LFQ of the O(N) Non Linear Sigma Models

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(In this talk I would consider the LFQ of a class of NLSM's)

PLAN

- **A Recap of the Model and Motivations**
- **Quantization: Instant Form Theory**
  Construction of GI models and their Quantization
- **Gauged–NLSM with SR**
- **Gauged-NLSM with FR (IF and in FF)**
  (and its quantization on the LF)
- **Conclusions**

It is a (1+1)D model involving the non linear constraint

It is a Constrained System

Systems with Constraints play a very important role in the description of fundamental laws of nature.

Most of the Systems of Physical Importance are Constrained Systems e.g. ED, QED,QCD, EW-Theories , String Theories and D-Brane Actions are all Constrained Systems.

- An important class of 2D field theories is represented by NLSM's and a class of gauged NLSM's, and the complex versions of these models

- Further these models can also be considered with and without the topological terms

- In these O(N)- NLSM's, the field sigma is a real N component field

- and the $CP^{(N-1)}$ models are the complex versions of the NLSM's

- The motivations for a study of the above models comes from the fact that the O(N) NLSM’s in (1+1)-D’s have some striking qualitative similarities with QCD. Some of the common features of both the FT’s are e.g., renormalizability and asymptotic freedom.

- They provide a laboratory for various non-perturbative techniques e.g., the 1/N - expansion, operator product expansions and the low energy theorems
and they are very important in the context of string field theories and D-brane actions where they appear in the classical limit.

- A class of gauged NLSM's and the complex versions of these models namely, the $CP\uparrow(N-1)$ models are also well known to possess an infinite number of conservation laws and also to be asymptotically free and to possess 1/N – expansions

    - They also provide a laboratory for testing several interesting theoretical ideas.

Non Linear Sigma Model in 2D defined by

UK+DSK+HJWMK Helv Phys Acta 66(1993)752-794

$$L^N = \frac{1}{2}\partial_\mu \sigma_k \partial^\mu \sigma_k + \lambda(\sigma_k{}^2 - 1); \qquad \mathrm{k} = 1,2,3\ldots\ldots, \mathrm{N}$$

$$= \frac{1}{2}(\partial_0 \sigma_k^2 - \partial_1 \sigma_k^2) + \lambda(\sigma_k{}^2 - 1); \qquad \mathrm{k} = 1,2,3\ldots\ldots, \mathrm{N}$$

$\sigma \equiv [\sigma_k (x,t), k=1,2,\ldots\ldots N]$ is a multiplet of N-real scalar fields in 2D and $\lambda(x,t)$ is another scalar field. The field $\sigma(x,t)$ maps the 2D space- time into the N-dimensional internal manifold whose coordinates are $\sigma_k (x,t)$.

$$\sigma_k \begin{cases} \text{O(N) - Vector} \\ \text{O(2) - Scalar} \end{cases} \qquad \lambda \begin{cases} \text{O(N) - Scalar} \\ \text{O(2) - Scalar} \end{cases}$$

Canonical conjugate momenta resp. to σ_k *and* λ *are*

$$\pi_k := \frac{\partial L^N}{\partial(\partial_0 \sigma_k)} = \partial_0 \sigma_k; \qquad p_\lambda := \frac{\partial L^N}{\partial(\partial_0 \lambda)} = 0$$

Total Hamiltonian density :

$$H_N^T = \frac{1}{2}\pi_k^2 + \frac{1}{2}\partial_1 \sigma_k^2 - \lambda(\sigma_k^2 - 1) + p_\lambda \omega$$

GNISM

$\exists$ one PC and three SC's

$$\chi_1 := p_\lambda \approx 0$$ PC

$$\chi_2 := \{\chi_1, H_T^N\}_p = (\sigma_k^2 - 1) \approx 0$$ SC

$$\chi_3 := \{\chi_2, H_T^N\}_p = 2\sigma_k \pi_k \approx 0$$ SC

$$\chi_4 := \{\chi_3, H_T^N\}_p = (2\pi_k^2 + 4\lambda\sigma_k^2 + 2\sigma_k[\partial_1\partial_1\sigma_k]) \approx 0$$ SC

The symbol $\approx$ denotes a weak equality in the sense of Dirac, and it implies that these constraints hold as a strong equalities only on the reduced hypersurface of the constraints and not in the rest of the phase space of the classical theory (and similarly one can consider it as a weak operator equality for the corresponding quantum theory)

The matrix of Poisson brackets among the constraints χ_i possesses Gauss anomalies and therefore non singular and thus its inverse exists .

LVGC $\partial_\mu j^\mu \neq 0$

$\exists$ VGA

Thus theory is a Gauge Non Invariant Theory

The non vanishing elements of the inverse of the matrix are

$$(T^{-1})_{12} = -(T^{-1})_{21} = \left(\frac{2\pi_k^2 - 4\lambda\sigma_k^2 - \sigma_k\partial_1\partial_1\sigma_k}{4\sigma_k^2\sigma_k^2}\right)\delta(z-z') - \left(\frac{1}{4\sigma_k^2}\right)\partial_1\partial_1\delta(z-z')$$

$$(T^{-1})_{13} = -(T^{-1})_{31} = \left(\frac{-\sigma_k\pi_k}{4\sigma_k^2\sigma_k^2}\right)\delta(z-z')$$

$$(T^{-1})_{14} = -(T^{-1})_{41} = \left(\frac{1}{4\sigma_k^2}\right)\delta(z-z')$$

$$(T^{-1})_{23} = -(T^{-1})_{32} = \left(\frac{-1}{4\sigma_k^2}\right)\delta(z-z')$$

Finally, the nonvanishing equal-time commutation relations are

$$[\pi_l(x), \pi_m(y)] = \frac{i}{\sigma_k^2}[\sigma_l(x)\pi_m(y) - \pi_l(x)\sigma_m(y)]\delta(x-y)$$

$$[\sigma_l(x), \pi_m(y)] = i[\delta_{lm} - \frac{\sigma_l(x)\sigma_m(y)}{\sigma_k^2}]\delta(x-y)$$

Construction of GI models corresponding to GNI model

GISM

Two methods

By calculating Stueckelberg/WZ kind of a term [introduced by E.C.G. Stueckelberg (HPA 1941)] in the context of renormalization properties of massive gauge theories.

In this method we enlarge the HS of the theory by introducing a new field called Stueckelberg Field through the redefinition of fields in the original Lag density L^N

Then after performing the changes in L^N we obtain the modified Lag density

$$L^I = L^N + L^S$$

describes a GI th.

appropiate ST

original Lag density

By terminating the chain of SC constraints which follows from a single PC

In this method we modify Hamiltonian density so that chain of constraints terminates at the desired point.

This method is however applicable only if a chain of SC constraint follow from a single PC

We reformulate a GNI theory into GI theory without any change in its physical contents.

FFSM

Theory on the Light-Front

PAM Dirac RMP21(1949)392

S.J. Brodsky et al PR301(1998)299

In the LFQ we define the Light –Cone coordinates a la Dirac:

$$x^{\pm} := \frac{1}{\sqrt{2}}\left(x^{0} \pm x^{1}\right)$$

And then rewrite all the quantities involved in the Lag. Density of the theory in terms of $x^{\pm}$ instead of x^{0} and x^{1}

The idea of quantizing on the Light-Cone as a characteristic surface is due to Dirac who called it **Front Form dynamics** as opposed to the usual space-like surface quantization or the Instant-Form dynamics

FFSM

The theory on the LF

UK+DSK IJTP(2002)1941-1956

$$S = \int L \, dx^+ dx^-$$

$$L = [\partial_+ \sigma_k \partial_- \sigma_k + \lambda(\sigma_k^2 - 1)]$$

The theory is seen to possess a set of THREE constraints

$$\chi_1 = p_\lambda \approx 0 \qquad (PC)$$
$$\chi_2 = (\pi_k - \partial_- \sigma_k) \approx 0 \qquad (PC)$$
$$\chi_3 = (\sigma_k . \sigma_k - \partial_- \sigma_k) \approx 0 \qquad (SC)$$

$$S_{\alpha\beta}(w^-, z^-) := \{\chi_\alpha(w^-), \chi_\beta(z^-)\}_p \quad \text{is singular}$$

The Total Hamiltonian density

$$H_T = \int dx^- \left[\lambda(\sigma_k^2 - 1) + p_\lambda u + (\pi_k - \partial_- \sigma_k) v\right]$$

where u and v are LMF's

The constraints form a set of FCC's and the theory is a GI th. and is invaiant under the LVGT

$$\delta\sigma_k = \beta(x,t); \quad \delta\pi_k = \partial_{-}\beta(x,t); \quad \delta v = \partial_{+}\beta(x,t)$$

$$\delta\lambda = \delta u = \delta p_{\lambda} = 0 = \delta\pi_u = \delta\pi_v = 0$$

$$\beta \equiv \beta(x^{-}, x^{+}) \ \textit{is gauge parameter}$$

FFSM

The theory could now be quantized under suitable GFC's

It is Imp to record here that usual original O(N)-NLSM is a GNI theory in the IF, however in the FF it describes a GI theory.

In the Front Form theory there does not exist any problem with respect to operator ordering as one encounters in the case of IF theory because product of canonical variables appear in the CR's and in the expressions for the constraints.

This problem is however be solved if one demands that all the fields and all the canonical momenta are Hermitian operators and that canonical commutation relation be consistent with the Hermiticity of these operators.

J maharana , PLB128(1993)411;
Ann Ins.Henri Poincare (XXXIX)(1993)193

GSM-SR
Ins Form

Gauged Non Linear Sigma Model

UK IJTP40(2001)491-506

The NLSM studied so far do not involve any gauge fields in the th.

Corresponding to these models, if we consider the models involving the gauge field, we obtained the so called gauged-NLSM

The O(N)-GNLSM is described by the Lagrangian density in (1+1)D

$$L = \frac{1}{2}\partial_\mu\sigma_k\partial^\mu\sigma_k + \lambda(\sigma_k^2 - 1) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - eA_\mu\partial^\mu\sigma_k + \frac{1}{2}e^2A_\mu A^\mu$$

$$= \frac{1}{2}(\partial_0\sigma_k^2 - \partial_1\sigma_k^2) + \lambda(\sigma_k^2 - 1) + \frac{1}{2}(\partial_0 A_1 - \partial_1 A_0)^2$$

$$- e(A_0\partial_0\sigma_k - A_1\partial_1\sigma_k) + \frac{1}{2}e^2(A_0{}^2 - A_1{}^2)$$

GSM-SR Ins Form

$$H_T^I = \frac{1}{2}[\pi_k^2 + E^2 + (\partial_1\sigma_k)^2 + e^2 A_0{}^2] + E\partial_1 A_0 + eA_0\pi_k - eA_1(\partial_1\sigma_k)$$
$$- \lambda(\sigma_k^2 - 1) - \frac{1}{2}e^2[(A_0 - A_1)^2] + \pi_0 u + p_\lambda w$$

The theory possesses FIVE constraints:

$$\chi_1 = \pi_0 \approx 0$$
$$\chi_2 = p_\lambda \approx 0$$
$$\chi_3 = (\partial_1 E - e\pi_k) \approx 0$$
$$\chi_4 = (\sigma_k^2 - 1) \approx 0$$
$$\chi_5 = (2\sigma_k\pi_k + 2eA_0\sigma_k) \approx 0$$

Here $\pi_{0,}\, p_{\lambda,}\, \pi_{k,}$ *and* $E(:= \pi^1)$ *are the canonical momenta corresponding to* A_0, λ, σ_k *and* A_1 *resp.*

The set of above constraints form a set of FCC's and the th. is a GI th.

The divergence of the vector current density $\partial^\mu j_\mu = 0$ under the gauge constraint $\lambda \approx 0$ which is, in fact, equivalent to temporal or time axial kind of a gauge for the coordinate λ.

GSM-SR
Ins Form

The nonvanishing ETCR's of the theory under the gauge $\lambda \approx 0$ are finally obtained as

$$[A_0(x), \pi_0(y)] = \frac{-i}{e^2}\delta(x-y)$$

$$[A_1(x), \pi_k(y)] = \frac{-i}{e}\partial_1\delta(x-y)$$

$$[A_1(x), E(y)] = i\delta(x-y)$$

$$[A_0(x), A_1(y)] = \frac{i}{e^2}\partial_1\delta(x-y)$$

$$[\pi_0(x), \pi_k(y)] = \frac{-i}{e}\delta(x-y)$$

GSM-SR
F Form

Gauged NLSM: LFQ

UK, IJTP(2001)1561-80

$$S = \int L \, dx^+ dx^-$$

$$L = \partial_+ \sigma_k \partial_- \sigma_k + \lambda(\sigma_k^2 - 1) + \frac{1}{2}(\partial_+ A^+ - \partial_- A^-)^2$$

$$- e[A^-(\partial_- \sigma_k) + A^+(\partial_+ \sigma_k)] + [\mathrm{e}^2 A^- A^+]$$

Canonical Hamiltonian density

$$H_c = \frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial_- A^-) - eA^-(\partial_- \sigma_k) - \lambda(\sigma_k^2 - 1) - e^2 A^+ A^-$$

Constraints

$$\Omega_1 = \phi_1 = \Pi^+ \approx 0 \qquad \text{(PC)}$$

$$\Omega_2 = \psi_1 = \Pi_\lambda \approx 0 \qquad \text{(PC)}$$

$$\Omega_3 = \chi_1 = (\pi_k - \partial_- \sigma_k + eA^+) \approx 0 \qquad \text{(PC)}$$

$$\Omega_4 = \{\phi_1, H_T^N\} = [\partial_- \Pi^- + e(\partial_- \sigma_k) + e^2 A^+] \approx 0 \qquad \text{(SC)}$$

$$\Omega_5 = \{\psi_1, H_T^N\} = (\sigma_k^2 - 1) \approx 0 \qquad \text{(SC)}$$

GSM-SR
F Form

The matrix of PB's among the constraints $S_{\alpha\beta}$ is singular with few nonvanishing elements

$$S_{33} = [-2\partial_{-}\delta(w^{-} - z^{-})]$$

$$S_{35} = -S_{53} = [-2\sigma_{k}\delta(w^{-} - z^{-})]$$

$$S_{44} = [-2e^{2}\partial_{-}\delta(w^{-} - z^{-})]$$

$\Rightarrow \exists$ No Inverse and the set of constraints is Ist Class
i.e., The theory is a Gauge Theory.

Invariant under the following LVGT

$$\delta\sigma_{k} = e\beta(x,t); \quad \delta A^{+} = \partial_{-}\beta(x,t); \quad \delta A^{-} = \partial_{+}\beta(x,t); \quad \delta\lambda = -\partial_{+}\beta(x,t)$$

$$\delta u_{1} = \partial_{+}\partial_{+}\beta(x,t); \quad \delta u_{2} = -\partial_{+}\partial_{+}\beta(x,t); \quad \delta u_{3} = e\partial_{+}\beta(x,t);$$

$$\delta\Pi u_{1} = \delta\Pi u_{2} = \delta\Pi u_{3} = \delta\Pi_{k} = \delta\Pi_{\lambda} = \delta\Pi^{+} = \delta\Pi^{-} = 0$$

Total Hamiltonian density

$$H_T = H_c + \Pi^{+} u_1 + \Pi_{\lambda} u_2 + (\Pi_k - \partial_{-} \sigma_k + eA^{+}) u_3$$

The nonvanishing ELCTCR's of the theory under the Gauge $A^- \approx 0$ and $\lambda \approx 0$ are finally obtained as ;

$$[A^+(x^-),\Pi^-(y^-)] = 3/2\, i\, \delta(x^- - y^-)$$
$$[A^+(x^-),\Pi_k(y^-)] = 1/2\, i\partial_-\, \delta(x^- - y^-)$$
$$[A^+(x^-),A^+(y^-)] = [-1/2\, e^2]\, i\, \partial_-\, \delta(x^- - y^-)$$
$$[\Pi^-(x^-),\Pi^-(y^-)] = [-e^2/4]\, \mathcal{E}(x^- - y^-)$$
$$[\Pi^-(x^-),\Pi_k(y^-)] = 1/2\, ie\, \delta(x^- - y^-)$$
$$[\Pi_k(x^-),\Pi_k(y^-)] = [-1/2]\, i\partial_-\, \delta(x^- - y^-)$$

In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z[J_k]$ of the theory, in the presence of the external sources J_k e.g., as follows

$$Z[J_k] = \int [d\mu] \exp\left[i \int d^2x \left[J_k \Phi^k + \mathcal{L}_{IO}\right]\right]$$

Where the Phase space variables of the theory are:
$\Phi_k \equiv (\sigma_k, \lambda, A^+, A^-, u_1, u_2, u_3)$ with the corresponding respective canonical conjugate momenta : $\Pi_k \equiv (\Pi_k, \Pi_\lambda, \Pi^-, \Pi^+, \Pi_{u_1}, \Pi_{u_2}, \Pi_{u \ldots}$

The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under gauge-fixing is obtained as :

$$[d\mu] = [4e\delta(x^- - y^-)\partial_- \delta(x^- - y^-)]\,[d\sigma_k][d\lambda][dA^+][dA^-][du_1][du \ldots$$
$$[du_3][d\Pi_k][d\Pi_\lambda][d\Pi^-][d\Pi^+][d\Pi_{u_1}][d\Pi_{u_2}][d\Pi_{u_3}][\delta \ldots$$
$$[\delta(\Pi_\lambda \approx 0)]\,[(\delta(\Pi_k - \partial_- \sigma_k + eA^+) \approx 0)]$$
$$[\delta(\partial_- \Pi^- + e\partial_- \sigma_k + e^2 A^+) \approx 0)]$$
$$[\delta((\sigma_k^2 - 1) \approx 0)]\,[\delta(A^- \approx 0)][[\delta(\lambda \approx 0)]$$

Gauged NLSM with FR
(present work)

GSM-FR
Inst Form

The Gauged O(N)-NLSM with a new regularization called as the Faddeevian regularization

The mass-like term for the vector gauge boson A_μ is different that of the gauged O(N)-NLSM with the standard regularization

In the Faddeevian regularization in the Instant form it is defined by the Lagrangian density [(1+1)D]

$$L_{gsm}^{N} = \frac{1}{2}\partial_\mu\sigma_k\partial^\mu\sigma_k + \lambda(\sigma_k^2 - 1) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - eA_\mu\partial^\mu\sigma_k + \frac{1}{2}e^2A_\mu M^{\mu\nu}A_\nu$$

$$= \frac{1}{2}(\partial_0\sigma_k^2 - \partial_1\sigma_k^2) + \lambda(\sigma_k^2 - 1) + \frac{1}{2}(\partial_0A_1 - \partial_1A_0)^2$$

$$- e(A_0\partial_0\sigma_k - A_1\partial_1\sigma_k) + \frac{1}{2}e^2[(A_0 - A_1)^2 - 4A_1{}^2]$$

$$M^{\mu\nu} = \begin{bmatrix} 1 & -1 \\ -1 & -3 \end{bmatrix}$$

SM+PM, Zeit.f.phys C67,525(1995);
Ann. phys. 241, 68(1995)

UK, Helv. Phy acta(1998)
IJTP(2001); Can. J. Phys. 80(2002)

This theory corresponds to a regularization different from those involved in the class of models studied earlier

The 1st term corresponds to a massless boson which is equivalent to a massless fermion

The 2nd term is the usual term involving the nonlinear constraint $[(\sigma_k^2 - 1) \approx 0]$ and the auxiliary field

The 3rd term is the KE term of the EM vector gauge field A_μ

The 4th term represents the coupling of the sigma field to the EM field

The last term involving only the e.m. field A_μ may be regarded as a signature of the regularization.

This term for the e.m. field A_μ has been explicitly derived by SM+PM (z.f.Phys C67(1993)) using the Pauli-Villars method of regularization and they have obtained the effective action with the above unconventional mass term.

The mass term for A_μ arises from the regularization ambiquities associated with the definition of current and contains the fermionic one loop effects.

For the gauged O(N)-NLSM with the Standard regularization the mass term for A_μ is

$$\frac{1}{2} a e^2 A_\mu A^\mu$$

Regularization parameter

where a is the regularization parameter [PRL(1985)]

This is the so called standard regularization

The new regularization used by P. Mitra [PLB(1992)] has been called by him Faddeevian Regularization.

This theory is in accordance with the Faddeev's Picture of anomalous gauge theories
Faddeev PL B145,81(1984),
Faddeev+Shatashivily PL B167 .,225(1986)

We recall that in a true GI theory the matrix of the PB's of the constraints is a NULL matrix.

Faddeev visulaized a situation where anomalies make the PB of the Gauss law constraint with itself nonvanishing.

If this happens, the constraints become 2nd class and the GI is lost

Faddeev argued that there would be more physical d.o.f than in the case of GI theories because NO GFC's would be needed to quantize the theory.

In the O(N)-NLSM with the new (so called Faddeevian) regularization considered by Mitra & coworkers the Faddeev's mechanism works: namely , the constraints become 2nd class through an anomaly in the PB of the Gauss Law constraints with itself

The mass like term does not have the Lorentz-Invariance and therefore the theory lacks manifest Lorentz.

However Poincare generators of the theory defined on the constraint hypersurface are seen to satisfy the Poincare algebra.

GSM-FR
Inst Form

In view of this, the theory despite the lack of manifest Lorentz-covariace, is seen to be implicitly Lorentz-Invariance.

$$\text{For} \quad L^{N}_{gsm} \ , \ \partial_\mu j^\mu \neq 0$$

$$\Rightarrow \quad \text{NO VGS at the classical level}$$

Total Hamiltonian density

$$H_N^T = \frac{1}{2}(\pi_k^2 + E^2 + \partial_1\sigma_k^2 + e^2 A_0^2) + E\partial_1 A_0 + eA_0\pi_k - eA_1\partial_1\sigma_k$$

$$-\lambda(\sigma_k^2 - 1) - \frac{1}{2}e^2[(A_0 - A_1)^2 - 4A_1^2] + \pi_0 u + p_\lambda v$$

The theory is seen to possess a set of SIX 2nd Class constraints

$$\Omega_1 = \varphi_1 = \pi_0 \approx 0 \qquad \text{(PC)}$$

$$\Omega_2 = \psi_1 = p_\lambda \approx 0 \qquad \text{(PC)}$$

$$\Omega_3 = \{\varphi_1, H_T^N\} = (\partial_1 E - e\pi_k - e^2 A_1) \approx 0 \qquad \text{(SC)}$$

$$\Omega_4 = \{\psi_1, H_T^N\} = (\sigma_k^2 - 1) \approx 0 \qquad \text{(SC)}$$

$$\Omega_5 = \{\varphi_2, H_T^N\} = (-e^2 E + 2e^2 \partial_1 A_1 + 2e\lambda\sigma_k) \approx 0 \qquad \text{(SC)}$$

$$\Omega_6 = \{\psi_2, H_T^N\} = (2\sigma_k \pi_k + 2e\sigma_k A_0) \approx 0 \qquad \text{(SC)}$$

Here $\pi_0, p_\lambda, E(\equiv \pi^1)$ and π_{k} are the momenta can. conjugate resp. to A_0, λ, A_1 and σ_{k}

$M_{\alpha\beta}(z,z') := \{\Omega_\alpha, \Omega_\beta\}_p$ *is now nonsingular*

$\Rightarrow$ *That* Ω_i' *are 2nd−class and that the theory is GNI*

model-C
GISM-FR
Inst F-ST

Construction of GI theory (Model-I)

Enlarge the H.S. of GI th. by introducing a new field θ through the redefinition of fields σ_k ,λ and A^μ in L_{gsm}^N Through the Stueckelberg Transformations

$$\sigma_k \to \Sigma_k = \sigma_k - \theta \quad and \quad \lambda \to \Lambda = \lambda + \partial_0 \theta$$
$$A^\mu \to \mathcal{A}^\mu = A^\mu + \partial^\mu \theta$$

$$L_{gsm}^N \to L^I = L^N + L^S$$

$$L^S = \left(\frac{1+2e}{2}\right)\left\{(\partial_0\theta)^2 - (\partial_1\theta)^2\right\} - (1+e)\left(\partial_0\sigma_k\partial_0\theta - \partial_1\sigma_k\partial_1\theta\right) + \lambda(\theta^2 - 2\sigma_k\theta)$$
$$+ (\partial_0\theta)(\sigma_k^2 + \theta^2 - 2\sigma_k\theta - 1) + e[\mathrm{A}_0(\partial_0\theta) - \mathrm{A}_1(\partial_1\theta)] + \frac{1}{2}e^2\left(\partial_0\theta - \partial_1\theta\right)^2$$
$$+ e^2(\mathrm{A}_0 - \mathrm{A}_1)(\partial_0\theta - \partial_1\theta) - 2e^2(\partial_1\theta)^2 - 4e^2\mathrm{A}_1(\partial_1\theta)$$

L^I describes a GI theory

Check for the GI of the new theory

The ELE's for $L^I \Rightarrow \partial_\mu j^\mu = \partial_0 j^0 + \partial_1 j^1 = 0$

model-C
GISM-FR
Inst Form

$\Rightarrow$ that L^I possesseses (at the classical level) VGS,
(s.t. $\exists$ NO VGA)

L^I is seen to possess a set of 5 constraint s, three PC's and two SC's.

$$\eta_1 = \varphi_1 = \pi_0 \approx 0 \qquad \text{(PC)}$$

$$\eta_2 = \psi_1 = p_\lambda \approx 0 \qquad \text{(PC)}$$

$$\eta_3 = \chi_1 = [\pi_\theta + (1+e)\pi_k - \sigma_k^2 - \theta^2 + 2\sigma_k\theta + 1 + e^2(\partial_1\theta + A_1) \approx 0 \qquad \text{(PC)}$$

$$\eta_4 = \{\varphi_1, H_T^N\} = [\partial_1 E - e\pi_k - e^2(A_1 + \partial_1\theta)] \approx 0 \qquad \text{(SC)}$$

$$\eta_5 = \{\psi_1, H_T^N\} = [(\sigma_k^2 - 1) + (\theta^2 - 2\sigma_k\theta)] \approx 0 \qquad \text{(SC)}$$

The matrix of PB's of the constraint s η_i is seen to be singular
$\Rightarrow$ that the constraint s η_i form a set of Ist class constraint s
and that the theory L^I is GI

model-C
GISM-FR
Inst Form

Generator of LVGT

(Charge operator J^0)

$$J^0 = \int j^0 dx$$

$$\begin{aligned} j^0 = {} & [\beta[\partial_0\sigma_k - eA_0 - (1+e)\partial_0\theta] - \partial_1\beta(\partial_0 A_1 - \partial_1 A_0) \\ & + \beta(1+2e)\partial_0\theta - (1+e)\partial_0\sigma_k + (\sigma_k - \theta)^2 - 1 + eA_0 \\ & + e^2(A_0 - A_1 + \partial_0\theta - \partial_1\theta)] \end{aligned}$$

(Current operator J^1)

$$J^1 = \int j^1 dx$$

$$\begin{aligned} j^1 = {} & \beta[-\partial_1\sigma_k + eA_1 - (1+e)\partial_1\theta] + \partial_0\beta(\partial_0 A_1 - \partial_1 A_0) \\ & + \beta[-(1+2e)\partial_1\theta + (1+e)\partial_1\sigma_k - eA_1 - e^2(\partial_0\theta - \partial_1\theta) \\ & - e^2(A_0 - A_1) - 4e^2(A_1 + \partial_1\theta) \end{aligned}$$

model-C
GISM-FR
Inst Form

L^I is invariant under the LVGT's

$$\delta\sigma_k = \beta(x,t); \qquad \delta\lambda = -\partial_0\beta(x,t); \qquad \delta\theta = \beta(x,t);$$

$$\delta A_0 = -\partial_0\beta(x,t) \qquad \delta A_1 = -\partial_1\beta(x,t)$$

The GI theory L^I could now be quantized by fixing the gauge

It is possible also to recover the physical contents of the GNI theory L^N_{gsm} from the GI theory L^I under some special gauge choices and accordingly we choose the GFC's :

$$\rho_1 = \theta \approx 0$$

$$\rho_2 = [2\sigma_k\pi_k + 2eA_0\sigma_k - \pi_\theta - (1+e)\pi_k - e^2 A_1] \approx 0$$

$$\rho_3 = [-e^2 E + 2e^2\partial_1 A_1 + 2e\lambda\sigma_k] \approx 0$$

The addition of $L{\uparrow}S$ to $L{\uparrow}N$ enlarges only the unphysical part of the full Hilbert Space of the theory $L{\uparrow}N$, without modifying the physical contents of the GNI theory $L{\uparrow}N$

Construction of GI theory (Model-II)

model-D
GISM-FR
Inst Form

We modify Hamiltonian Density

$$H_T^I = H_T^N - [(\partial_1 E - e\pi_k - e^2 A_1)A_0 - \frac{1}{2}\pi_k^2]$$

$$= \frac{1}{2}[\pi_k^2 + E^2 + (\partial_1\sigma_k)^2 + e^2 A_0{}^2] + E\partial_1 A_0 + eA_0\pi_k - eA_1(\partial_1\sigma_k)$$

$$- \lambda(\sigma_k^2 - 1) - \frac{1}{2}e^2[(A_0 - A_1)^2 - 4A_1{}^2] + \pi_0 u + p_\lambda v$$

$$- [(\partial_1 E - e\pi_k - e^2 A_1)A_0 - \frac{1}{2}\pi_k^2]$$

and the associated Lag density with the help of HE's

$$\boxed{L_T^I = \frac{1}{2}[(\partial_0 A_1)^2 + (\partial_1\sigma_k)^2] + eA_1(\partial_1\sigma_k) + \lambda(\sigma_k^2 - 1) - \frac{3}{2}e^2 A_1{}^2}$$

model-D
GISM-FR
Inst Form

The new GI model is seen to possses Three Ist Class Constraints

$$\Omega_1 = \varphi_1 = \pi_0 \approx 0 \qquad \text{(PC)}$$

$$\Omega_2 = \psi_1 = p_\lambda \approx 0 \qquad \text{(PC)}$$

$$\Omega_3 = \{\psi_1, H_T^I\} = (\sigma_k^2 - 1) \approx 0 \quad \text{(SC)}$$

$$and \qquad \mathrm{M}_{\alpha\beta} = O_{3\mathrm{x}3} \quad (null\ matrix)$$

$$\Rightarrow \qquad \text{the theory is a Pure GI theory}$$

Now this theory can be quantized as before

FFSM (FR)-GI

The gauged NLSM with FR on the LF is defined by the action

$$S = \int L\, dx^{+} dx^{-}$$

$$L = \partial_{+}\sigma_k \partial_{-}\sigma_k + \lambda(\sigma_k^2 - 1) + \frac{1}{2}(\partial_{+}A^{+} - \partial_{-}A^{-})^2$$

$$- e[A^{-}(\partial_{-}\sigma_k) + A^{+}(\partial_{+}\sigma_k)] + [2\mathrm{e}^2 A^{-}A^{+} - \mathrm{e}^2 (A^{-})^2]$$

The theory is seen to possess a set of three PC's constraints and two secondary Gauss-Law constraints:

$$\Omega_1 = \varphi_1 = \Pi^{+} \approx 0$$

$$\Omega_2 = \psi_1 = \Pi_\lambda \approx 0$$

$$\Omega_3 = \chi_1 = (\Pi_k - \partial_{-}\sigma_k + eA^{+}) \approx 0$$

$$\Omega_4 = \{\varphi_1, H_T^N\} = [\partial_{-}\Pi^{-} + e(\partial_{-}\sigma_k) + 2e^2(A^{+} - A^{-})] \approx 0$$

$$\Omega_5 = \{\psi_1, H_T^N\} = (\sigma_k^2 - 1) \approx 0$$

Here $\Pi^{+}, \Pi_\lambda, \Pi^{-}$ and Π_k are the LC can momenta conjugate resp. to A^{-}, λ, A^{+} and σ_k

FFSM (FR)GI

$$S_{\alpha\beta}(w^-, z^-) := \{\Omega_\alpha(w^-), \Omega_\beta(z^-)\}_p \quad \text{is singular}$$

The above constraints forms a set of FCC's and the theory is a GI th. and is invaiant under the LVGT

$$\delta\sigma_k = e\beta(x,t); \quad \delta A^+ = \partial_-\beta(x,t); \quad \delta A^- = \partial_+\beta(x,t);$$

$$\delta u = \partial_+\partial_+\beta(x,t); \quad \delta w = e\partial_+\beta(x,t); \quad \delta\lambda = \delta v = 0;$$

$$\delta\Pi_k = \delta\Pi_\lambda = \delta\Pi^+ = \delta\Pi^- = \delta\Pi_u = \delta\Pi_v = \delta\Pi_w = 0$$

Total Hamiltonian density

$$H_T^I = \frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial_- A^-) - eA^-(\partial_-\sigma_k) - \lambda(\sigma_k^2 - 1)$$

$$- 2e^2 A^- A^+ + e^2 (A^-)^2 + \Pi^+ u + \Pi_\lambda v + (\Pi_k - \partial_-\sigma_k + eA^-)w$$

where $u(x,t), v(x,t)$ *and* $w(x,t)$ *are Lagrange multiplier fields and*

$$H_T = \int H_T dx^-$$

The vector gauge current of the theory $J^{\mu} \equiv (J^{+}, J^{-})$ *is*

$$J^{+} = \int j^{+} dx^{-} = \int dx^{-} [(\partial_{-}\sigma_{k} - eA^{+}) e\beta + \partial_{-}\beta(\partial_{+}A^{+} - \partial_{-}A^{-})]$$

$$J^{-} = \int j^{-} dx^{-} = \int dx^{-} [(\partial_{+}\sigma_{k} - eA^{-}) e\beta + \partial_{+}\beta(\partial_{+}A^{+} - \partial_{-}A^{-})]$$

It implies that the theory possesses a local vector gauge symmetry.

The LF theory could now be quantized under the appropiate LC gauges, accordingly we choose GFC's

$$\varsigma = \lambda \approx 0$$

The divergence of the vector current density

$$\partial_{\nu} j^{\nu} = \partial_{+} j_{-} + \partial_{-} j_{+} = 2e\lambda\sigma_{k}\beta = 0$$

under the gauge $\lambda = 0$

FFSM
GI

The nonvanishing ELCT-CR's of the theory could now be obtained following DQR

$$[\sigma_k(x), \Pi_k(y)] = i[\delta(x-y) - \frac{\sigma_k}{e}\varepsilon(x-y)]$$

$$[A^-(x), \Pi_k(y)] = \frac{i}{2e}\partial_-\delta(x-y)$$

$$[A^-(x), \Pi^-(y)] = (i)\delta(x-y)$$

$$[A^+(x), \Pi^-(y)] = (i)\delta(x-y)$$

$$[A^-(x), A^-(y)] = (i)\partial_-\delta(x-y)$$

$$[A^-(x), A^+(y)] = (-i)\delta(x-y)$$

$$[\Pi_k(x), \Pi_k(y)] = \frac{4\sigma_k^2}{e^2}\delta(x-y)$$

In the path integral formulation , the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z[J_k]$ of the theory in the presence of the external sources : J_k e.g., as follows

$$Z[J_k] = \int [d\mu] \ \exp[i\int dx \ [J_k \Phi^k + \Pi_k(\partial_+ \sigma_k) + \Pi_\lambda(\partial_+ \lambda) + \Pi^+(\partial_+ A^-) + \Pi^-(\partial_+ A^+) + \Pi_u(\partial_+ u) + \pi_v(\partial_+ v) + \pi_w(\partial_+ w) - H_T^I \ ]$$

where the Phase space variables of the theory are:

$\Phi_{\mathrm{k}} \equiv (\sigma_k, \lambda, A^+, A^-, u, v, w)$ with the corresponding respective canonical conjugate momenta : $\Pi_{\mathrm{k}} \equiv (\Pi_k, \Pi_\lambda, \Pi^-, \Pi^+, \Pi_u, \Pi_v, \Pi_w)$.

The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under this gauge-fixing is obtained as:

$$
\begin{aligned}
[d\mu] = {} & [[(-4)e^6(\partial_\_\delta(x^- - y^-))^2(\delta(x-y))^4]^{\frac{1}{2}}[d\sigma_k][d\lambda][dA^-][dA^+] \\
& [du][dv][dw][d\Pi_k][d\Pi_\lambda][d\Pi^+][d\Pi^-][d\Pi_u][d\Pi_v][d\Pi_w] \\
& [\delta(\Pi^+ \approx 0)][\delta(\Pi_\lambda \approx 0)][\delta(\Pi_k - \partial_\_\sigma_k + eA^+) \approx 0)] \\
& [\delta(\partial_\_\Pi^- + e(\partial_\_\sigma_k) + 2e^2(A^+ - A^-)) \approx 0)][\delta(\sigma_k^2 - 1) \approx 0)] \\
& + [\delta(\lambda \approx 0)]
\end{aligned}
$$

Conclusions

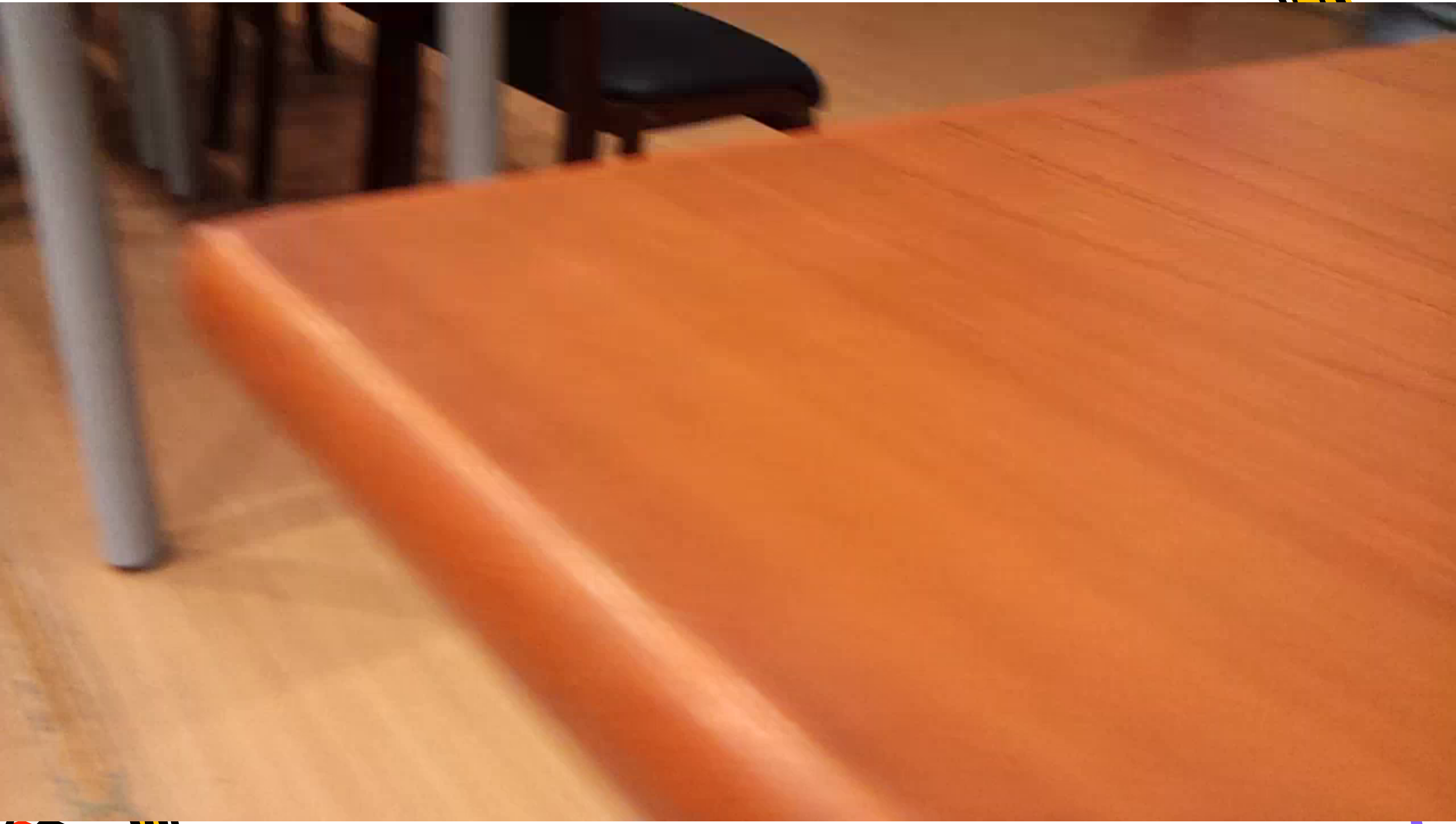
- Considered the O(N)-NLSM
  ----IF-GNI, where it was possible to construct 2-GI models
  for this theory and
  the LF model is already GI.

- Presented a Gauged O(N)-NLSM with SR
  ----and discussed its Quantization in IF and in FF

- Presented a Gauged O(N)-NLSM with FR
  ----and discussed Hamiltonian quantization
  of the model on the LF.
  --- Constructed two GI versions of the O(N)-NLSM
  and discussed their quantization.
  --- possible to recover physical contents of GNI Theory

A Vary Vary Happy Birthday James

THANK YOU

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