) motion introduces multiple spurious copies of intrinsic eigenspectrum
- Introducing a Lagrangian multiplier term ΛP_{cm}^{-} to P^{-}

  ⇨ shift up copies with excited cm motion

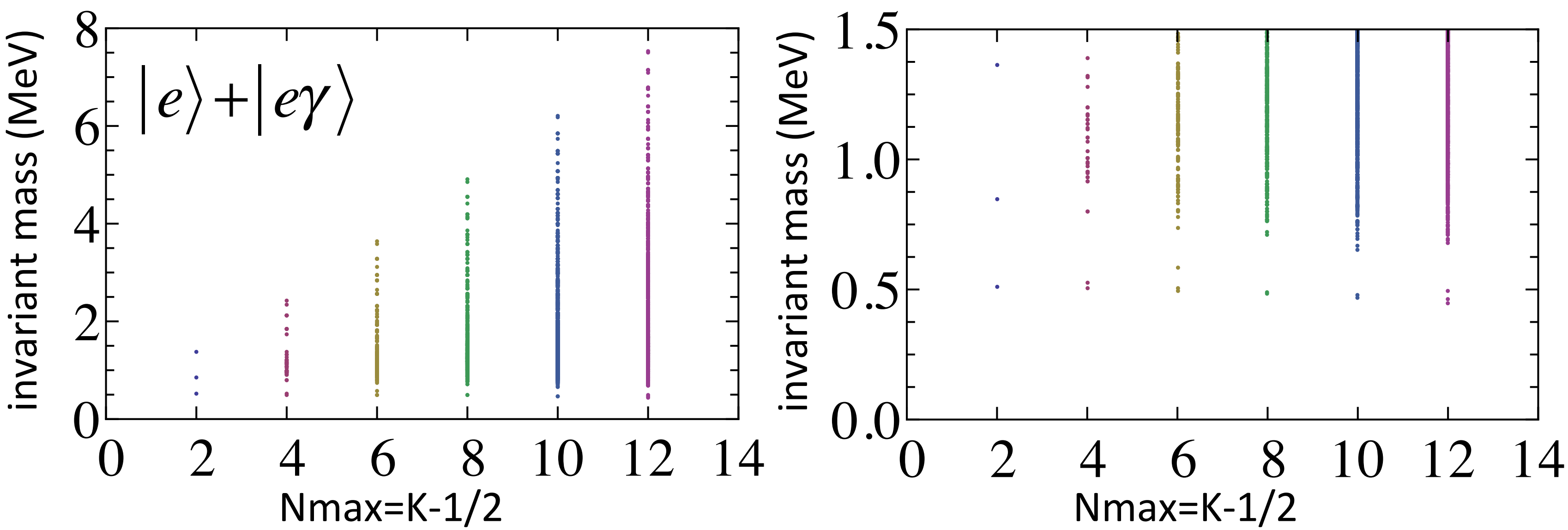
  – requires exact factorization between intrinsic and cm motion

  – achieved by N_{max} truncation in the 2D-HO basis

Renormalization in BLFQ

- Renormalization necessary for obtaining physical results
  - need both mass and charge renormalization
- Renormalization in BLFQ is different from that in perturbation theory
  - sector dependent vs. order-of-coupling dependent counter-terms
  - complication due to broken Ward identity

Eigenspectrum of QED (N_f=1)



- Single electron(bound state) + eγ scattering states (continuum)
- Larger basis covers wider QED spectrum
- Multiple "copies" of intrinsic spectrum introduced by cm motion
- Renormalization needed for obtaining physical electron mass

General Procedure for BLFQ

1. Derive LF-Hamiltonian from Lagrangian
2. Construct basis states $|\alpha\rangle$
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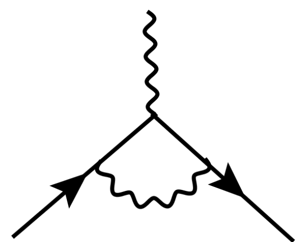
Evaluate Electron g-2 with BLFQ Approach

- Electron anomalous magnetic moment

$$a_e \equiv \frac{g-2}{2} = F_2(q^2 \to 0)$$

- In pert. theory, the following loop gives the Schwinger's result

$$a_e = \frac{\alpha}{2\pi} \quad \left(\alpha = \frac{1}{137}\right)$$



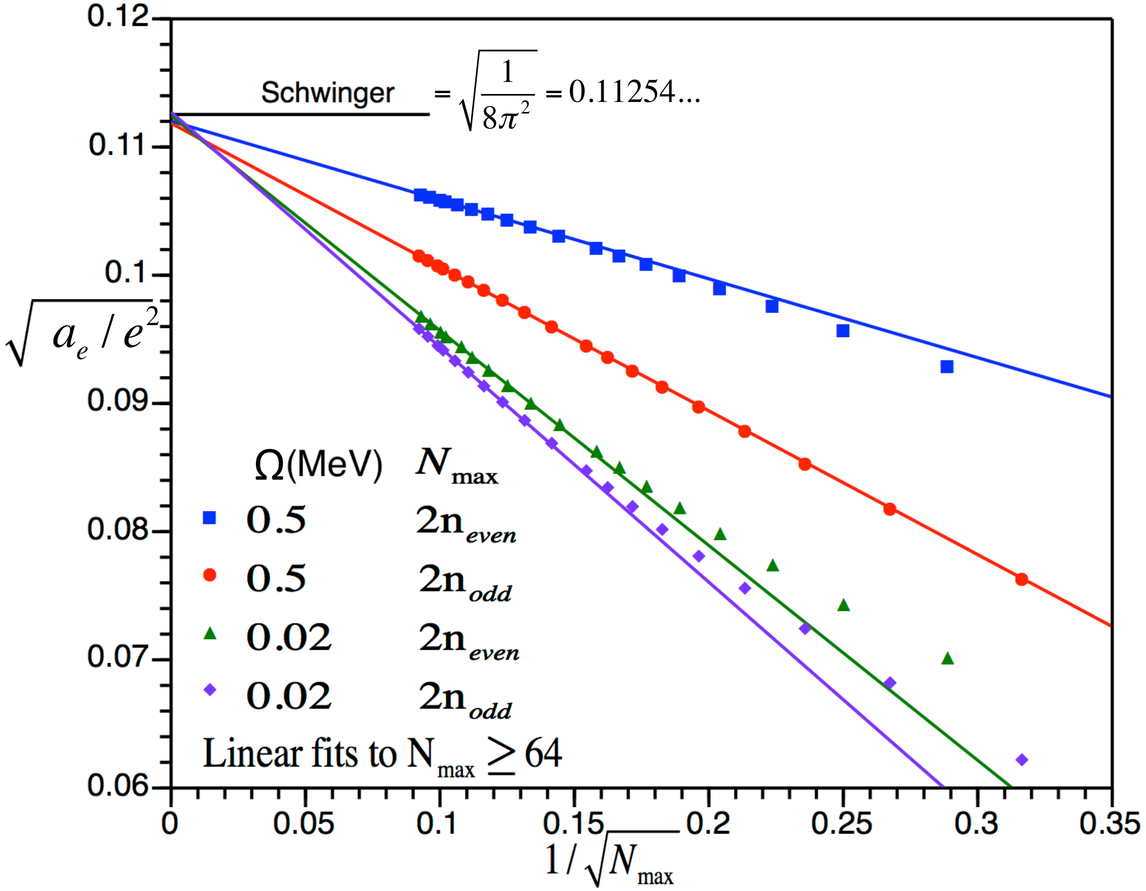
[Schwinger 1948]

- In BLFQ , we first solve for the physical electron state $|e_{phys}\rangle$, and then use it to sandwich the F_2 operator

$$a_e = \langle e_{phys} | \hat{F}_2(q^2 \to 0) | e_{phys} \rangle$$

Numerical Results for Electron g-2

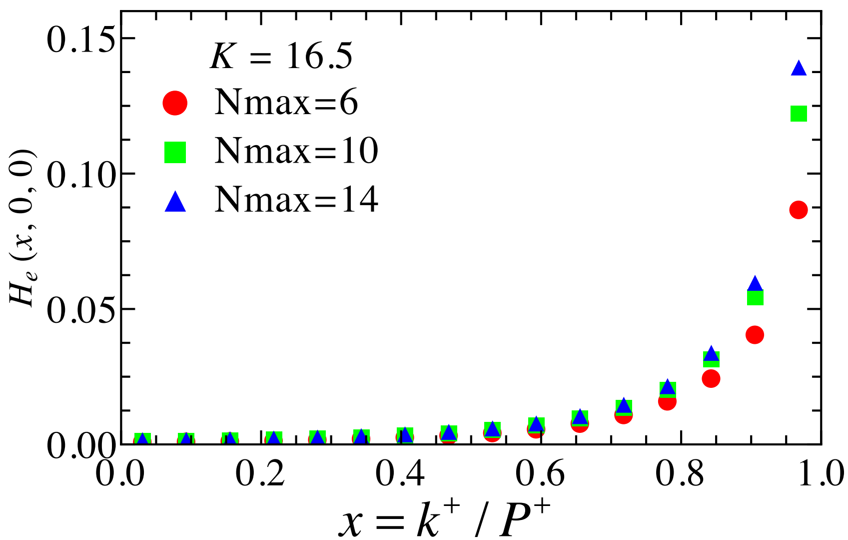
[Zhao, Honkanen, Maris, Vary, Brodsky, 2012]

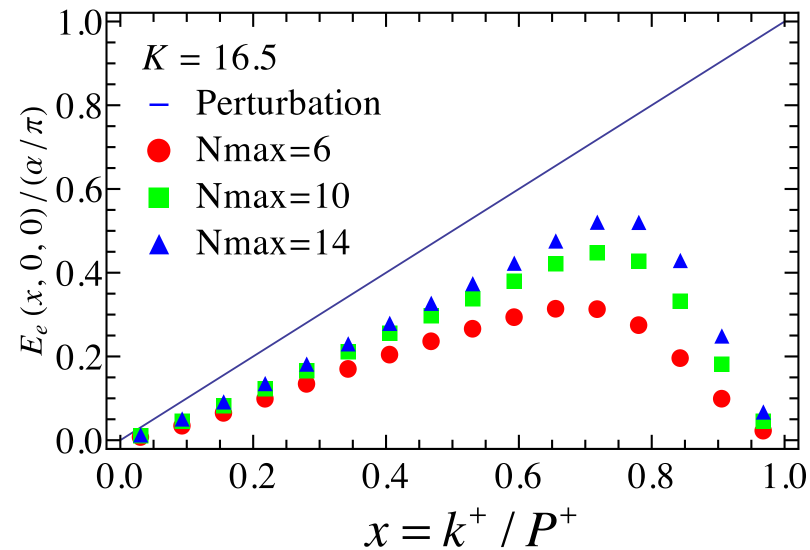


- As Nmax ➔∞, results approach Schwinger result
- Less than 1% deviation from Schwinger's result (by linear extrapl.)
- Convergence over wide range of Ω's (by a factor of 25!)

Generalized Parton Distribution for Electron

- $$H(x,0,\vec{q}) = \left\langle e^{\uparrow}_{phys}(\vec{q}) \middle| \int dy^- e^{ixP^+y^-/2} \overline{\psi}(0)\gamma^+\psi(y) \middle| e^{\uparrow}_{phys}(0) \right\rangle \Big|_{y^+=0, y_\perp=0}$$
- $$\frac{q_1 - iq_2}{2m_e} E(x,0,\vec{q}) = \left\langle e^{\downarrow}_{phys}(\vec{q}) \middle| \int dy^- e^{ixP^+y^-/2} \overline{\psi}(0)\gamma^+\psi(y) \middle| e^{\uparrow}_{phys}(0) \right\rangle \Big|_{y^+=0, y_\perp=0}$$





- Peak in H(x,0,0) around x=1 signals infrared divergence
- E(x,0,0) approaches perturbative results as basis size increases

Conclusion and Outlook (I)

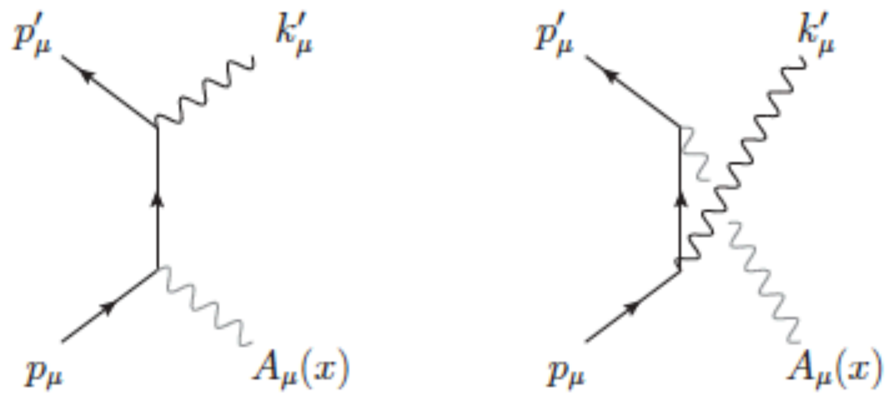
- Basis Light-Front Quantization (BLFQ) approach
  - first-principles nonperturbative method for quantum field theory
  - access light-front wavefunction of bound states
  - initial application to QED reproduces the Schwinger result for anomalous magnetic moment

---

- Bound states in QED, underway…
- Apply to QCD and study hadron spectrum and structure, such as form factors, generalized parton distribution function (GPDs), transverse momentum distribution (TMDs)…
- Finite temperature physics

'Nonlinear Compton scattering'

- Simplest laser-particle scattering process.
- $e^- \to e^- + \gamma$ within a laser field. Reiss, Nikishov, Ritus, Kibble...
- 10^{20} photons in a laser: model as a background field.



- Perturbation theory:
- Looks like ordinary Compton
- $\sigma \propto$ Klein-Nishina $\times \tilde{A}^2$

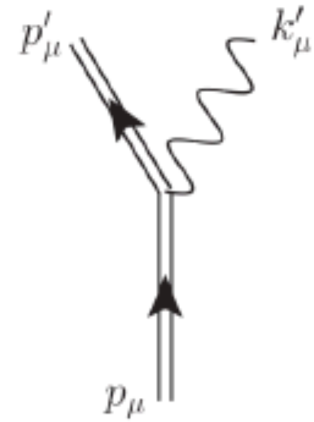
- But! High intensity $\implies$ background should be treated nonperturbatively.

At high intensity

- Fermions become ‘dressed’ by the background.
-  = exact propagator in background.
- Most analytic progress for plane waves. Volkov, 1935

  Harvey, Heinzl, Ilderton PRA 79 (2009) 063407

  Heinzl, Seipt, Kämpfer, PRA 81 (2010) 022125

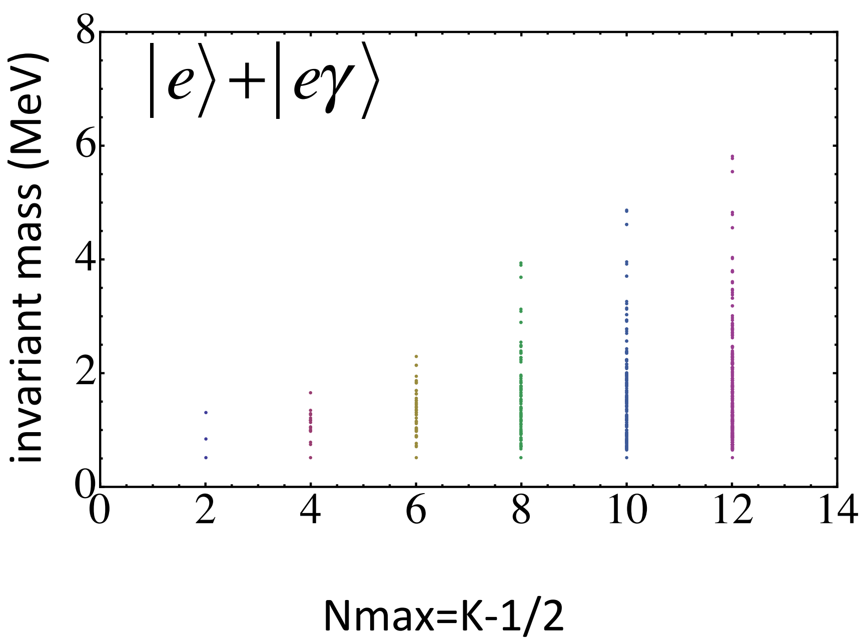


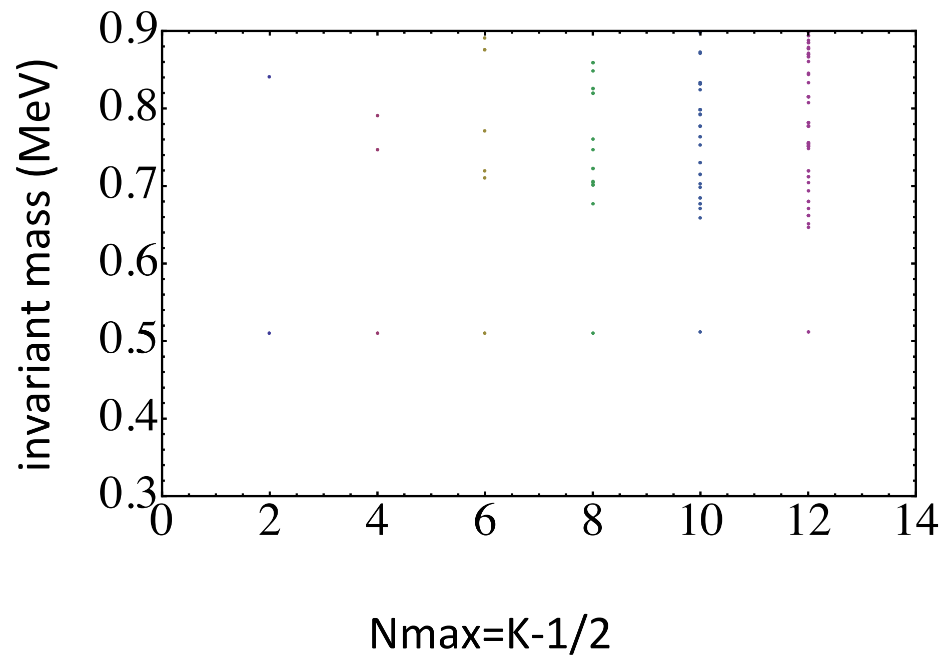
☹ Few analytic results for realistic background fields.
(Need solution of Dirac equation in background.)

☹ Few results for $1 \to 3$, $2 \to 2$... scattering. Ilderton, PRL 106 (2011) 020404
(Even plane wave calculations become very complex.)

? A different approach needed ?

Eigenspectrum of QED (N_f=1)





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