Hot nuclear matter in intense magnetic field

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Heavy-ion Collisions in a nutshell

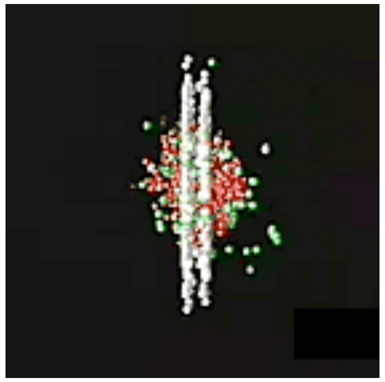
RELATIVISTIC HEAVY ION COLLISIONS

Relativistic Heavy Ion Collider at BNL; Au on Au at 200 GeV

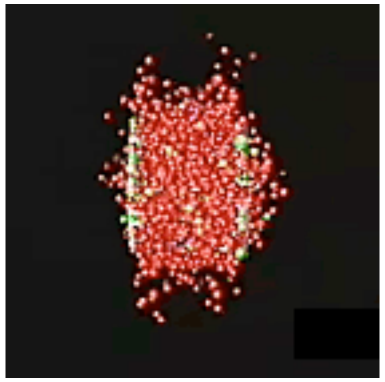
Large Hadron Collider at CERN; Pb on Pb at 5 TeV.

HEAVY ION COLLISIONS

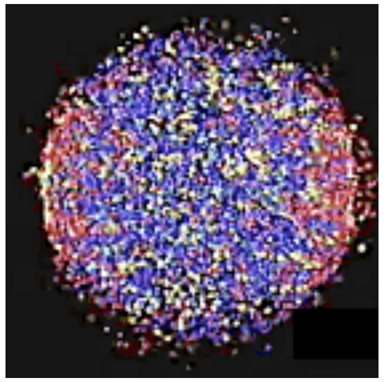
Ions about to collide



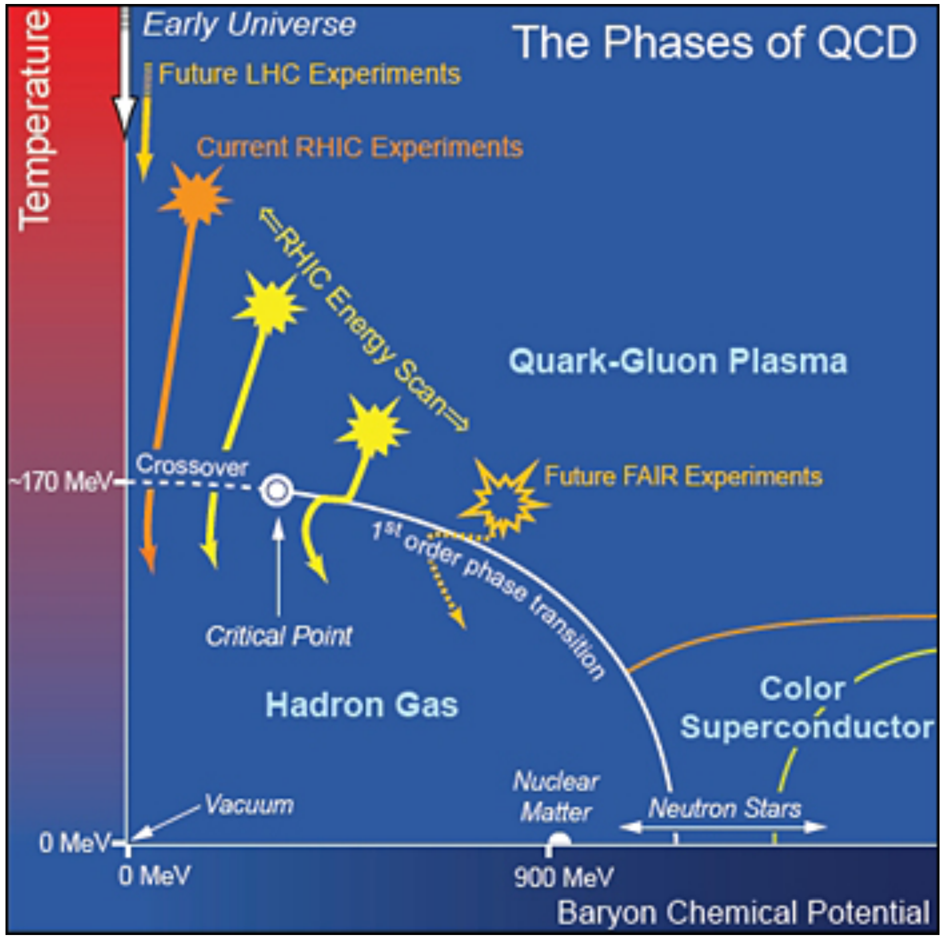
Ion collision



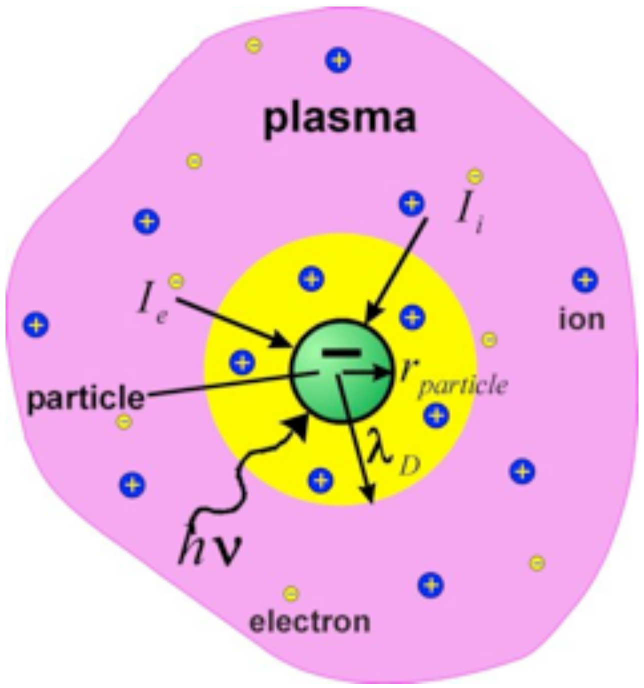
Quarks, gluons freed



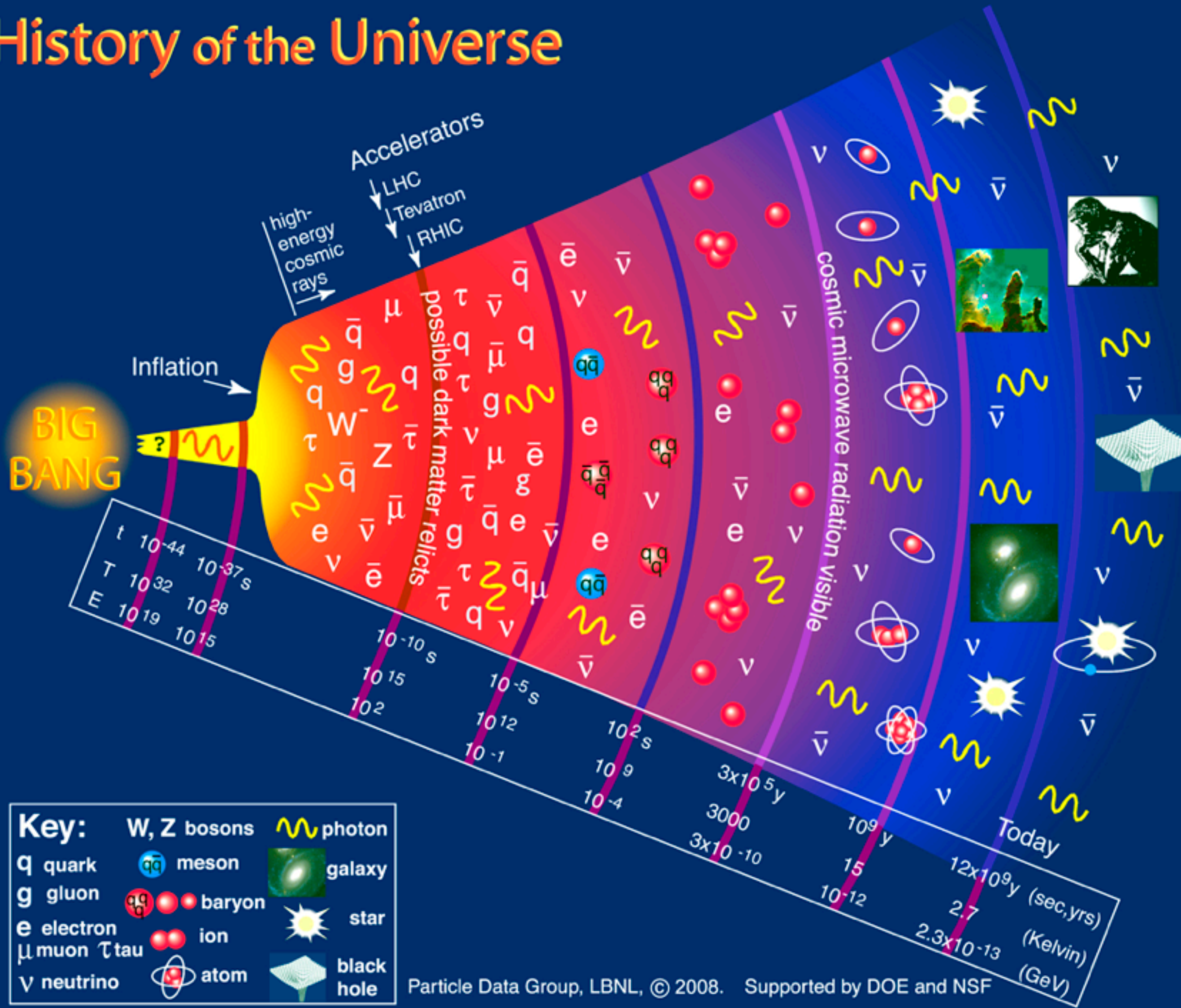
Plasma created



Debye Screening

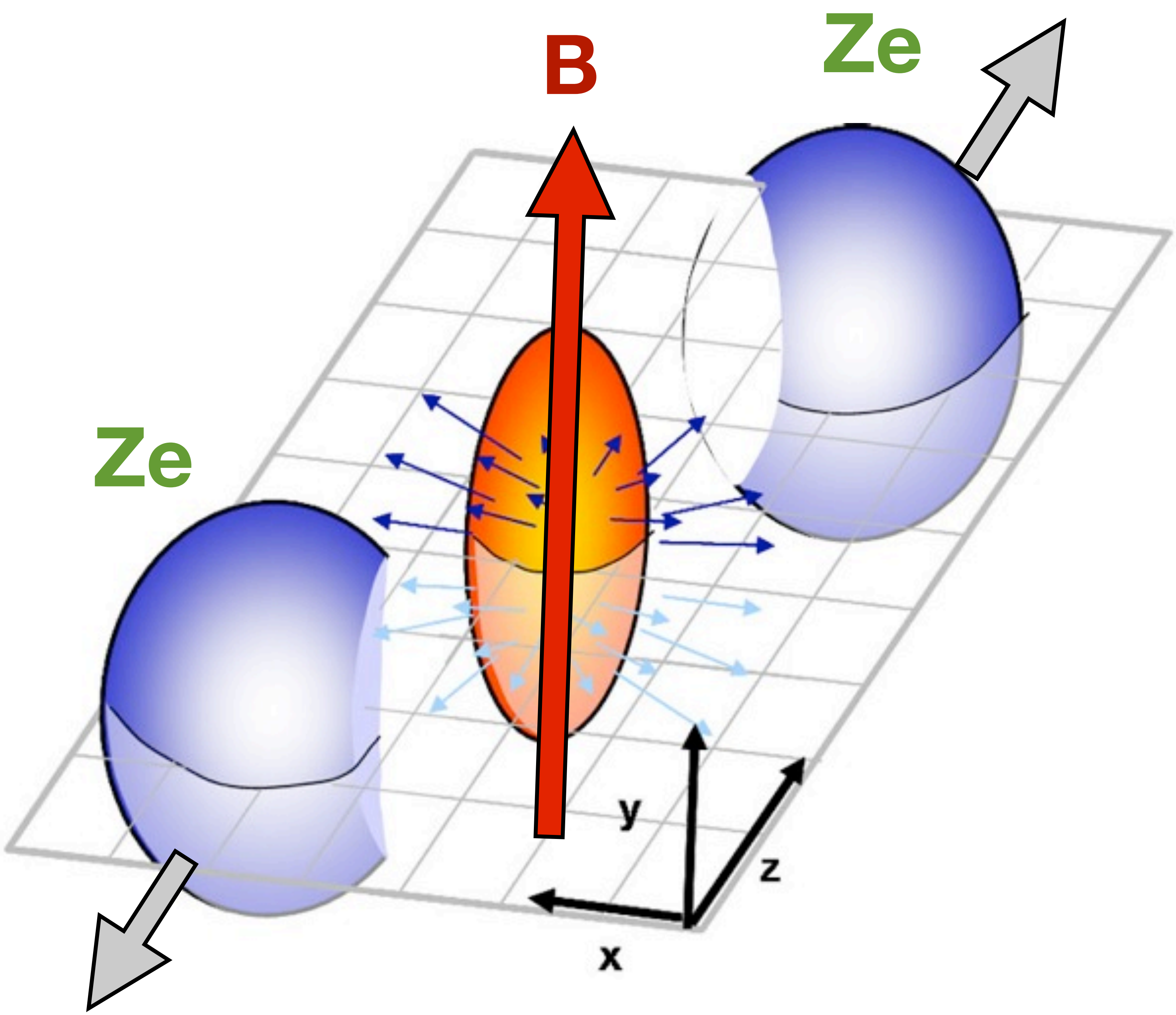


History of the Universe

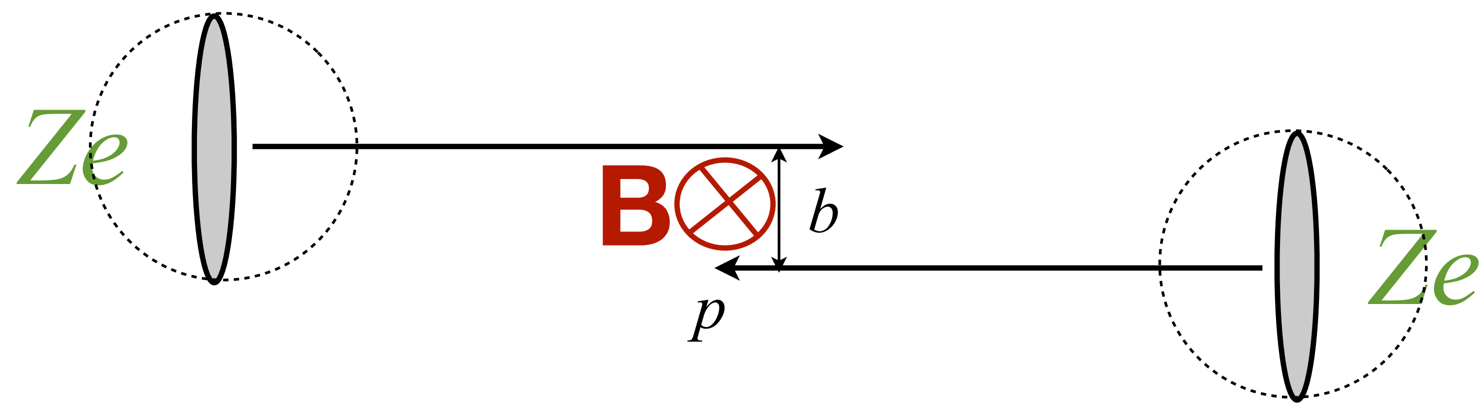


Origin and properties of magnetic field

ORIGIN



ORIGIN



To estimate the ELECTROMAGNETIC FILEDS created in heavy ion collisions take Lienard-Wiechert potentials, integrate over the charge distribution in nuclei.

Order of magnitude estimate: $B \sim Ze \dfrac{b}{R^3} \gamma$

$Z_{Au} = 79$, $b \sim R = 7\ fm$, $\gamma = 100 \Rightarrow$ $eB = (200\ MeV)^2 \approx m_\pi^2$ $\quad B \sim 10^{18}\ G$

EXAMPLES OF MAGNETIC FIELD STRENGTHS

10^{-4} G: Earth's magnetic field

50 G: Refrigerator magnet

10^{5} G: Modern MRI system

10^{6} G: Strongest pulse obtained in lab

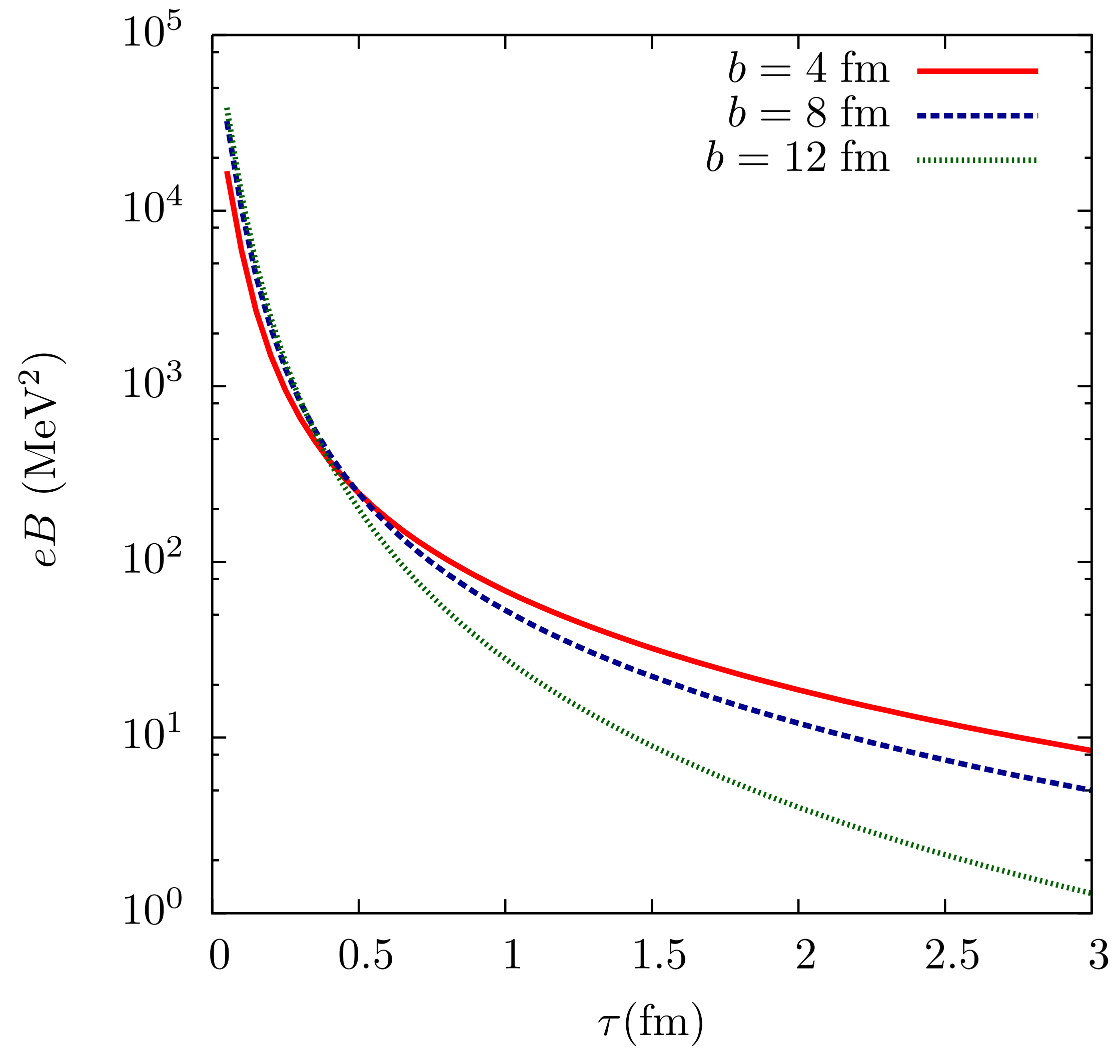
10^{10}-10^{13} G: Neutron stars

10^{13} G: Breakdown of superposition principle of electrodynamics

10^{12}-10^{15} G: Magnetars

10^{16}-10^{18} G: Heavy ion collisions

NUMERICAL SIMULATIONS

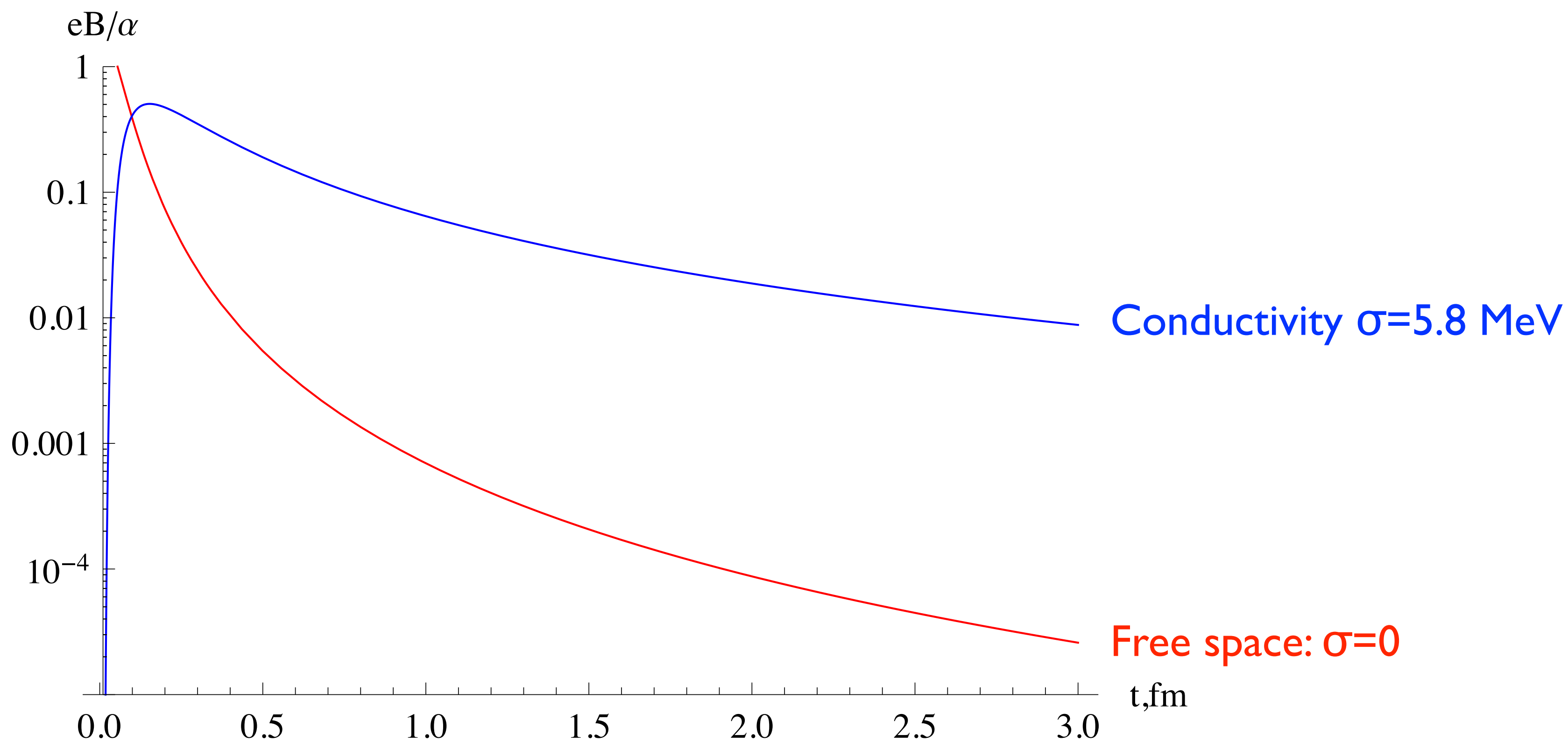


Kharzeev, McLerran, Warringa (2007)

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

ROLE OF CONDUCTIVITY

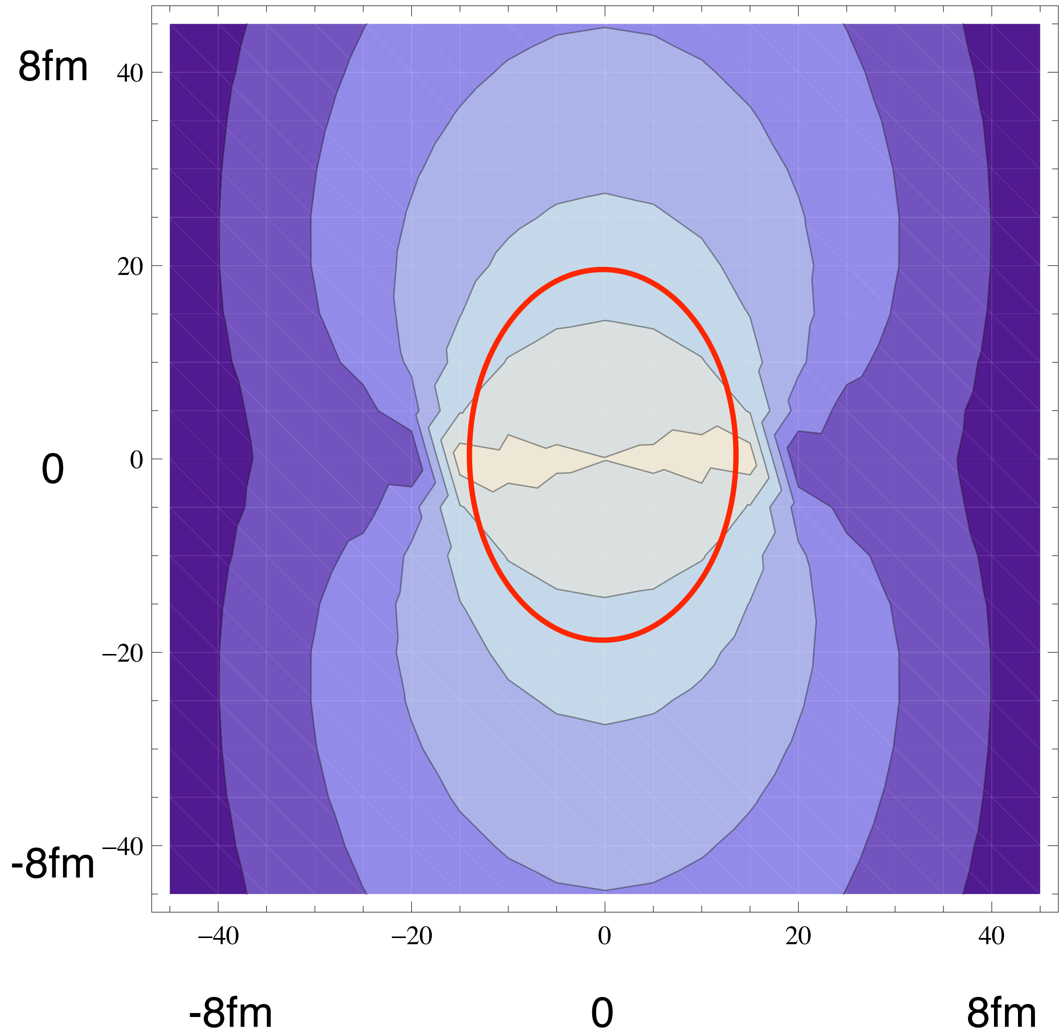
Due to finite electric conductivity of quark-gluon plasma magnetic field relaxes over time comparable to the plasma lifetime.



KT (2011)

SPATIAL DISTRIBUTION

Rather modest spatial variation ~20% in the QGP region

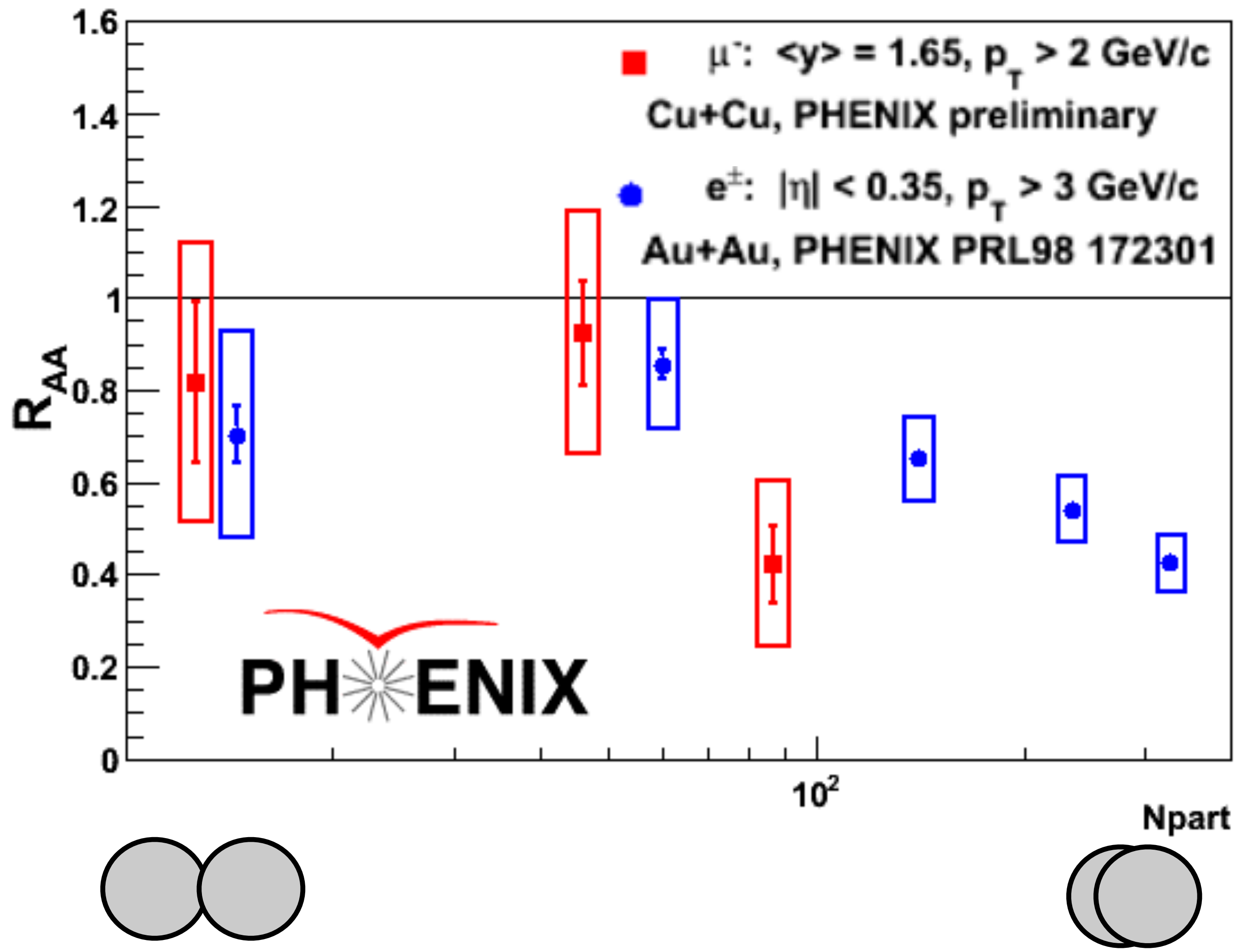


Magnetic field is constant in the 0th approximation

Quarkonium in magnetic field

WHY QUARKONIUM?

Quarkonium melts in plasma. Expect its suppression in heavy-ion collisions vs pp ⇒ "smoking gun" of plasma.



However, there are other mechanisms to destroy quarkonium.

QUARKONIUM IN MAGNETIC FIELD

Zeeman effect. Quarkonium state of total angular momentum J splits (in weak field) into states of different mass: $\Delta M = (eB/2m) g J_z$, where $J_z = -J, -J+1, \ldots, J$. For example J/ψ ($S=1, L=0, J=1$) $J_z = 0, \pm 1 \Rightarrow \Delta M = 0.15$ GeV (at LHC)

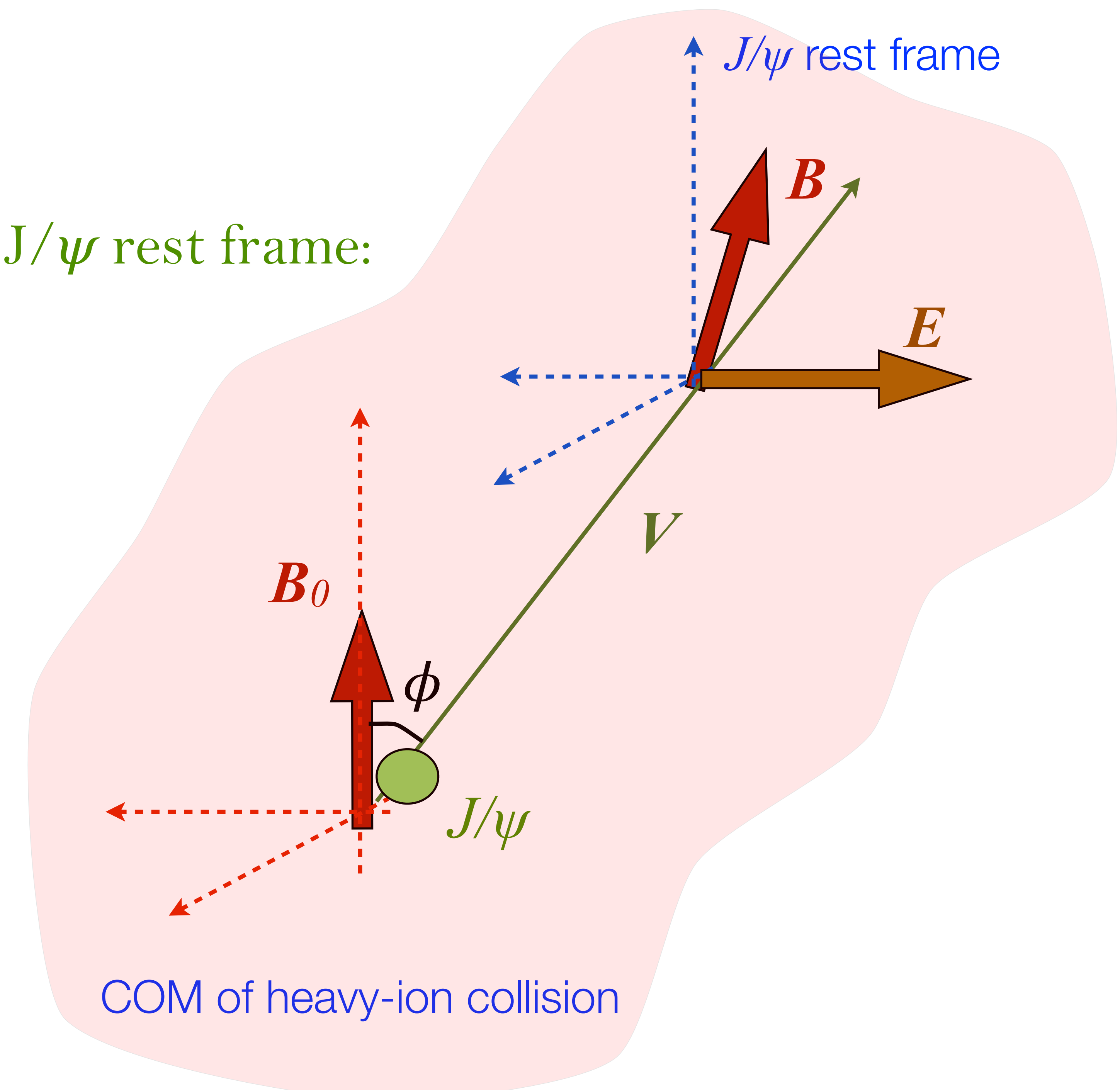
Distortion of the quarkonium potential due to high order effects. This is important $B \sim 3\pi m^2/e$ which is $3\pi/\alpha$ stronger than the Schwinger's field. Machet, Vysotsky (2010)

Lorentz ionization: in the quarkonium rest frame there are perpendicular electric and magnetic fields. Electric field renders quarkonium unstable with respect to decay into q and anti-q.

J/Ψ IN MAGNETIC FIELD

Consider J/ψ moving with velocity V in COM frame in field B.

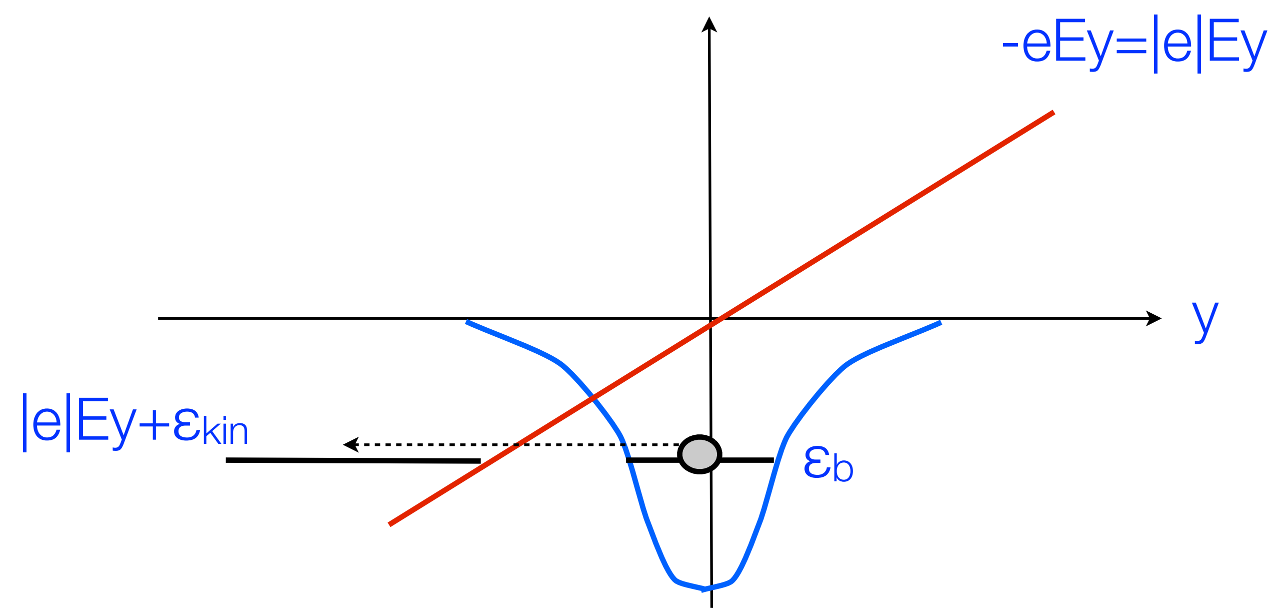
Boost to J/ψ rest frame:



J/Ψ IONIZATION IN ELECTRIC FIELD

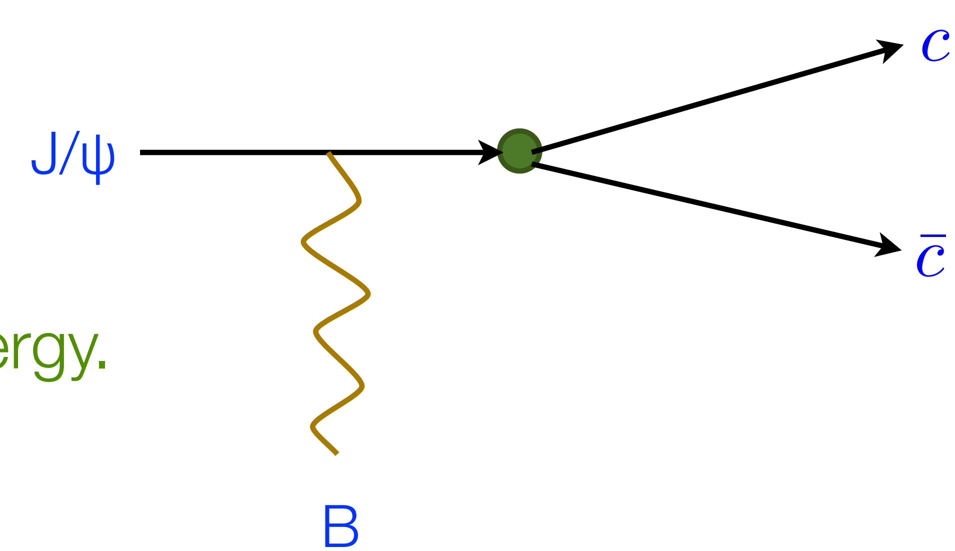
Quarkonium rest frame

There is finite quantum probability for the anti-quark ($e<0$) to tunnel through the potential barrier and go to $y \to -\infty$.



This is $J/\psi + E \to D^+D^-$ decay.

Lab frame

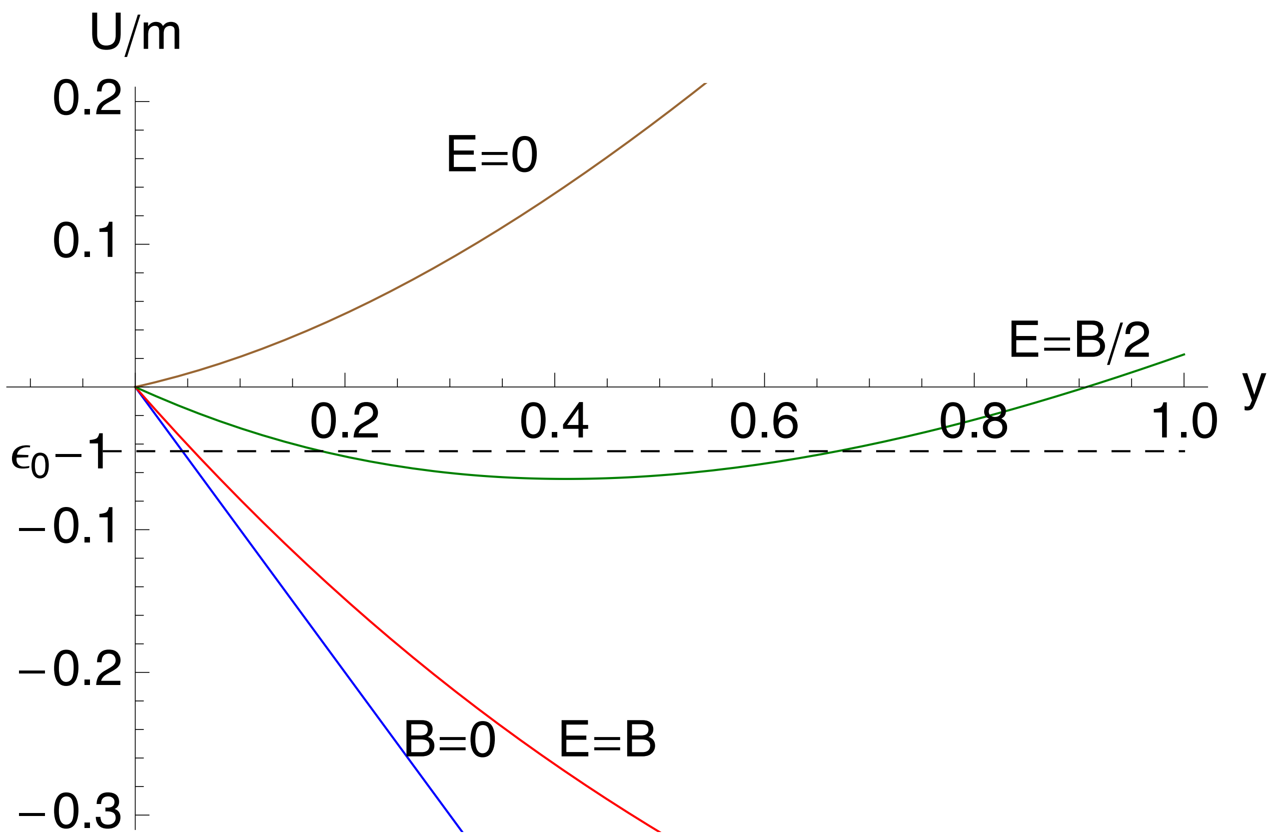


Magnetic field supplies momentum,
while moving quarkonium supplies energy.

J/Ψ IONIZATION: ROLE OF B

In the rest frame decay happens only due to electric field (magnetic field does no work). What is the role of magnetic field?

$$U(y) = \sqrt{m^2 + (p_x + eBy)^2 + p_y^2 + p_z^2)} - eEy - \sqrt{m^2 + p_x^2 + p_y^2}$$



Dissociation rate can be calculated in the WKB approximation as a tunneling rate of quark thru the potential barrier.

WKB APPROXIMATION

- Ionization probability = Transmission coefficient $w = e^{-2\int_0^{y_1}\sqrt{-p_y^2}dy} \equiv e^{-f}$

  where y_1 is the turning point.

- Extremum of f is $f_m = \dfrac{m^2\tau_0\rho}{eE\sqrt{1-\rho^2}}[1-\epsilon_0(\epsilon_0 - q_m\rho)]$

- Non-relativistic approximation: $p \ll m$, $\varepsilon_b/m \ll 1$, $\rho = E/B \ll 1$

$$f_m = \frac{2m^2(2\epsilon_b)^{3/2}}{3eE}g(\gamma) \qquad g(\gamma) = \frac{3\tau_0}{2\gamma}\left[1 - \frac{1}{\gamma}\left(\frac{\tau_0^2}{\gamma^2} - 1\right)^{1/2}\right]$$

Keldysh (1965)

where $\gamma = \dfrac{\sqrt{2\epsilon_b}}{\rho}$ is the adiabaticity parameter

Weak binding: $\varepsilon_b \ll \rho^2 \Rightarrow \gamma \ll 1$ $\qquad w = \exp\left\{-\dfrac{2}{3}\dfrac{(2\varepsilon_b m)^{3/2}}{meE}\right\}$

DISSOCIATION RATE

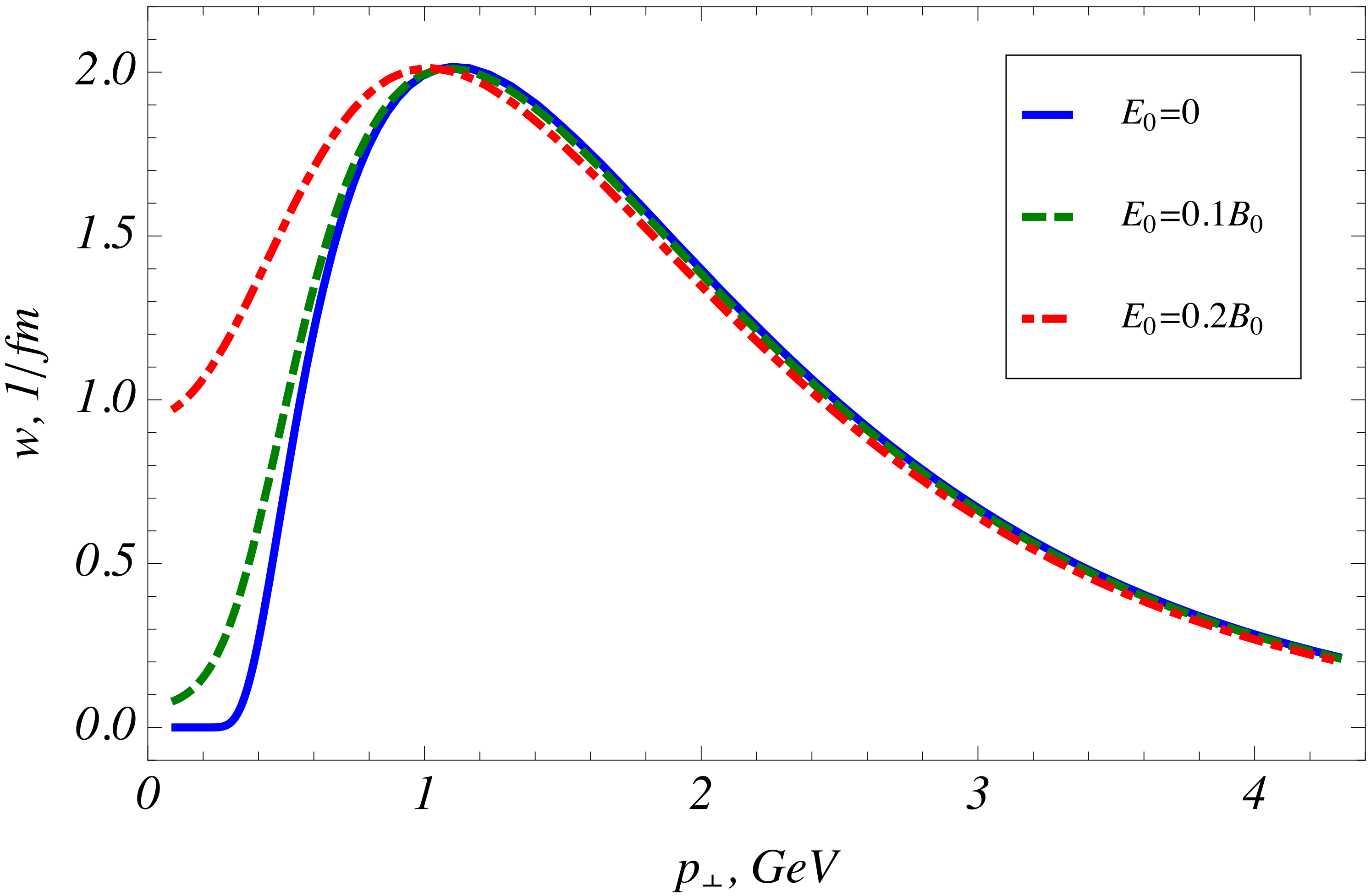
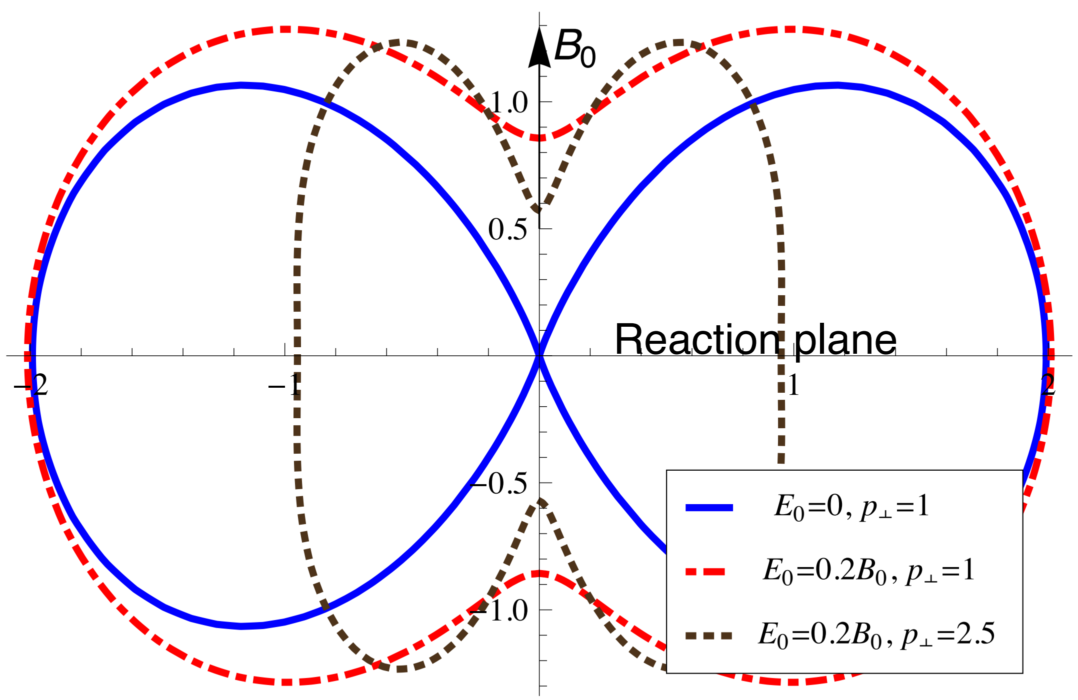


FIG. 1: Dissociation rate of J/ψ at $eB_0 = 15m_\pi^2$, $\phi = \pi/2$ (in the reaction plane), $\eta = 0$ (midrapidity) as a function of (a) $P_\perp$ at $\varepsilon_b = 0.16$ GeV and (b) ε_b at $P_\perp = 1$ GeV.

AZIMUTHAL ASYMMETRY

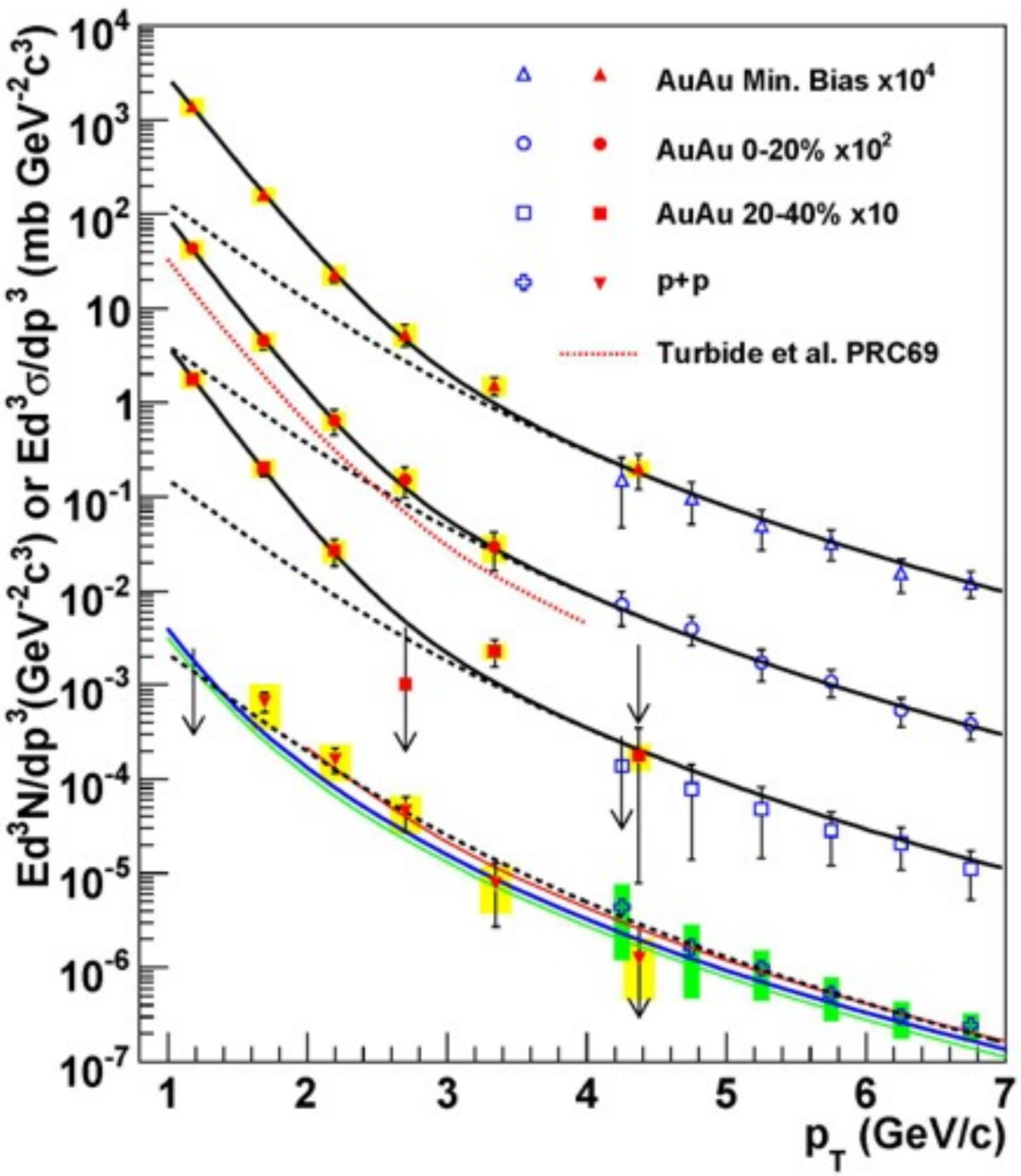
Spectrum of quarkonia surviving in EM field is proportional to survival probability $P=1-wt$

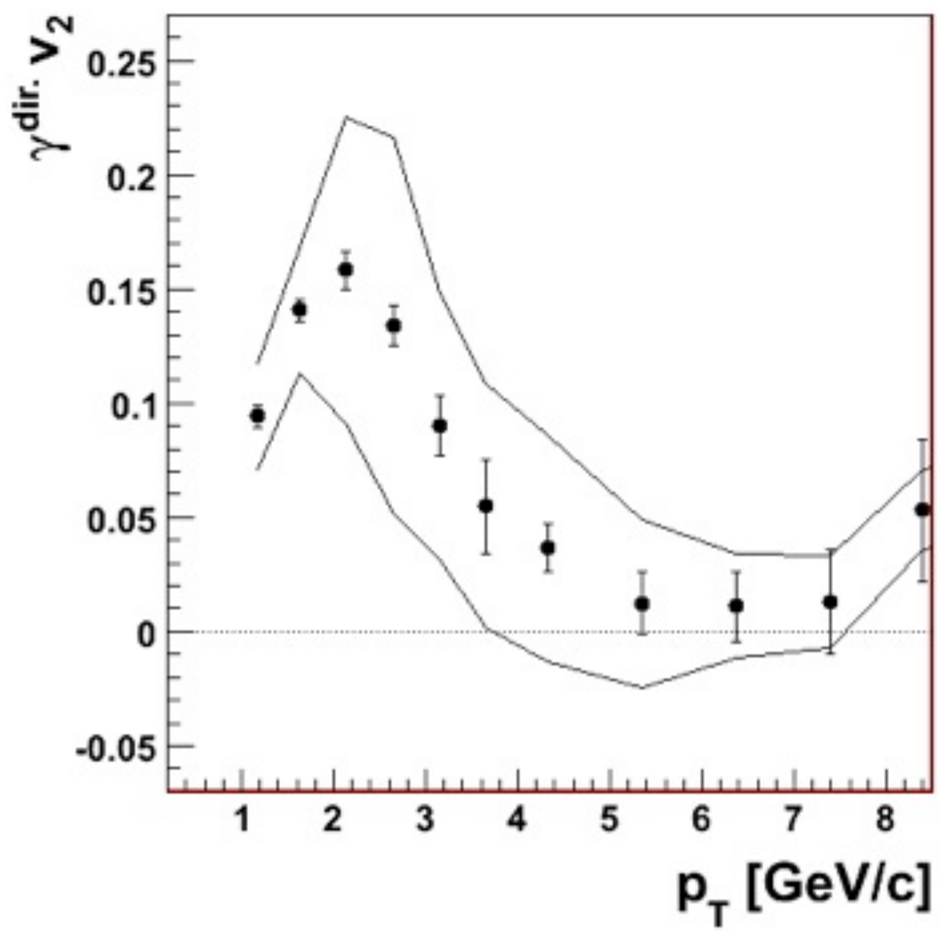


Magnetic field destroys J/ψ's. This effect grows with p_T and strongly depends on azimuthal angle.

Synchrotron radiation

UNEXPLAINED EXCESS OF PHOTONS IN HEAVY ION COLLISIONS



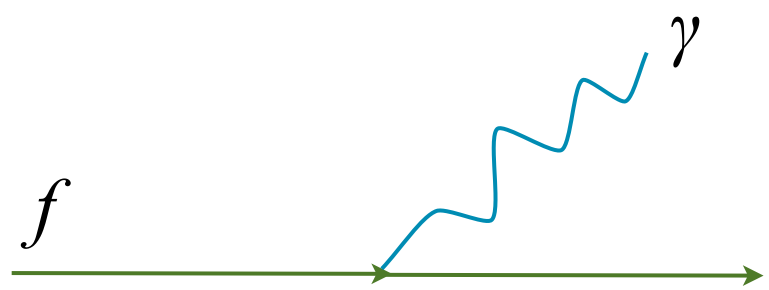


Azimuthal asymmetry of photons.

$$\frac{dN}{d\phi} = N_0[1 + 2v_2 \cos(2\phi) + 2v_4 \cos(4\phi) + \ldots]$$

ANGULAR DISTRIBUTION OF RADIATION

Synchrotron radiation:



$$f(e_f, j, p) \to f(e_f, k, q) + \gamma(\boldsymbol{k})$$

QGP is transparent to the emitted electromagnetic radiation because its absorption coefficient is suppressed by α^2.

Spacing between the Landau levels ~ $\boldsymbol{eB/\varepsilon}$, while their thermal width ~ $\boldsymbol{T}$. When $\boldsymbol{eB/\varepsilon \gtrsim T}$ it is essential to account for quantization of fermion spectra.

KINEMATICS

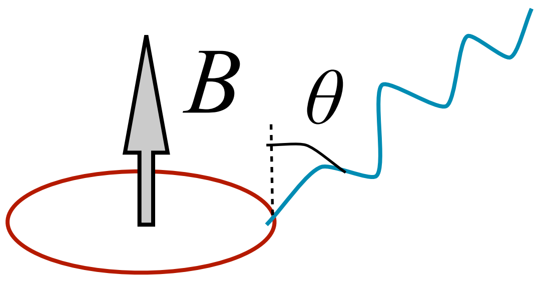
$$\varepsilon_j = \sqrt{m^2 + p^2 + 2je_fB}\,, \quad \varepsilon_k = \sqrt{m^2 + q^2 + 2ke_fB}$$

j (k) is the quantum number of Landau orbit of *initial (final)* charged fermion.

p (q) is the projection of *initial (final)* fermion momentum on the direction of *B*

Magnetic field does no work, thus energy is conserved. Magnetic Lorentz force has no component along the *B*-direction:

$$\varepsilon_j = \omega + \varepsilon_k\,, \quad p = q + \omega\cos\theta$$



Angular distribution of the power spectrum:

$$\frac{dI^j}{d\omega d\Omega} = \sum_f \frac{z_f^2\alpha}{\pi}\omega^2 \sum_{k=0}^{j} \Gamma_{jk} \left\{|\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2\right\} \delta(\omega - \varepsilon_j + \varepsilon_k)$$

Matrix elements are well-known functions of Laguerre polynomials.

Sokolov, Ternov (1968) and others

PHOTON NUMBER SPECTRUM

We are interested in the photon number spectrum radiated from QGP

$$\frac{dN^{\mathrm{synch}}}{dt d\Omega d\omega} = \sum_f \int_{-\infty}^{\infty} dp \frac{e_f B (2N_c) V}{2\pi^2} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \frac{dI^j}{\omega d\omega d\Omega} (2-\delta_{j,0}) f(\varepsilon_j)[1-f(\varepsilon_k)]$$

$$f(\varepsilon) = \frac{1}{e^{\varepsilon/T}+1}$$

Energy conservation $\quad \delta(\omega - \varepsilon_j + \varepsilon_k) = \sum_{\pm} \frac{\delta(p-p_\pm^*)}{\left|\frac{p}{\varepsilon_j} - \frac{q}{\varepsilon_k}\right|}$

$$\Rightarrow \quad p_\pm^* = \Big\{ \cos\theta (m_j^2 - m_k^2 + \omega^2 \sin^2\theta) \\ \pm \sqrt{[(m_j+m_k)^2 - \omega^2\sin^2\theta][(m_j-m_k)^2 - \omega^2\sin^2\theta]} \Big\} / (2\omega\sin^2\theta)$$

$$m_j^2 = m^2 + 2je_f B\,, \quad m_k^2 = m^2 + 2ke_f B$$

$p_\pm$ is real in two cases:

$$\text{(i)}\ m_j - m_k \geq \omega \sin\theta\,, \quad \text{or} \quad \text{(ii)}\ m_j + m_k \leq \omega \sin\theta$$

synchrotron radiation

one-photon pair annihilation

In case (i) the $j \to k$ transition must satisfy

$$\omega \leq \omega_{s,jk} \equiv \frac{m_j - m_k}{\sin\theta} = \frac{\sqrt{m^2 + 2je_f B} - \sqrt{m^2 + 2ke_f B}}{\sin\theta}$$

in particular $j=k$ transition is forbidden.

Spectral distribution of the synchrotron radiation rate per unit volume:

$$\frac{dN^{\text{synch}}}{V\,dt\,d\Omega\,d\omega} = \sum_f \frac{2N_c z_f^2 \alpha}{\pi^3} e_f B \sum_{j=0}^{\infty} \sum_{k=0}^{j} \omega (1 + \delta_{k0})\, \vartheta(\omega_{s,ij} - \omega) \int dp \sum_{\pm} \frac{\delta(p - p_\pm^*)}{\left|\frac{p}{\varepsilon_j} - \frac{q}{\varepsilon_k}\right|}$$

$$\times \left\{|\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2\right\} f(\varepsilon_j)[1 - f(\varepsilon_k)]\,,$$

SYNCHROTRON SPECTRUM

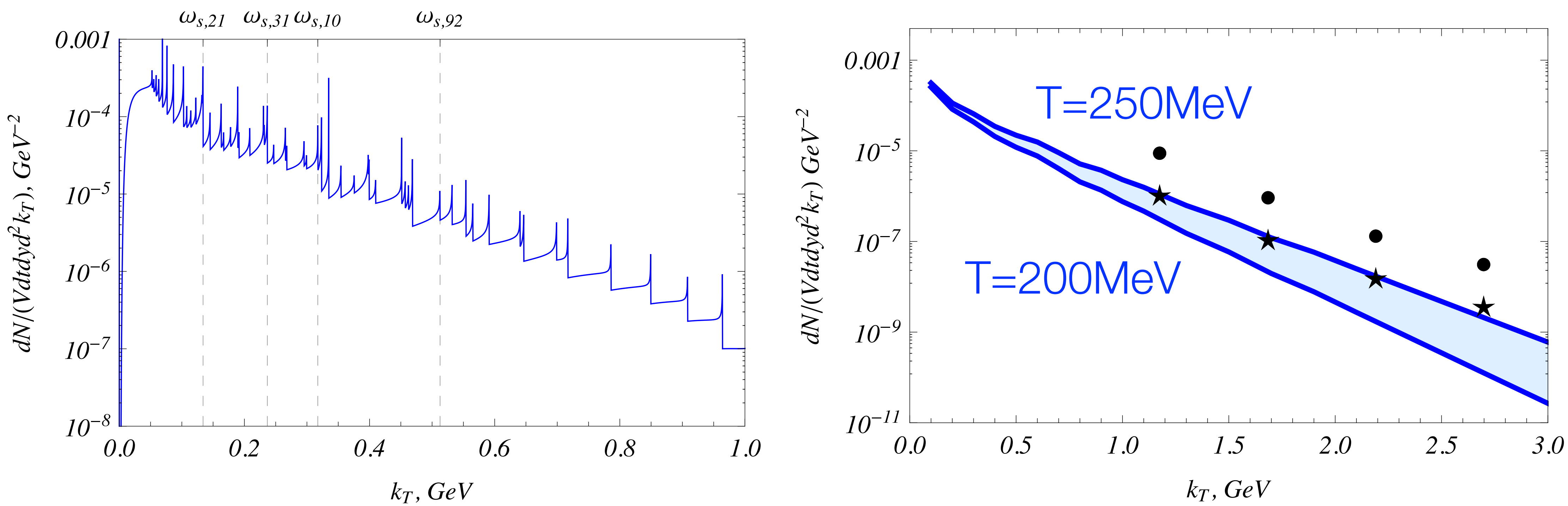
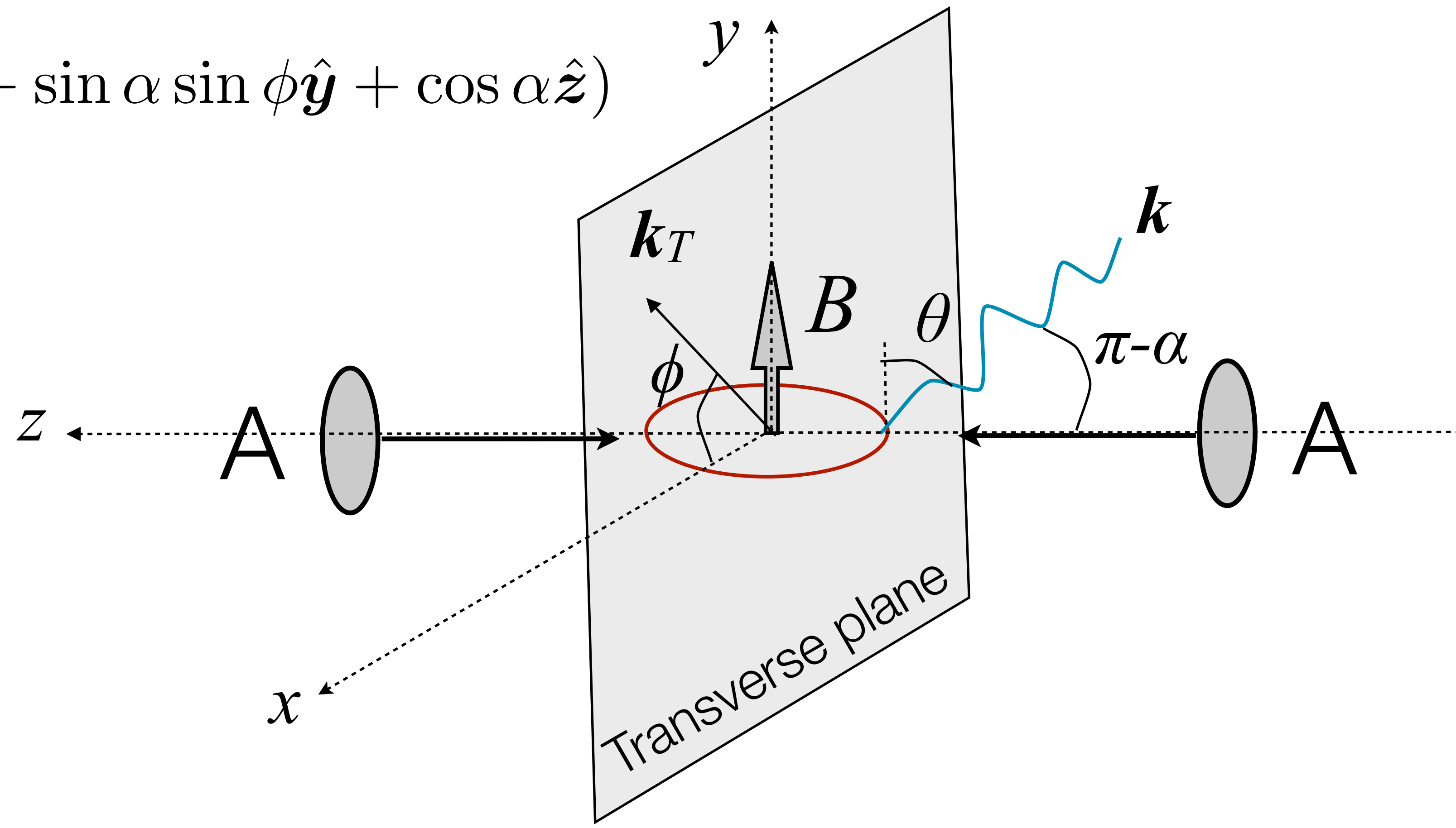


FIG. 1: Spectrum of synchrotron radiation by u quarks at $eB = m_\pi^2$, $y = 0$, $\phi = \pi/3$: (a) contribution of 10 lowest Landau levels $j \leq 10$; several cutoff frequencies are indicated; (b) summed over all Landau levels. $m_u = 3$ MeV, $T = 200$ MeV.

REFERENCE FRAMES

$$\boldsymbol{k} = \omega(\sin\alpha\cos\phi\hat{\boldsymbol{x}} + \sin\alpha\sin\phi\hat{\boldsymbol{y}} + \cos\alpha\hat{\boldsymbol{z}})$$



$$\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{y}} = \cos\theta \;\Rightarrow$$

$$\cos\theta = \sin\alpha\sin\phi$$

Thus, azimuthal dependence (ϕ) of the spectrum is an artifact of the frame choice!

$$k_\perp = \sqrt{k_x^2 + k_y^2} = \frac{\omega\cos\theta}{\sin\phi}\,, \quad y = -\ln\tan\frac{\alpha}{2}$$

$$\frac{dN^{\rm synch}}{dV\,dt\,d^2k_\perp dy} = \omega\frac{dN^{\rm synch}}{dV\,dt\,d^3k} = \frac{dN^{\rm synch}}{dV\,dt\,\omega d\omega d\Omega}$$

ANGULAR DISTRIBUTION OF SR

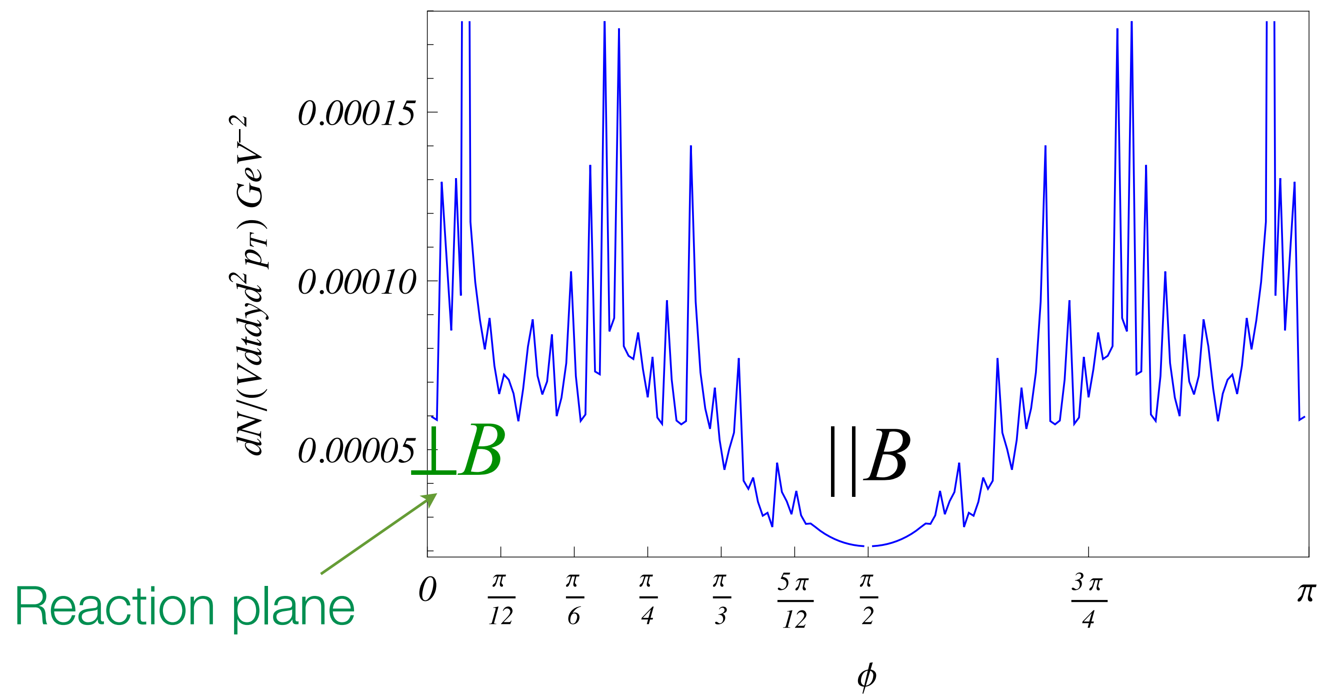


FIG. 2: Azimuthal distribution of synchrotron radiation by u-quarks at $k_\perp = 0.2$ GeV, $eB = m_\pi^2$, $y = 0$. $m_u = 3$ MeV.

This distribution implies that $v_2 > 0$ (to be calculated)

HOW MANY LANDAU LEVELS CONTRIBUTE?

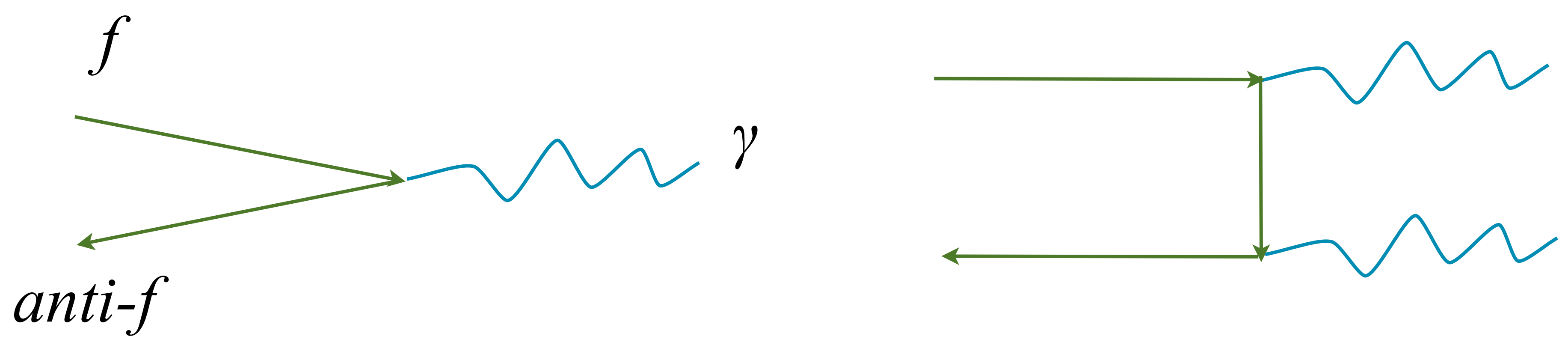
$$\frac{dN^{\text{synch}}}{dt d\Omega d\omega} = \sum_f \int_{-\infty}^{\infty} dp \frac{e_f B(2N_c)V}{2\pi^2} \sum_{j=0}^{j_{\max}} \sum_{k=0}^{j} \frac{dI^j}{\omega d\omega d\Omega}(2-\delta_{j,0}) f(\varepsilon_j)[1-f(\varepsilon_k)]$$

f	u	u	u	u
eB/m_π^2	1	1	15	15
T, GeV	0.2	0.2	0.4	0.4
ϕ	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
$k_\perp$, GeV	0.1	1	1	2
x	0.096	9.6	0.64	2.6
$j_{\max}$	30	40	8	12

TABLE I: The upper summation limit in (18) that yields the 5% accuracy. $j_{\max}$ is the highest Landau level of the initial quark that is taken into account at this accuracy. Throughout the table $y = 0$.

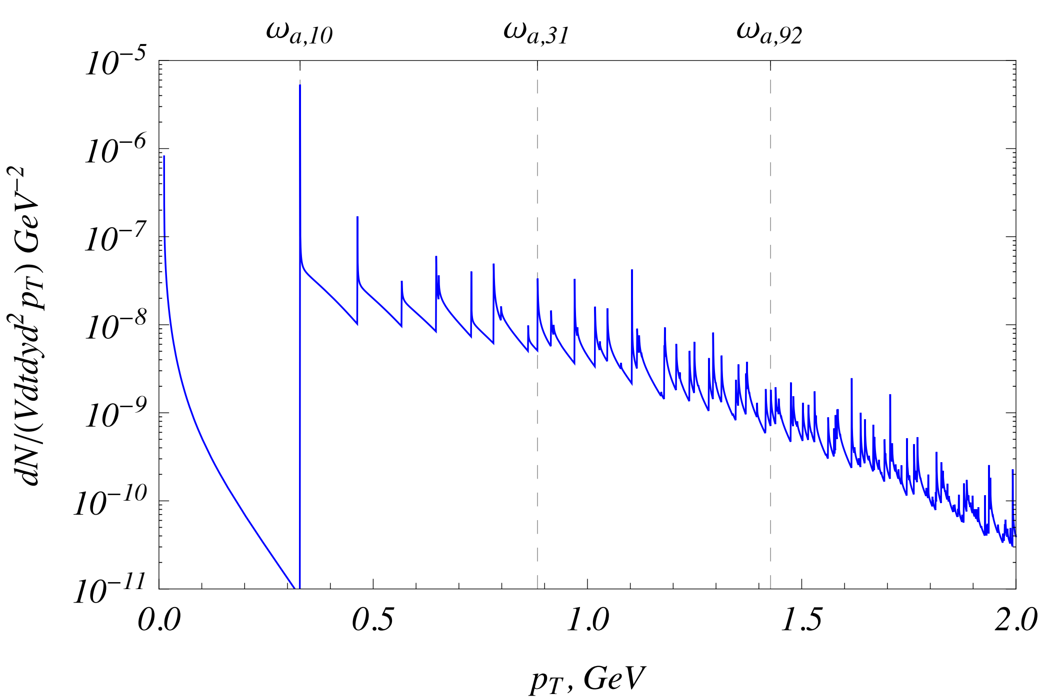
Large j,k correspond to quasi-classical limit.

PAIR ANNIHILATION

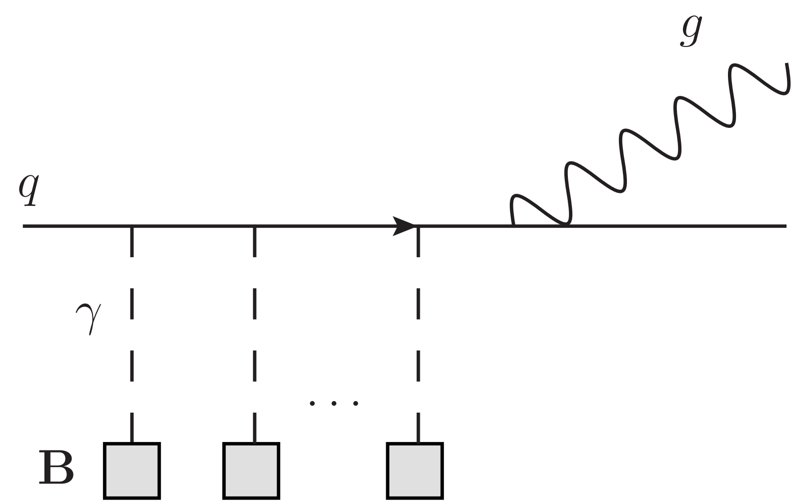


One and two-photon annihilation: At $eB \gg m^2$ one-photon annihilation dominates.

One-photon annihilation is a cross-channel of synchrotron radiation. The corresponding matrix elements are straightforward to calculate.



QUARK ENERGY LOSS DUE TO GLUON SYNCHROTRON RADIATION



- General formulas for synchrotron radiation simplify if quark is **ultra-relativistic** $\varepsilon >> m$ *before* and *after* gluon radiation.

  This always holds in week fields $eB \ll m^2$

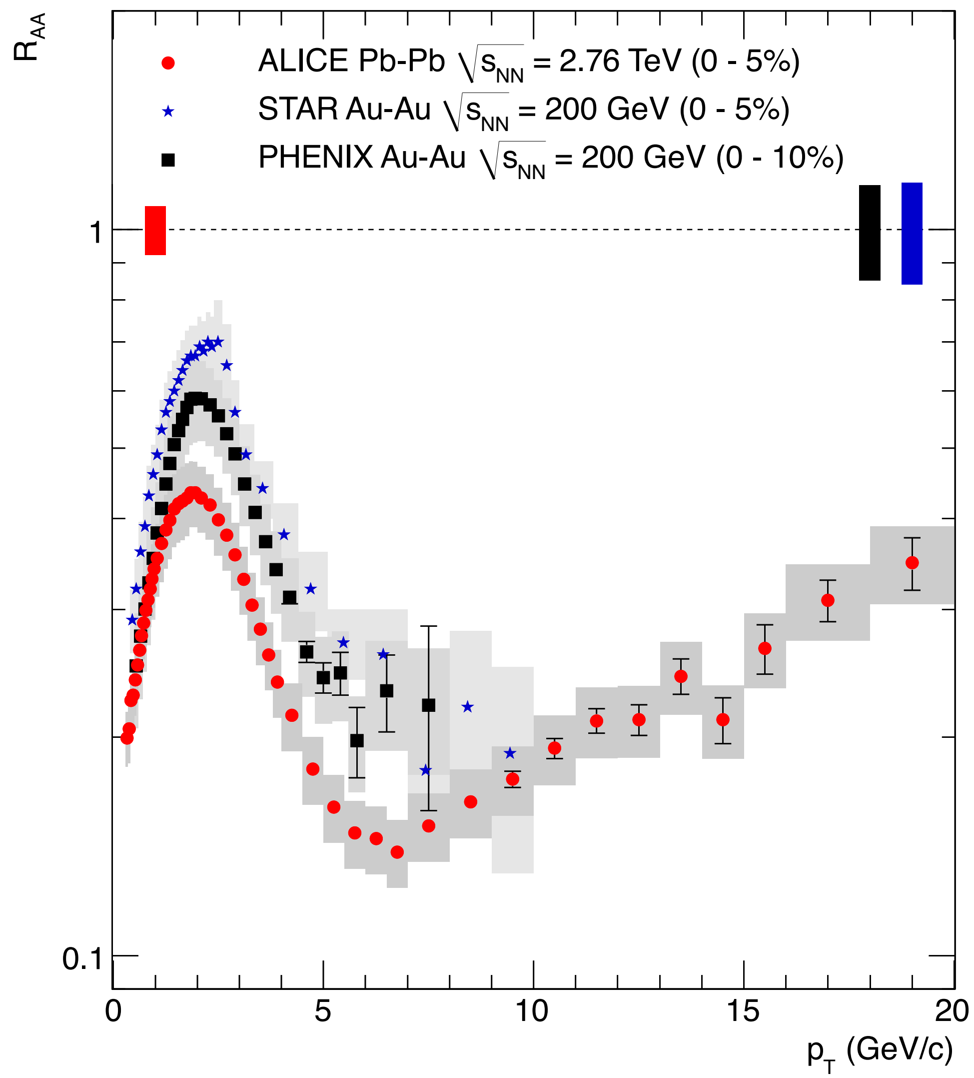
  In strong fields $eB \gg m^2$ this approximation breaks down at the threshold $\omega \sim \varepsilon$, i.e. gluon carries away almost all quark energy ⇒ energy loss in this approximation must satisfy $\Delta\varepsilon \ll \varepsilon$

- Synchrotron radiation is **quasi-classical** if

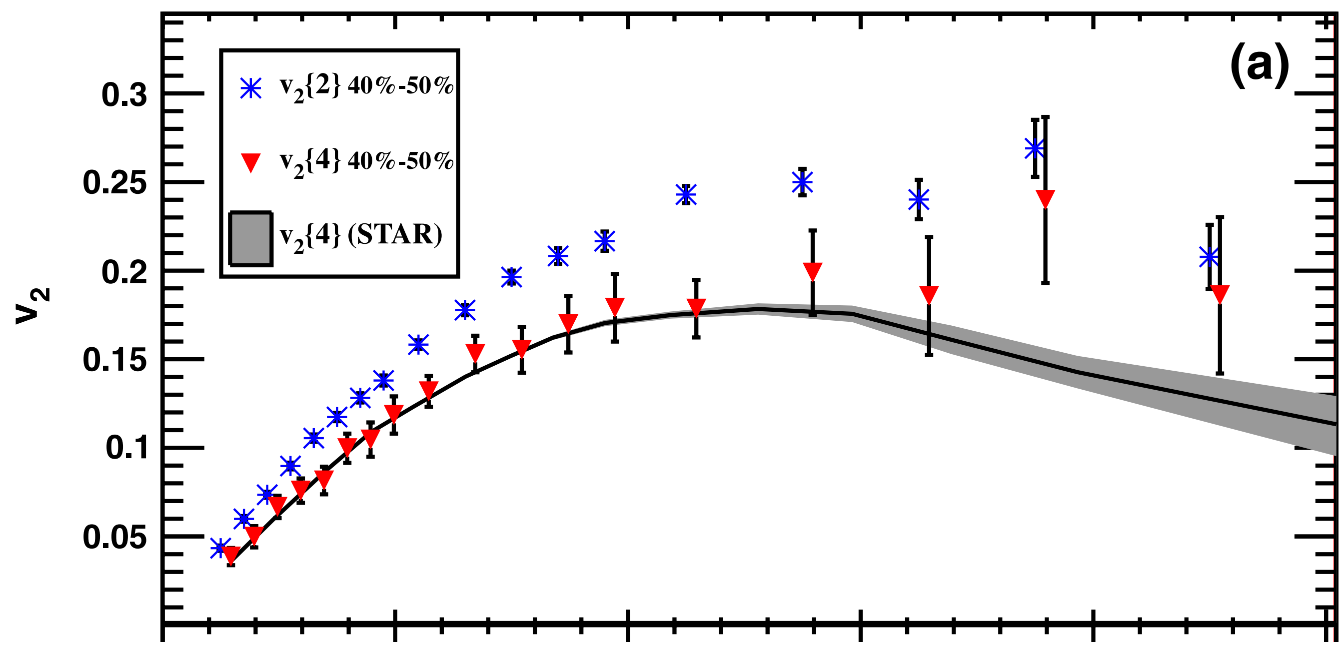
  1.Spacing between Landau levels eB/ε is much smaller than ε $\Rightarrow \varepsilon^2 \gg eB$

  2. Recoil due to gluon emission is small: $\omega \ll \varepsilon$ (i.e. far from the threshold)

RADIATION BY FAST QUARKS



arXiv:1012.1004 [nucl-ex]



Alice, PRL 105 (2010)
arXiv:1011.3914 [nucl-ex]

Synchrotron radiation contributes to quark anergy loss and azimuthal asymmetry.

ULTRA-RELATIVISTIC + QUASI-CLASSICAL LIMIT

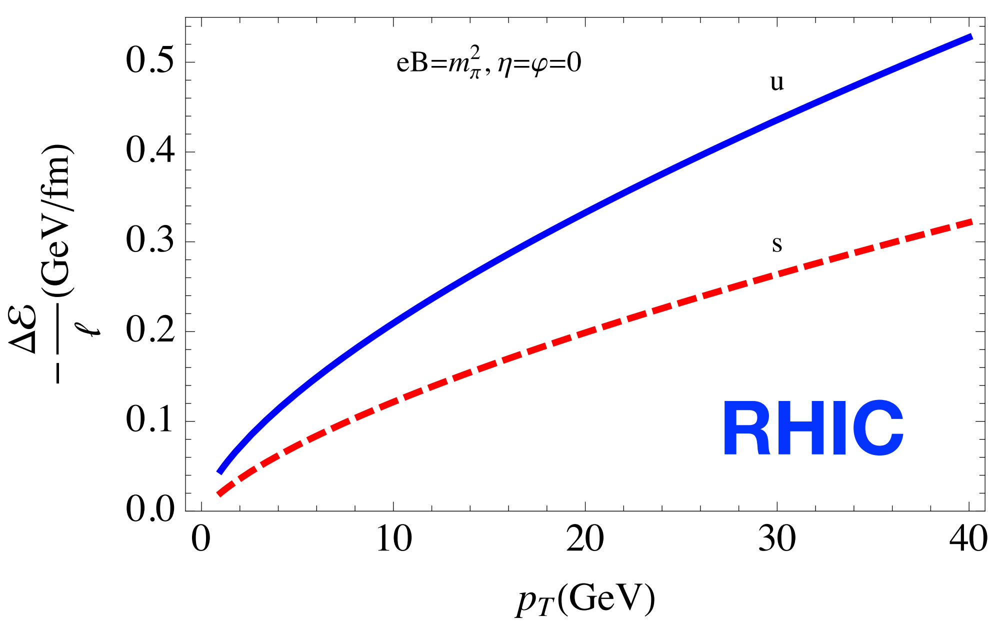
In the quasi-classical approximation $j \gg 1$, $k \gg 1$. Taking also the UR limit Laugerre polynomials reduce to Airy functions:

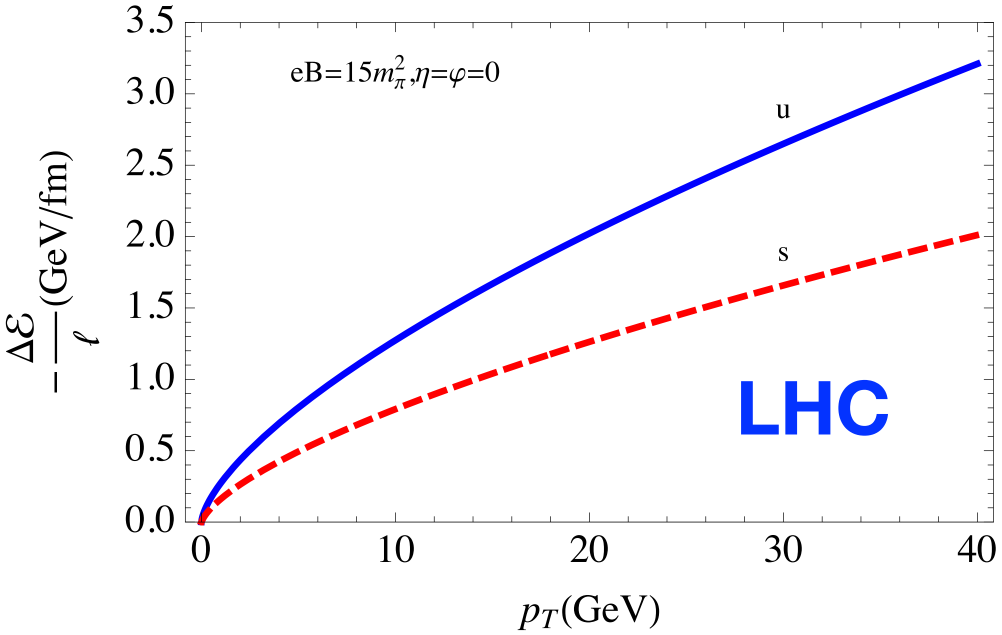
$$\frac{dI}{d\omega} = -\alpha_s C_F \frac{m^2\,\omega}{\varepsilon^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)\,d\xi + \left(\frac{2}{x} + \frac{\omega}{\varepsilon}\,\chi\,x^{1/2}\right)\mathrm{Ai}'(x)\right\}$$

Invariant parameter $$\chi^2 = -\frac{\alpha_{\mathrm{em}} Z_q^2 \hbar^3}{m^6}\,(F_{\mu\nu}p^\nu)^2 = \frac{\alpha_{\mathrm{em}} Z_q^2 \hbar^3}{m^6}(\mathbf{p}\times\mathbf{B})^2$$

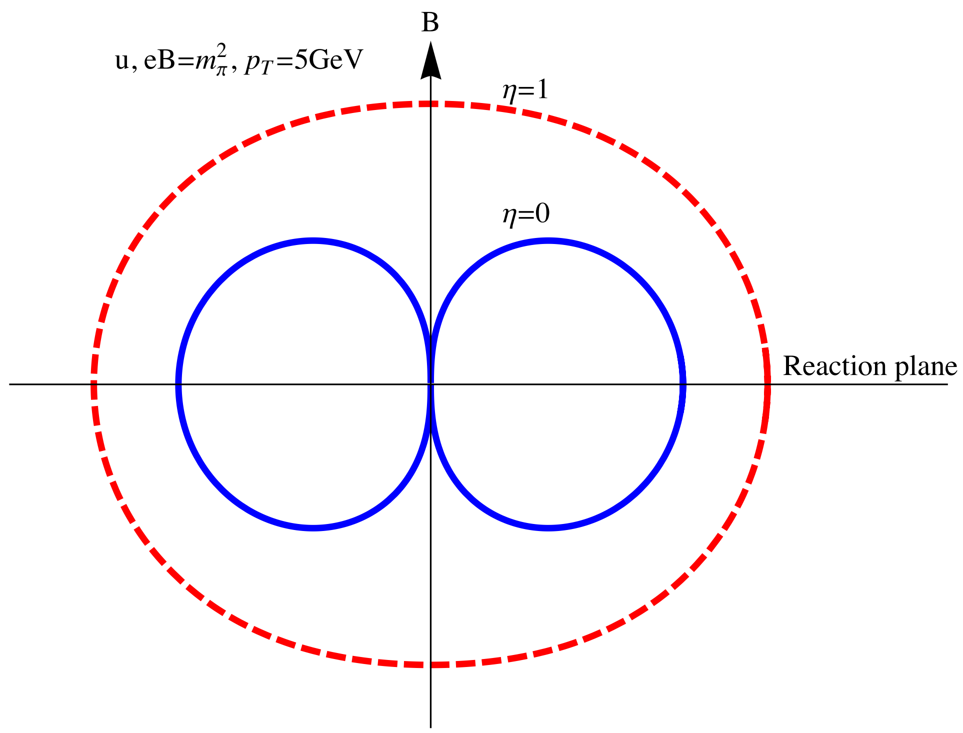
Energy loss $$\frac{d\varepsilon}{dl} = -\int_0^\infty d\omega \frac{dI}{d\omega} = \alpha_s C_F \frac{m^2\,\chi^2}{2}\int_0^\infty \frac{4 + 5\chi\,x^{3/2} + 4\chi^2\,x^3}{(1+\chi\,x^{3/2})^4}\,\mathrm{Ai}'(x)\,x\,dx$$

Energy loss in magnetic field





Azimuthal asymmetry:

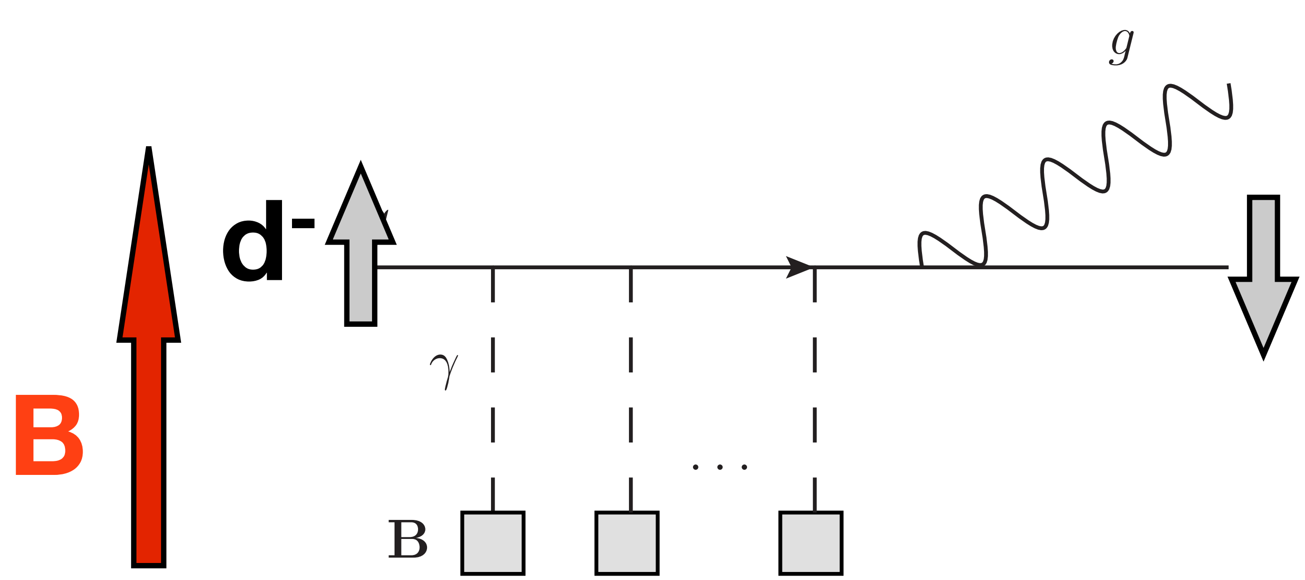


POLARIZATION OF LIGHT QUARKS

Spin-flip probability per unit time

$$w = \frac{5\sqrt{3}\alpha_s C_F}{16}\frac{\hbar^2}{m^2}\left(\frac{\varepsilon}{m}\right)^5\left(\frac{Z_q e\,|\boldsymbol{v}\times\boldsymbol{B}|}{\varepsilon}\right)^3\left(1-\frac{2}{9}(\boldsymbol{\zeta}\cdot\boldsymbol{v})^2-\frac{8\sqrt{3}}{15}\mathrm{sign}\,(e_q)\,(\boldsymbol{\zeta}\cdot\boldsymbol{b})\right)$$

Sokolov, Ternov (1964)



NR case: $H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -\left(\frac{geZ_q\hbar}{2m}\right)\boldsymbol{s}\cdot\boldsymbol{B}$

A very strong polarization of quarks and leptons $A = \frac{8}{5\sqrt{3}} = 92\%$

Other effects

- ✳ Synchrotron radiation — KT (2010,2012)
- ✳ Photon/dilepton production — KT (2010,2013)
- ✳ Azimuthal anisotropy of QGP — Mohaparta, Saumia, Srivastava (2011), KT (2011)
- ✳ Ionization of bound states (e.g. J/ψ) — Marasinghe, KT (2011)
- ✳ Chiral Magnetic Effect — Kharzeev (2006), Kharzeev, Zhitnitsky (2007), Kharzeev, McLerran, Warringa (2008), ...
- ✳ QCD phase diagram

SUMMARY

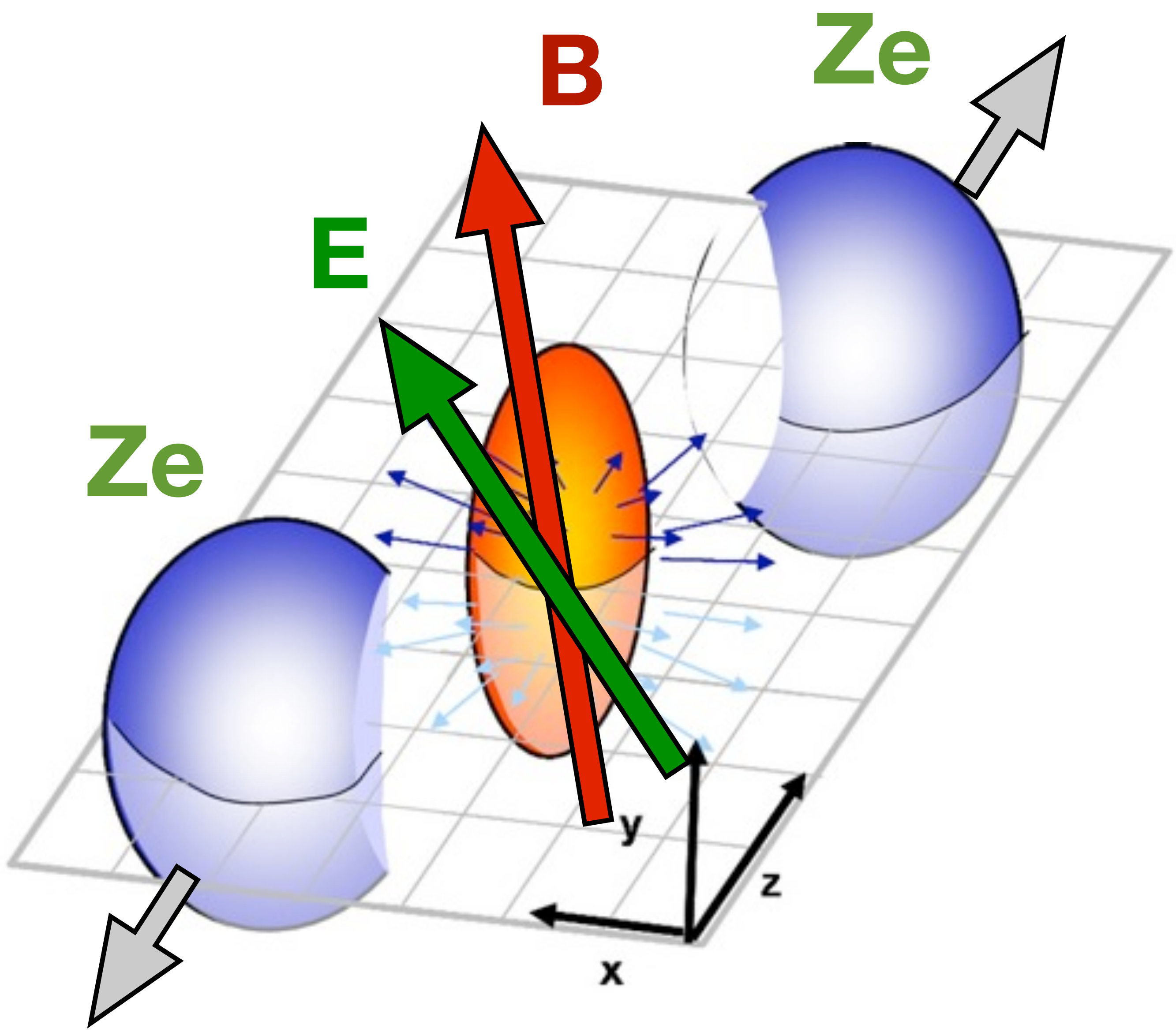
- Electromagnetic field produced in relativistic heavy-ion collisions is, probably, strongest in nature: $B\sim10^{18}\ G$

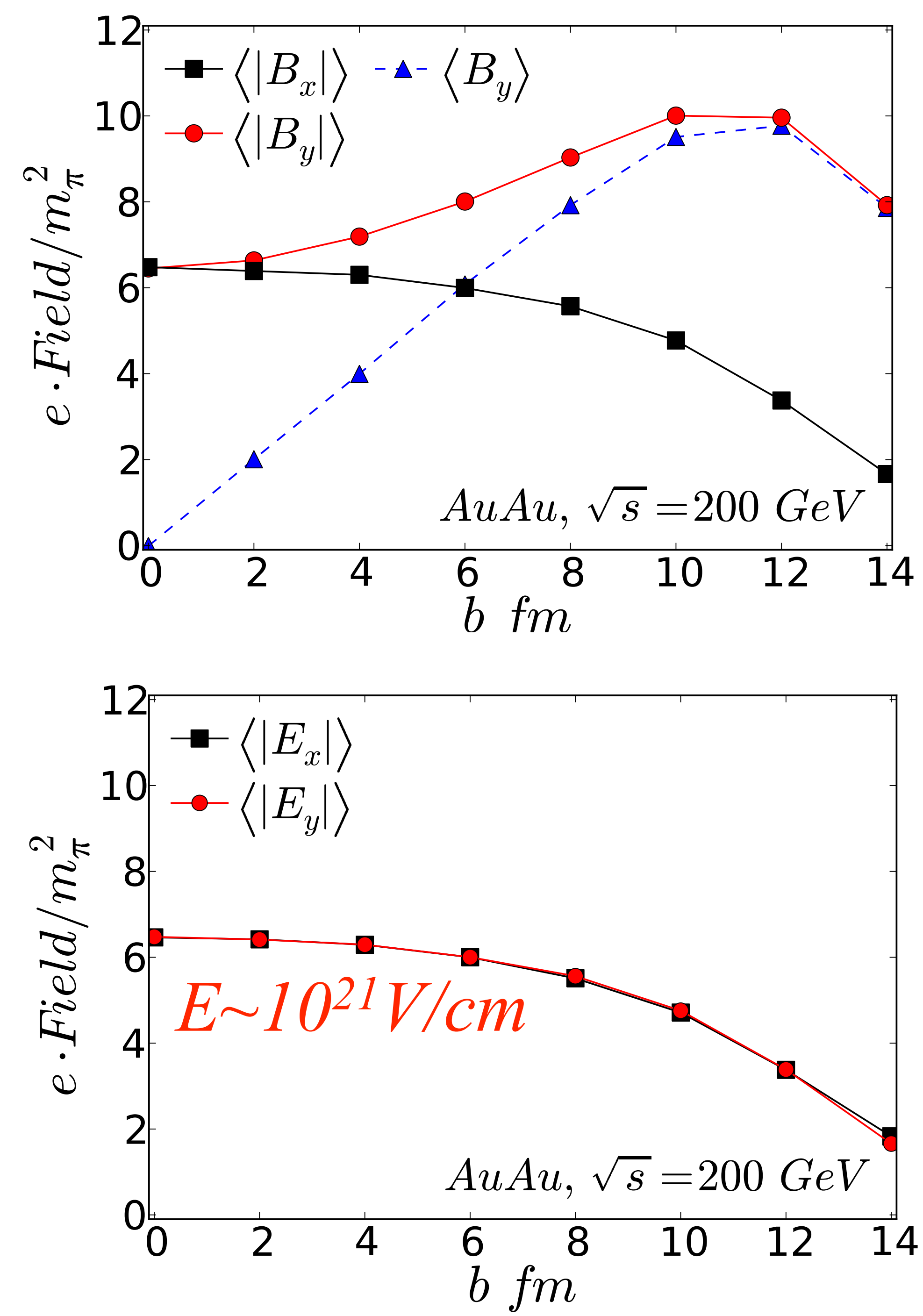
- Strong magnetic field can trigger a lot of novel phenomena that have never been observed before (some were predicted long time ago).

- We have a unique opportunity to study non-perturbative Quantum Electrodynamics in lab - a lot of cross-disciplinary applications.

Other stuff

NUCLEAR DENSITY FLUCTUATIONS





Fluctuations of nucleon positions generate other components of electromagnetic field.

Bzdak and Skokov (2011)

EXAMPLES OF STRONG ELECTRIC FIELDS

10^5 V/cm (10^7 W/cm^2): SLAC particle accelerator

10^{16} V/cm (10^{29} W/cm^2): Schwinger limit (instability of QED vacuum)

10^{21} V/cm (10^{39} W/cm^2): Heavy ions collisions