Ab initio description of light nuclei in the Berggren basis

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European Research Council

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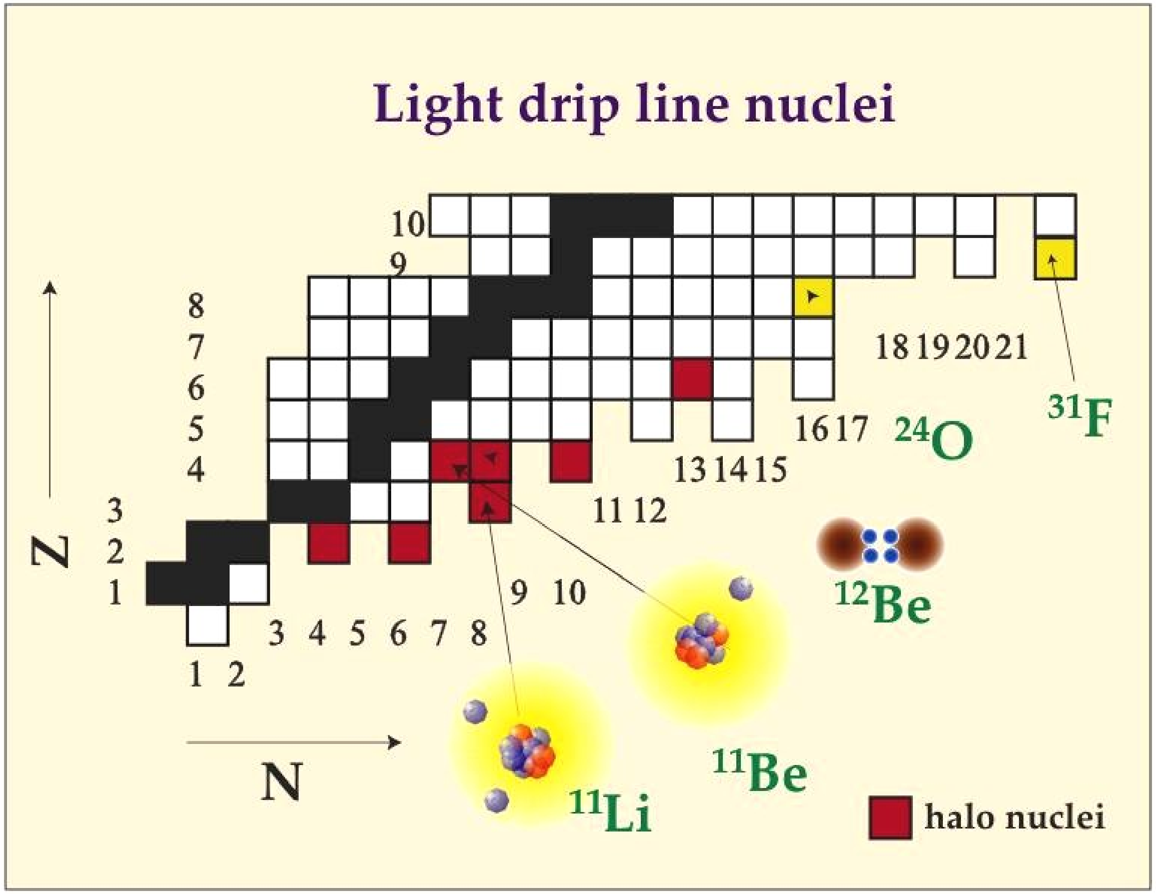
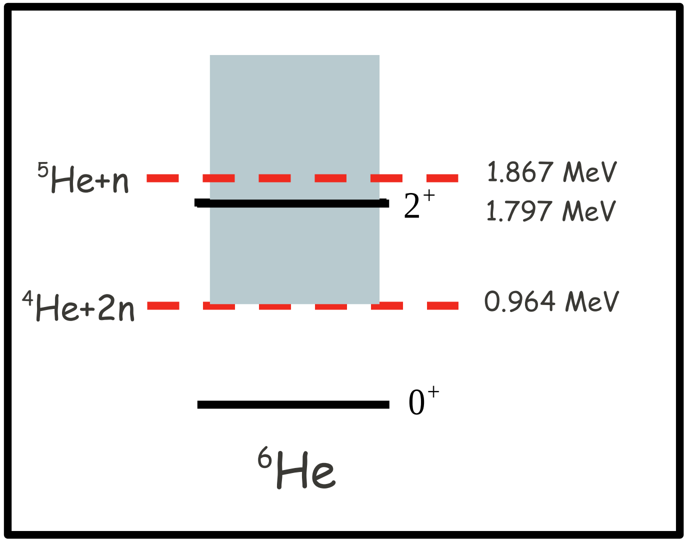
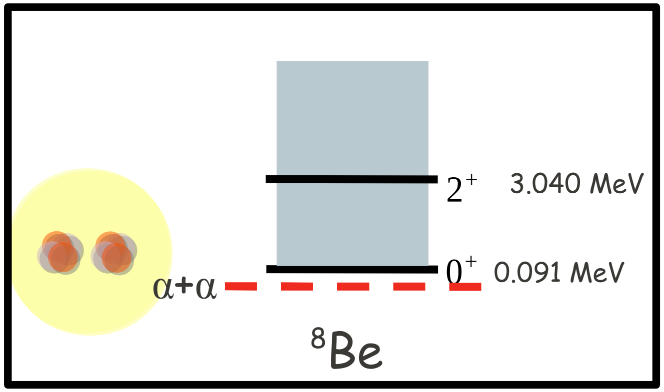
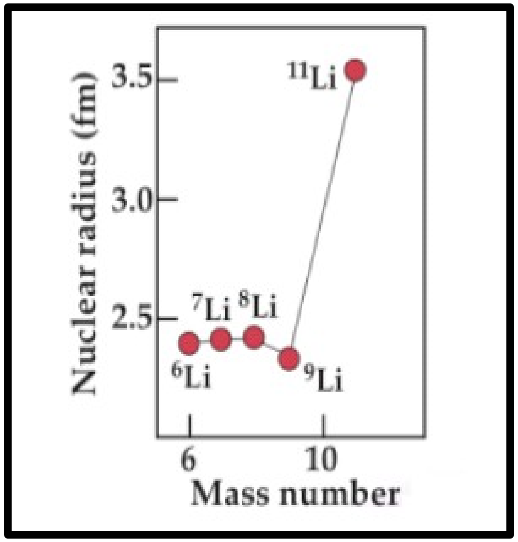
International Conference on Nuclear Theory in the Supercomputing Era, Ames, May 2013

Impact of continuum for nuclei far from stability

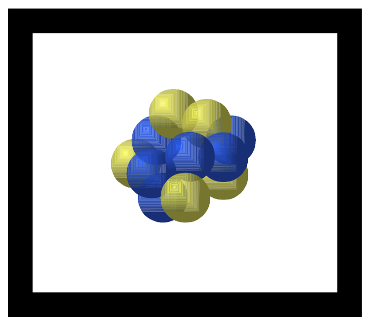


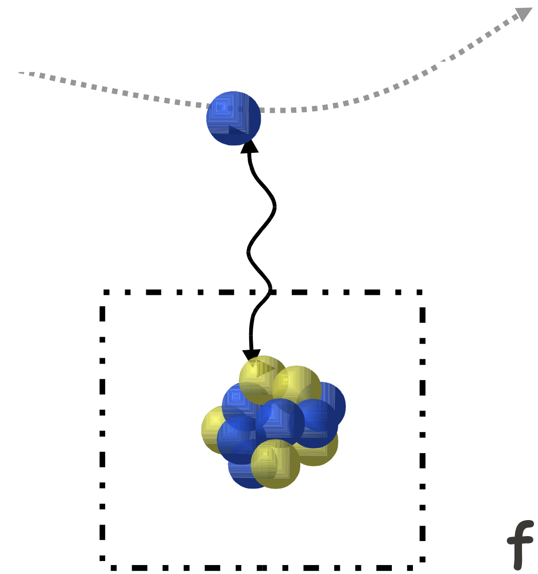




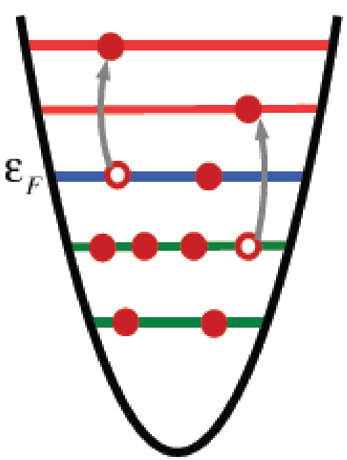


Closed quantum systems	Open quantum systems
(nuclei near the valley of stability)	(nuclei far from stability)



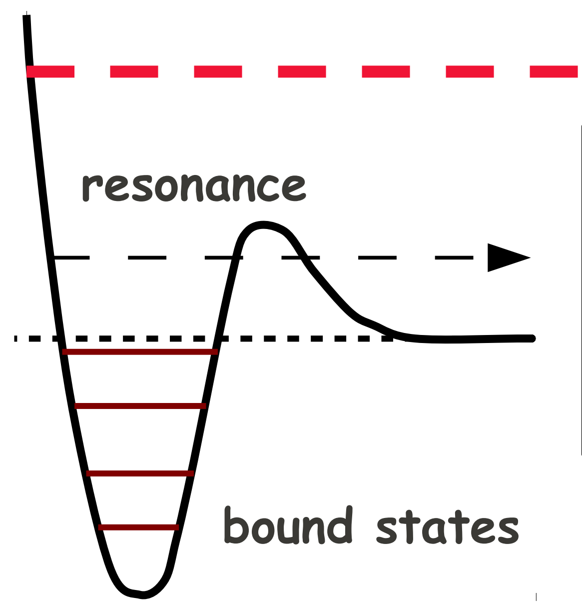


nucleons in bound shells



-> usually HO basis
-> exact treatment of the center of mass, analytical wavefunctions...

finite-depth potential

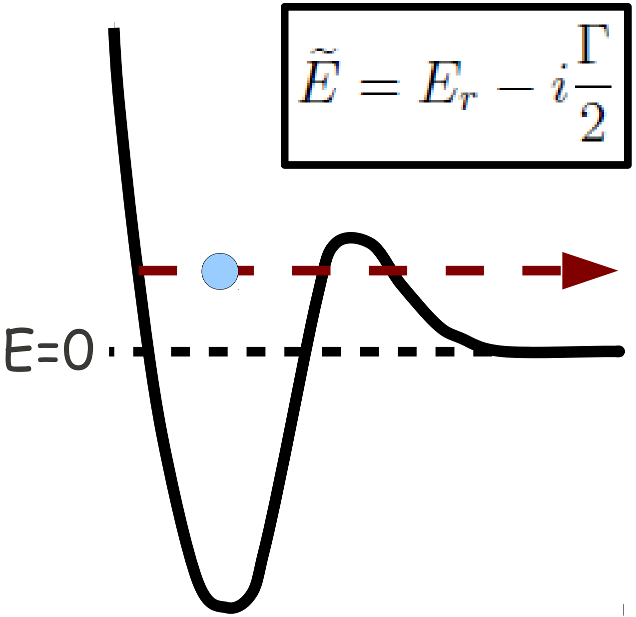


-> Woods-Saxon, Hartree-Fock potential.....
-> bound, resonant, continuum shells

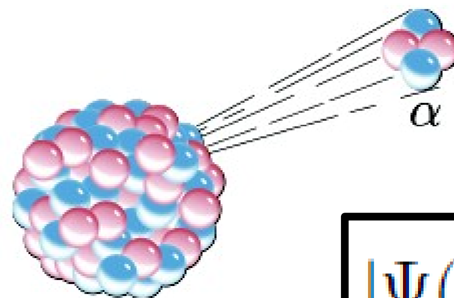
see also talks by G.Hagen, P. Navratil, M Caprio, M. Hjorth-Jensen

Gamow States

G. Gamow, Z. Phys. 51 (1928) 204



$$\tilde{E} = E_r - i\frac{\Gamma}{2}$$

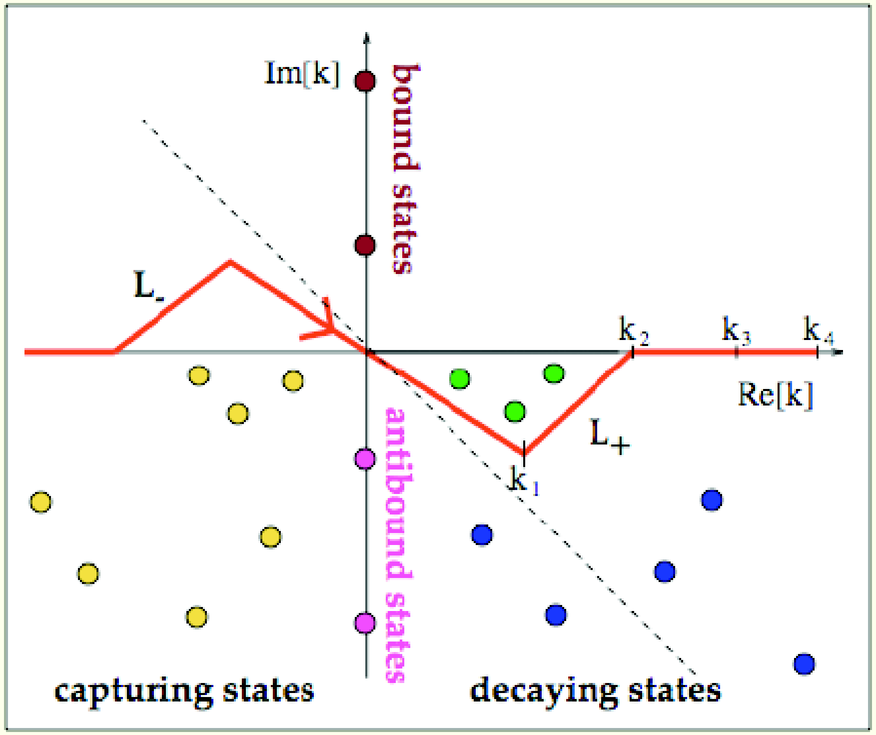


$$|\Psi(t,r)| \sim e^{-\frac{\Gamma t}{2\hbar}} e^{k_1 r}, r \to \infty$$

$$\Psi(t,r) = e^{\frac{-i\tilde{E}t}{\hbar}} \psi(r)$$

Gamow states and completeness relations

T. Berggren, Nucl. Phys. A109, 265 (1968); Nucl. Phys. A389, 261 (1982)
T. Lind, Phys. Rev. C47, 1903 (1993)



$$\sum_{n=b,r} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_{L_+} |u(k)\rangle\langle u(k^*)|\, dk = 1$$

particular case: Newton completeness relation

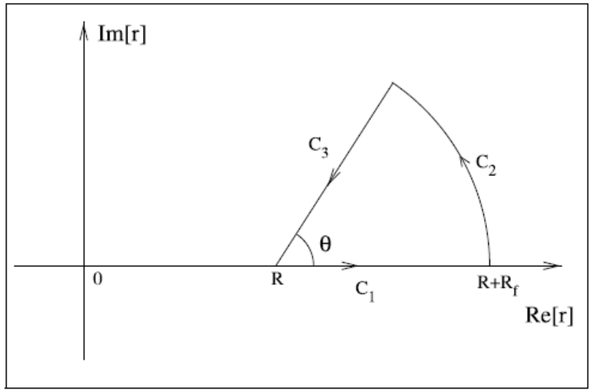
$$\sum_{n=b} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_{R} |u(k)\rangle\langle u(k^*)|\, dk = 1$$

Bound, resonant state

$$u(r) \to C_+ H^+_{l,\eta}(kr)$$

normalization of resonant states with external complex scaling :

$$N_i = \sqrt{\int_0^R u_i^2(r)\,dr + \int_0^{+\infty} u_i^2(R + x\cdot e^{i\theta})\, e^{i\theta}\, dx}$$

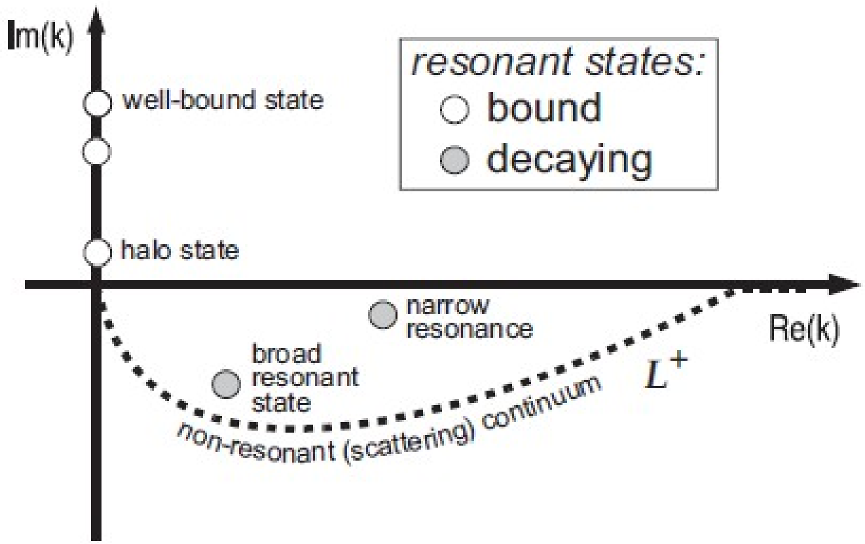


Complex scattering state

$$u(r) \to C_+ H^+_{l,\eta}(kr) + C_- H^-_{l,\eta}(kr)$$

Gamow Shell Model (GSM)

N. Michel *et al*, J.Phys. G36 (2009) 013101



"Shell Model in the Berggren basis"

i) *discretization* of continuum contour

$$\sum |u_{res}\rangle\langle u_{res}| + \sum_i |u_{ki}\rangle\langle u_{ki}| \simeq 1$$

ii) construction of many-body basis

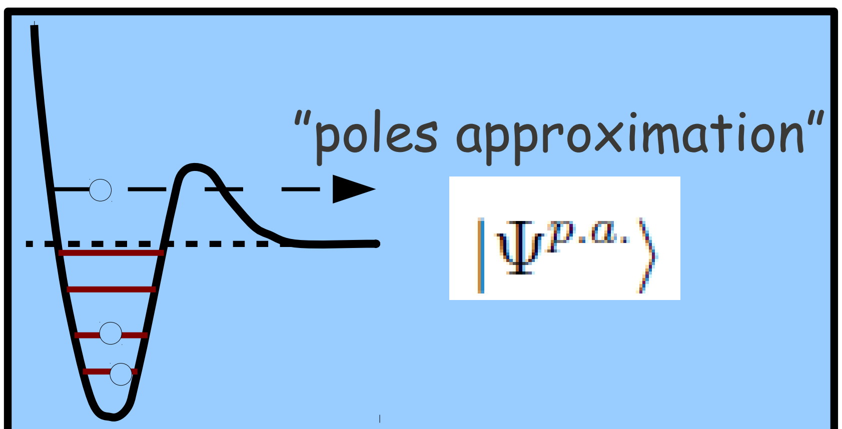
$$|SD_i\rangle = |u_{i1.......}u_{iA}\rangle$$

iii) construction of Hamiltonian matrix

$$\langle SD_i|H|SD_j\rangle$$

(complex-symmetric matrix)

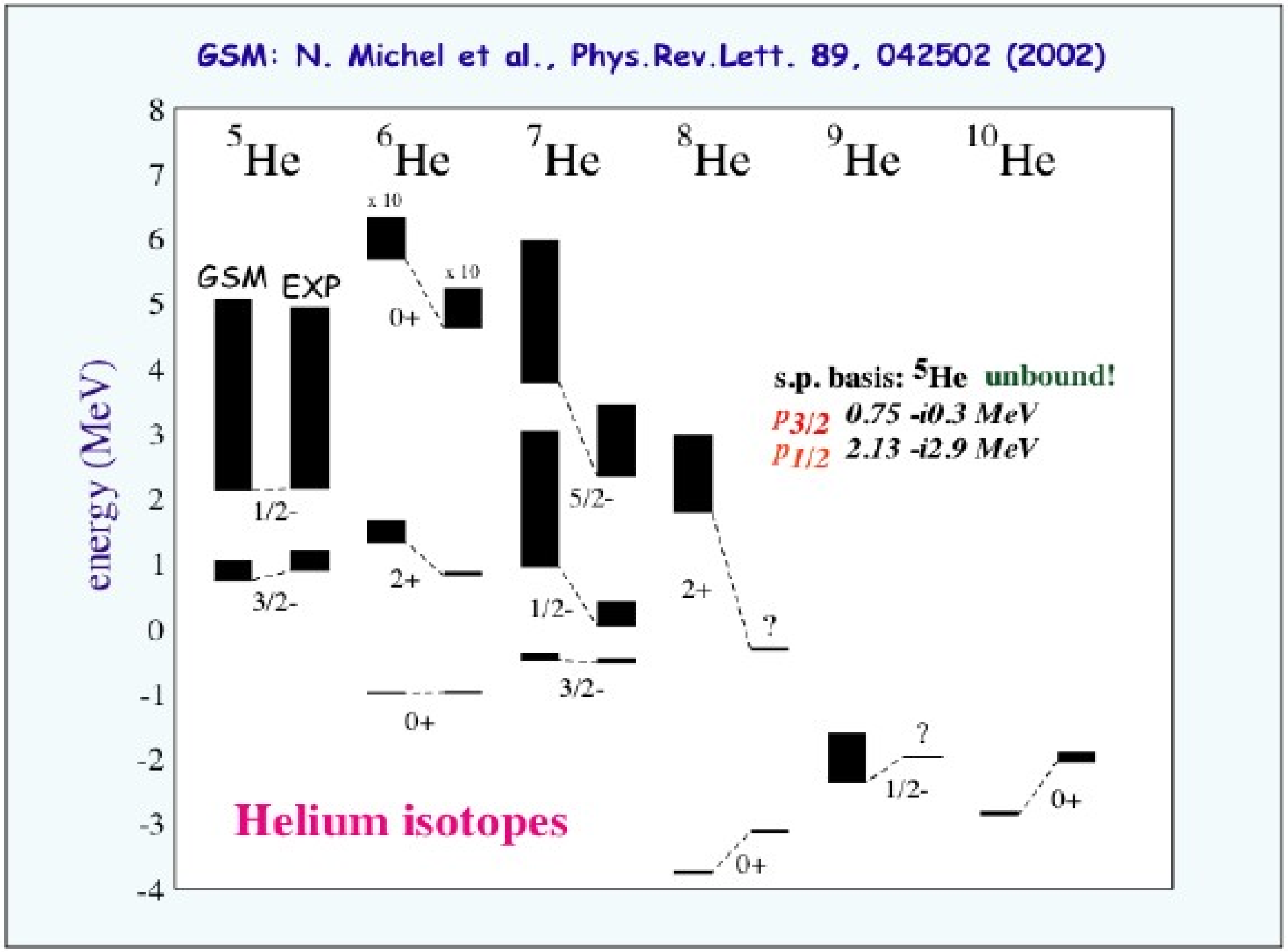
iv) -> many-body bound,resonant and continuum states



Many-body resonance states :

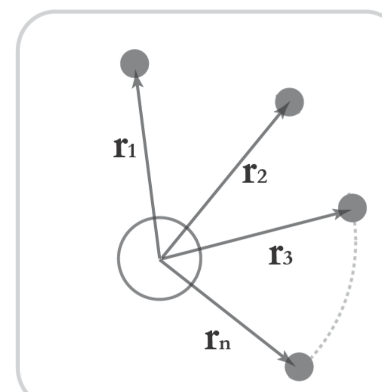
$$\max\{|\langle\Psi|\Psi^{p.a.}\rangle|\}$$

Helium chain (^{4}He core plus valence neutrons)



i) Woods-Saxon potential for (^{4}He-n)

ii) two-body zero-range force for (n-n)



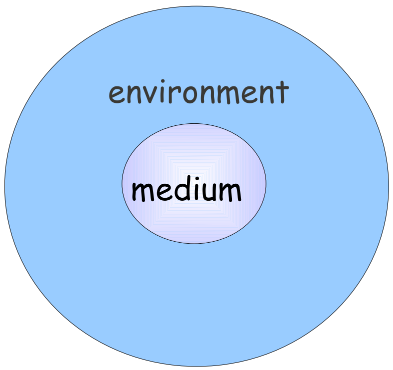
Pole approximation :

$p_{3/2}$, $p_{1/2}$ resonance (^{5}He g.s and 1st excited state)

Density Matrix Renormalization Group (DMRG)

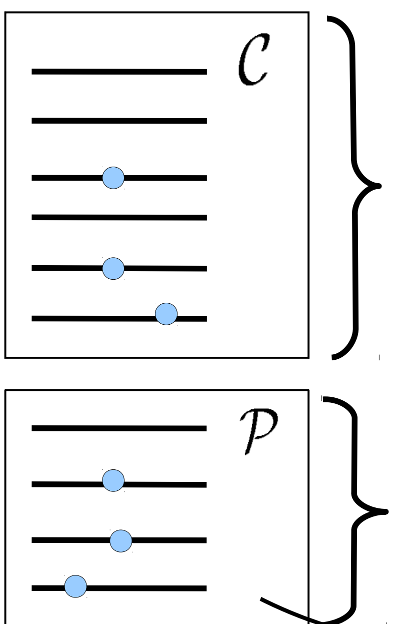
S. R. White, PRL. 69 (1992); PRB 48 (1993)
T.Papenbrock et al J.PG 31 (2005)
S.Pittel et al PRC 73 (2006)

quantum system



* separation into a "medium" and "environment"
* truncation of degrees of freedom in the environment

GSM-picture



continuum shells

poles

largest contribution to the GSM wave function

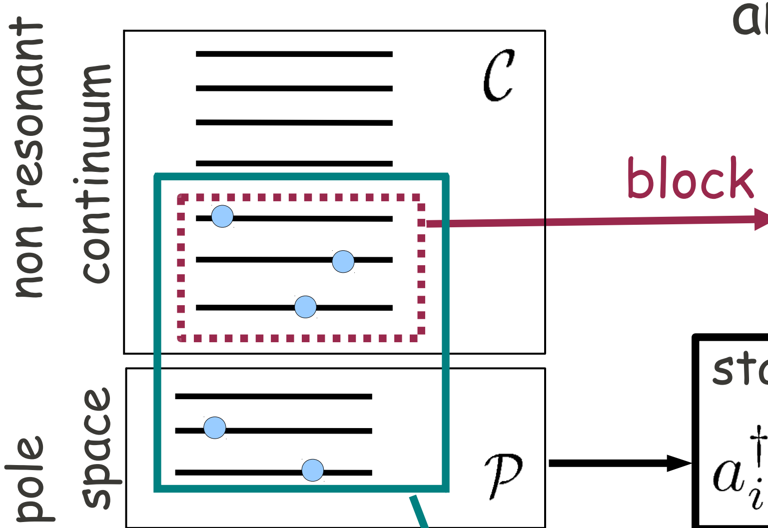
$$|\Psi\rangle^J = \sum_{p,c} \Psi_{pc} \left[|p\rangle^{J_p} |c\rangle^{J_c} \right]^J$$

DMRG

truncation among states with nucleons in the continuum

Warm up phase

Construction of 2nd quantization operators and states in $\mathcal{P}$ and $\mathcal{C}$



states with 0,1,...n nucleons

$$a_i^\dagger, (a_i^\dagger a_j^\dagger)^K, [(a_i^\dagger a_j^\dagger)^K \tilde{a}_k]^L \ldots$$

states with 0,1,...n nucleons

$$a_i^\dagger, (a_i^\dagger a_j^\dagger)^K, [(a_i^\dagger a_j^\dagger)^K \tilde{a}_k]^L \ldots$$

* diagonalization in the superblock ⟶ $|\Psi\rangle^J = \sum_{p,c} \Psi_{pc} (|p\rangle^{J_p} |c\rangle^{J_c})^J$

* singular value decomposition

$$\begin{cases} |\Psi'\rangle^J = \sum_{p,\alpha} \Psi_{p\alpha} (|p\rangle^{J_p} |\alpha\rangle^{J_c})^J \\ |\Psi'\rangle^J \simeq |\Psi\rangle^J \end{cases}$$

$|\alpha\rangle$: eigenvectors of density

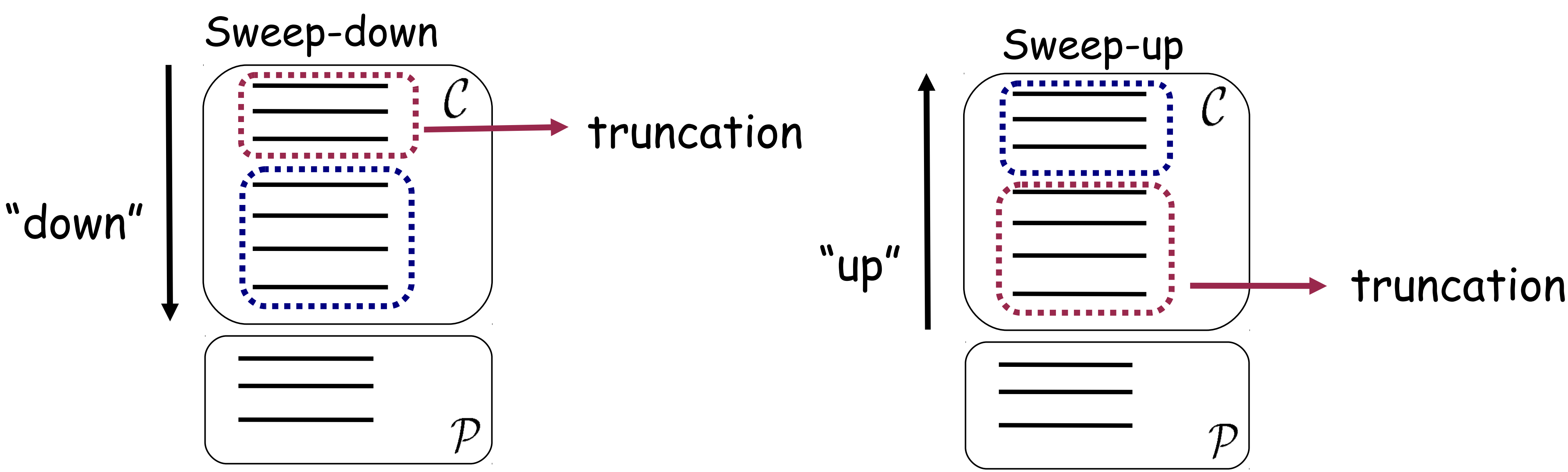
$$\rho^{J_c}_{c,c'} = \sum_p \Psi_{pc} \Psi_{pc'}$$

N_{opt} eigenstates of density with "largest" eigenvalues are kept:

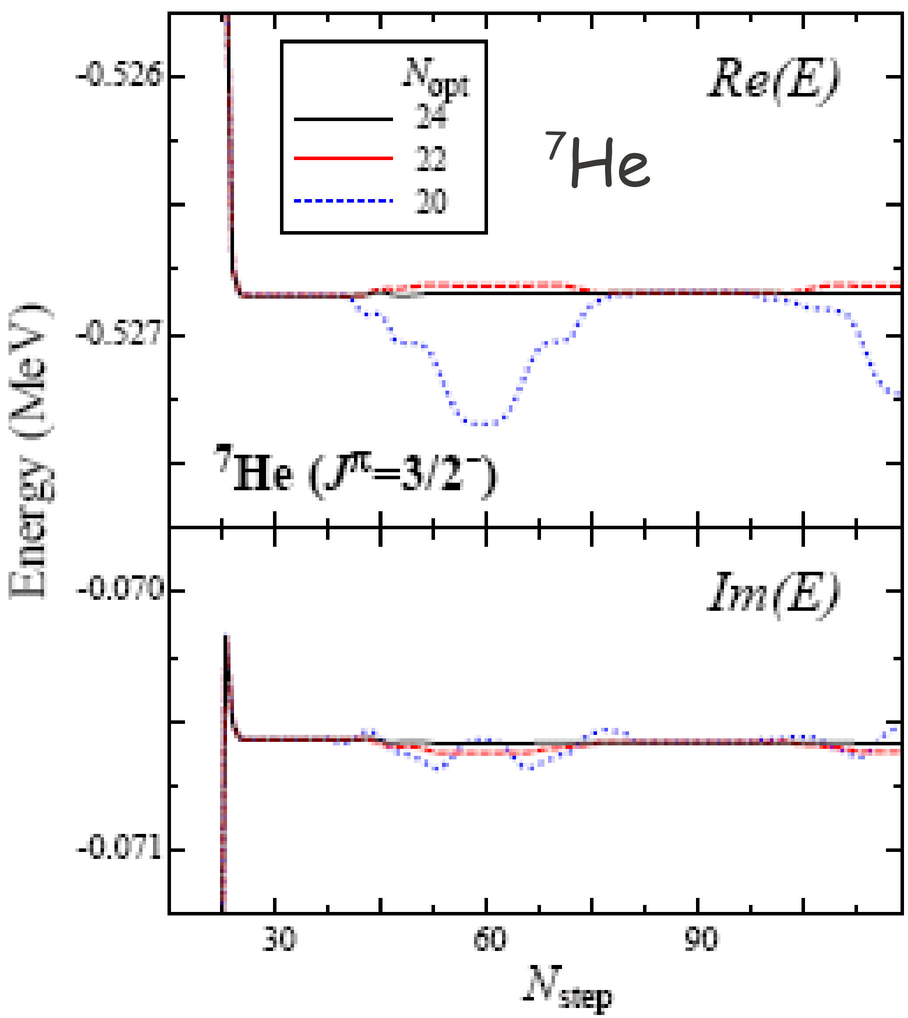
i) fixed N_{opt} or ii) $|1 - Re \left(\sum_{\alpha=1}^{N_{opt}} w_\alpha \right) | < \varepsilon$

In the warm up phase, continuum shells are added one by one until they all have been included.

sweeping phase



Convergence of the energy as a function of DMRG iteration



GSM description of ^7He :

^4He core + 3 neutrons

*Woods-Saxon potential + Gaussian V_{nn}

*resonance : $0p_{3/2}$, $0p_{1/2}$
*complex-continuum shells: $p_{3/2}$, $p_{1/2}$

(62 shells)

Full GSM dimension = 83,948
DMRG dimension = 1,143

(J.R *et al.*, PRL 97 (2006) 110603)

* All nucleons are active
* Expansion in the Berggren basis
* Density Matrix Renormalization Group technique

↓

Ab Initio description of light nuclear systems at and beyond the drip lines

"No Core Gamow Shell Model"

No Core Gamow Shell Model (NCGSM)

$$H = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m} + V_{NN,ij}$$

G. Papadimitriou *et al*, arXiv:1301.7140

i) NN potential:

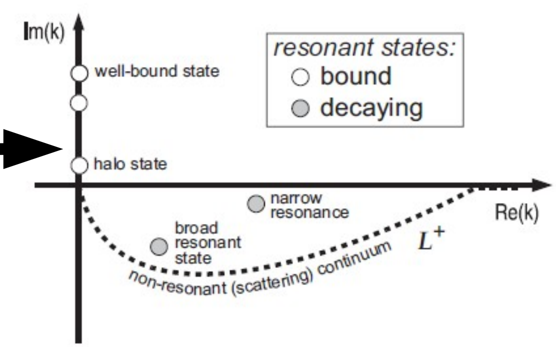
* AV18 (R.B. Wiringa et al PRC 51 (1995) 38)
* N^3LO (D.R.Entem et al PRC(R) 68 (2003) 041001)

} softened by v_{low-k} with Λ = 1.9 fm-1 (S. Bogner et al, Phys. Rep. 386 (2003) 1)

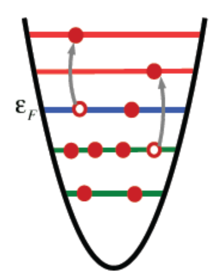
(For comparison with Faddeev, Faddeev-Yakubovsky and Coupled Cluster)

ii) single particle states:

a) s- and p-shells from Hartree-Fock potential ⟷

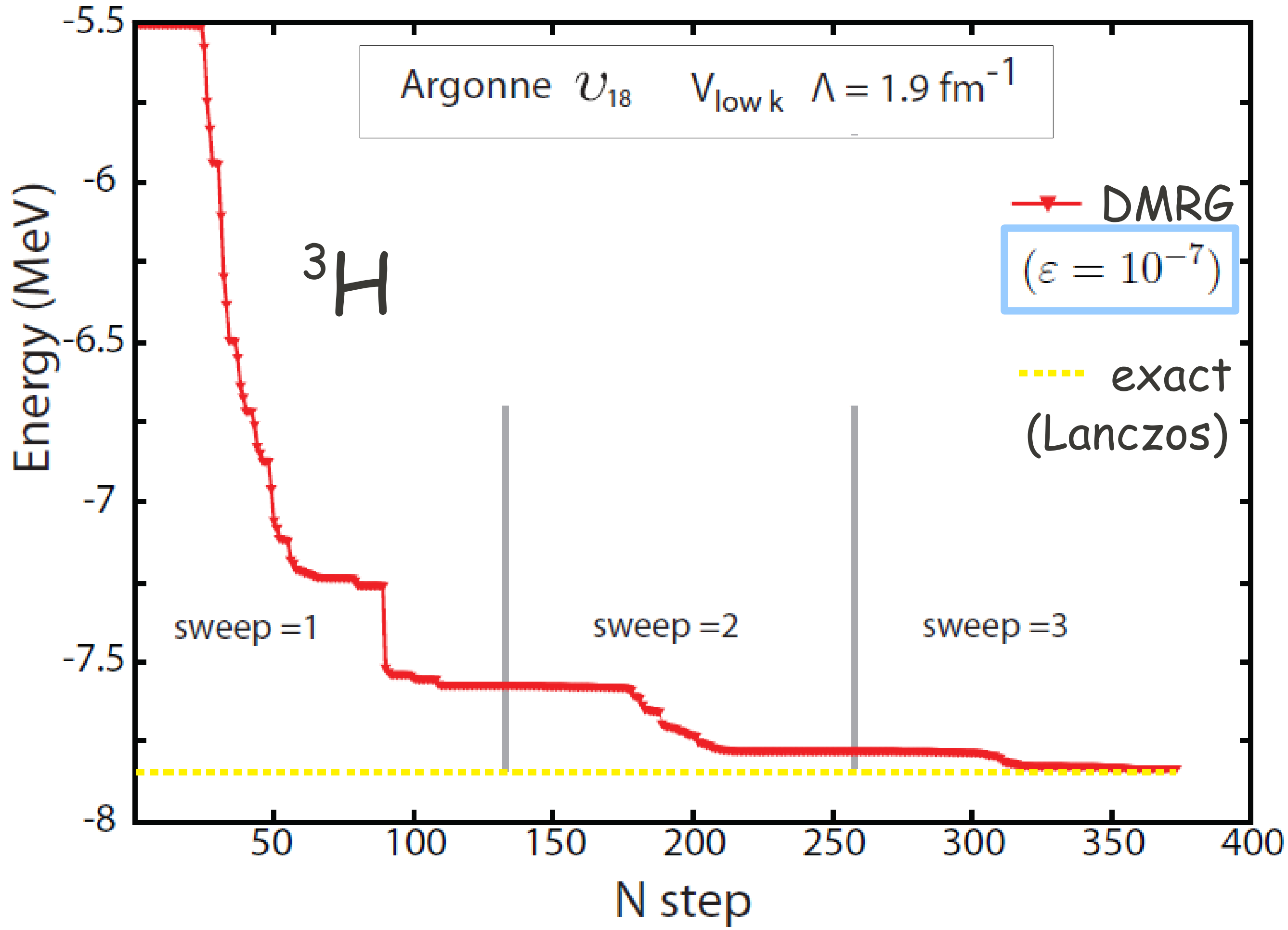


b) for l>1, shells of the Harmonic Oscillator



iii) Numerical resolution with DMRG

Calculations of ^3H, ^4He and ^5He



i) $s_{1/2}$ $p_{3/2}$ $p_{1/2}$ real-energy HF states

ii) $d_{5/2}$ $d_{3/2}$ H.O states

* $0s_{1/2}(p)$: E= -10.417 MeV
* $0s_{1/2}(n)$: E= -11.982 MeV

130 shells

J-scheme dimension

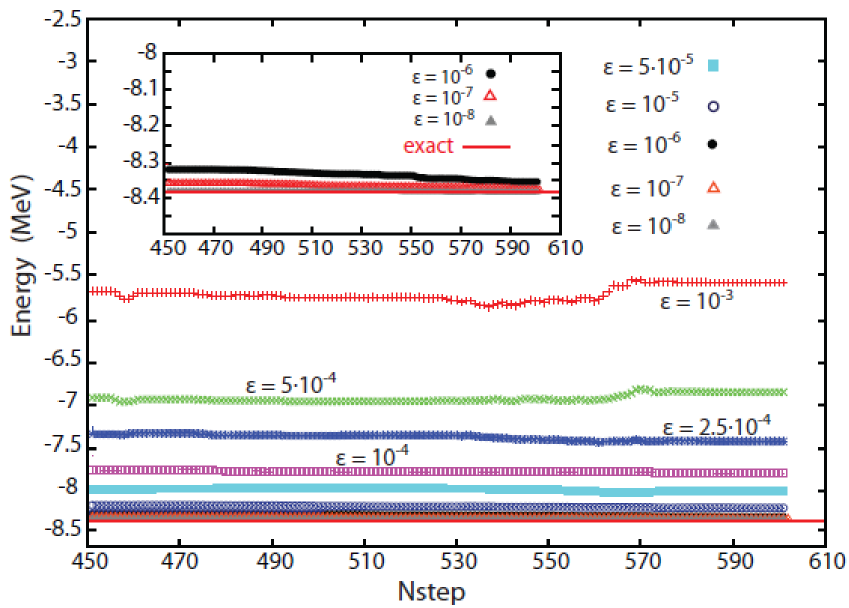
* Full NCGSM space: 96,883 ⟶ E_{exact}=-7.840 MeV
* DMRG ~ 1,200 ⟶ E_{DMRG}=-7.832 MeV

G. Papadimitriou *et al*, arXiv:1301.7140

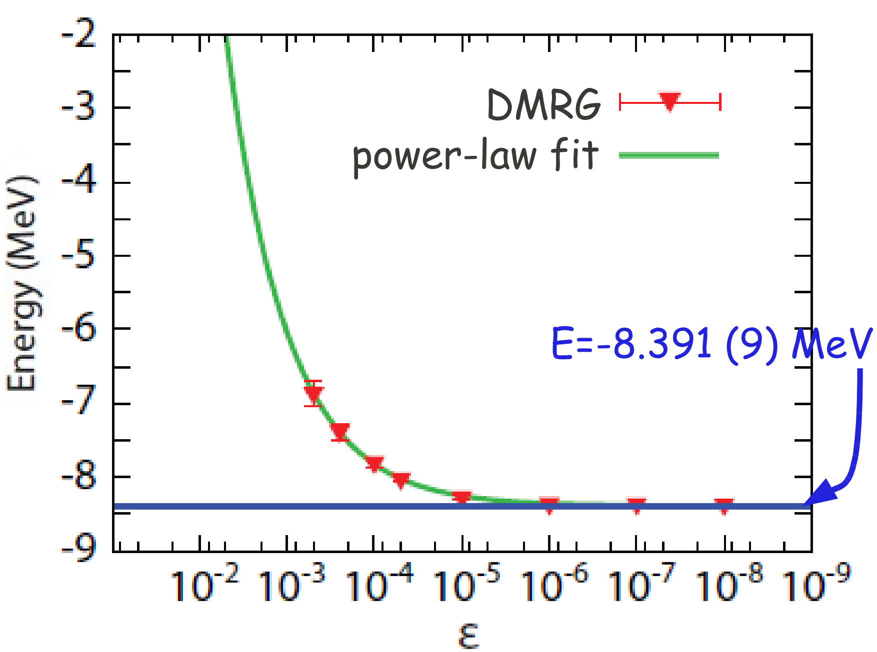
^3H binding energy

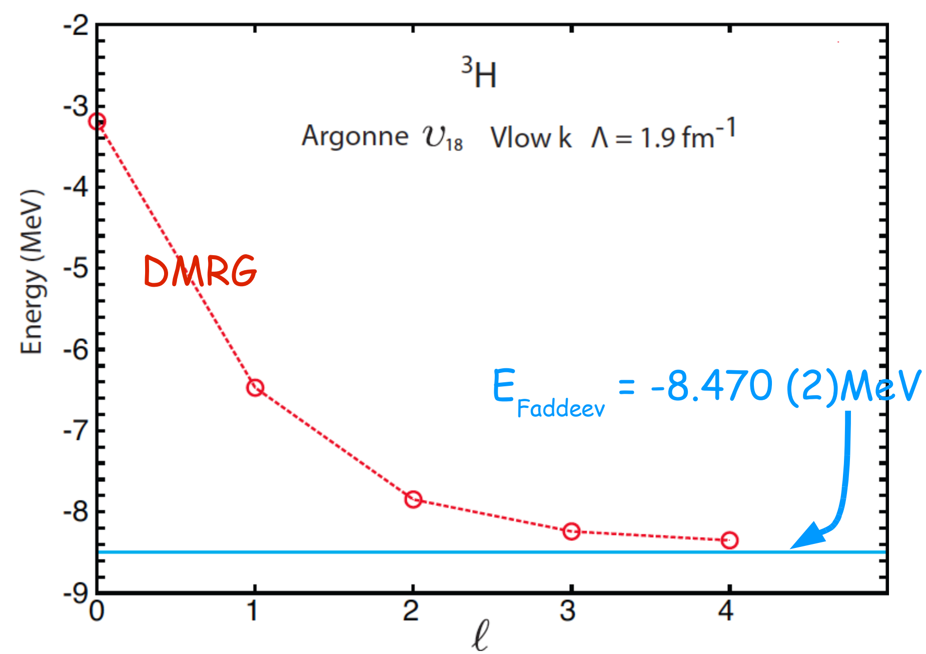
*spdfg shells
*full NCGSM dim: 123,835

Evolution of energy with iteration

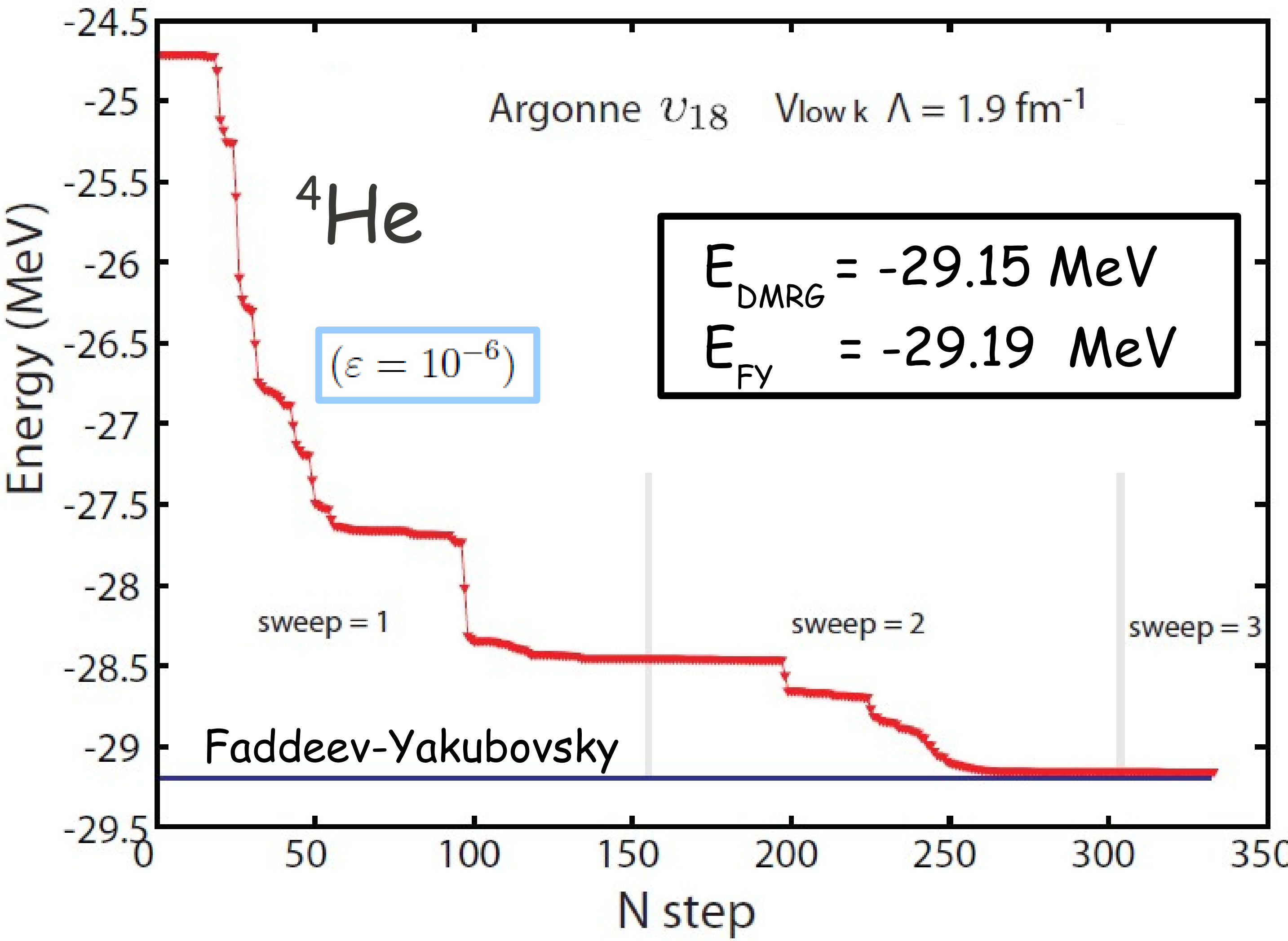


ϵ	dimension	energy (MeV)
$5.0 \cdot 10^{-4}$	99	-6.878
$2.5 \cdot 10^{-4}$	109	-7.396
10^{-4}	173	-7.821
$5.0 \cdot 10^{-5}$	308	-8.042
10^{-5}	600	-8.287
10^{-6}	1075	-8.357
10^{-7}	1909	-8.381
10^{-8}	2575	-8.388





(Faddeev result from Nogga et al, PRC 70 (2004) 061002(R))



i) $s_{1/2}$ $p_{3/2}$ $p_{1/2}$ real-energy HF states

ii) dfg H.O states

* $0s_{1/2}$(p) : E= -24.453 MeV
* $0s_{1/2}$(n) : E= -26.290 MeV

156 Shells

G. Papadimitriou *et al*, arXiv:1301.7140

J-scheme dimension

* Full NCGSM space: 6,230,512
* DMRG ~ 6000

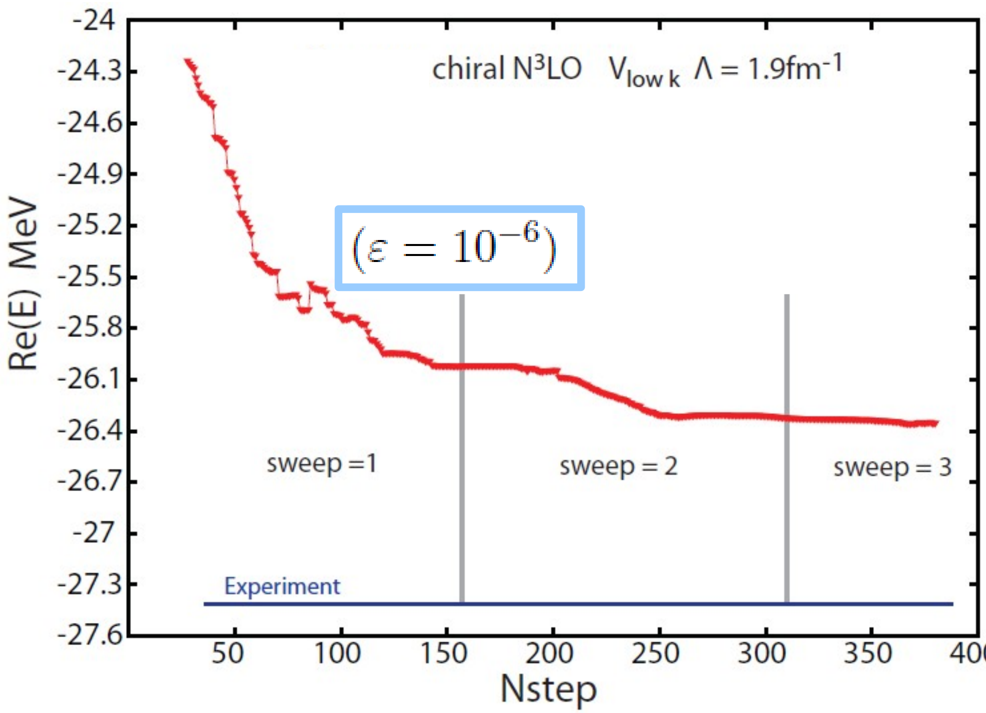
(FY result from Nogga et al, PRC 70 (2004))

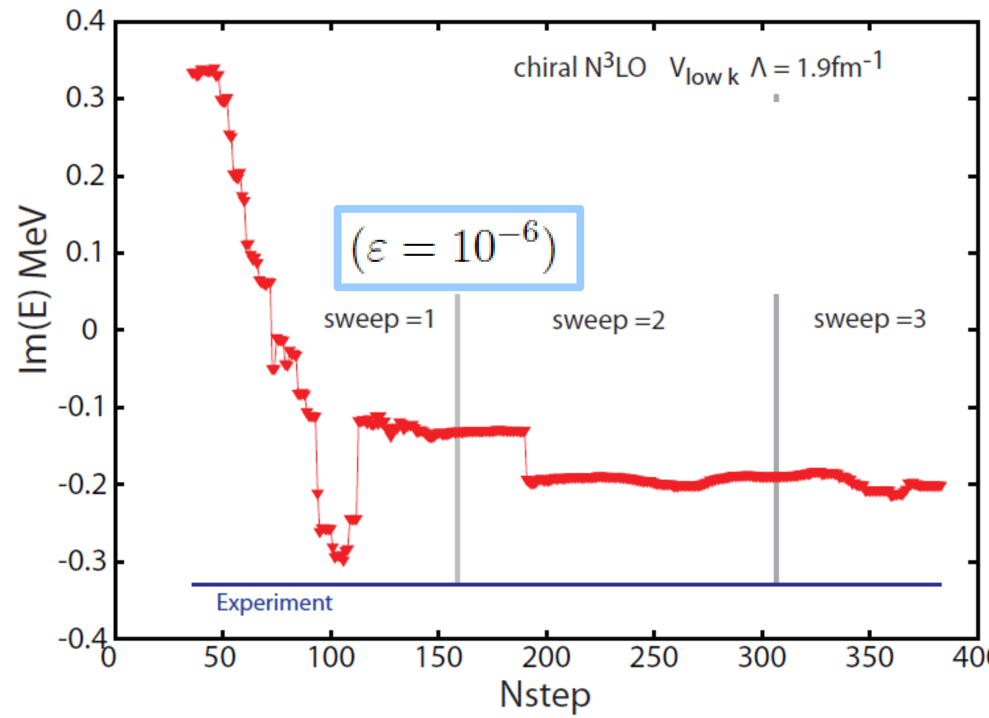
^5He

HF poles

* 0p3/2 (n): E= (1.194,-0.633) MeV
* 0s1/2(p) : E= -23.291 MeV
* 0s1/2(n) : E= -23.999 MeV

Inclusion of $p_{3/2}$ complex-continuum contour for neutron





157 Shells

J-scheme dimension

* Full NCGSM space: 1,379,196,439
* DMRG ~ 1.10^5

DMRG ($\varepsilon = 10^{-6}$)	Coupled Cluster (CCSD)
(-26.31,-0.20)	(-24.87,-0.16)

G. Hagen *et al*, PLB 656 (2007) 169.

Ab initio description of light nuclei in the Berggren basis

- coupling with the continuum states taken into account by expansion in the Berggren basis.
- application of the DMRG technique to the Gamow Shell Model and the No-Core Gamow Shell Model.

In development:

* implementation of truncations "N-particle N-hole" in DMRG
* Monte Carlo technique to select continuum shells

Happy Birthday James !