Nuclear Theory in the Supercomputing Era 2013

Origin and properties of strong inter-nucleon interactions

R. Machleidt, University of Idaho

In Honor of James P. Vary

Outline

- Historical overview
- The divers current status
- The EFT approach
- **Some open issues in chiral EFT:**
- Proper renormalization of chiral forces
- Sub-leading many-body forces
- **Conclusions**

Table 1. Eight Decades of Struggle: The Theory of Nuclear Forces

1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions ≡ multi-pion resonances: σ, ρ, ω, ... The One-Boson-Exchange Model
1970's	Refine meson theory: More sophisticated meson-exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Yukawa's Meson Theory!** *But, with Chiral Symmetry*

This was just the history in a nutshell.
The history was, of course, much richer.
Let me show this for the last decade.
It is best to distinguish between

- phenomenological and

- "first principal"

approaches.

Current phenomenological approaches/ models/potentials

- The Moscow potential, Kukulin et al.; hybrid model, short-range 6-quark bag, long-range meson-exchange.
- The high-precision NN potentials
  - Argonne V18 (1995)
  - Nijmegen (1994)
  - CD-Bonn (1996 & 2001)

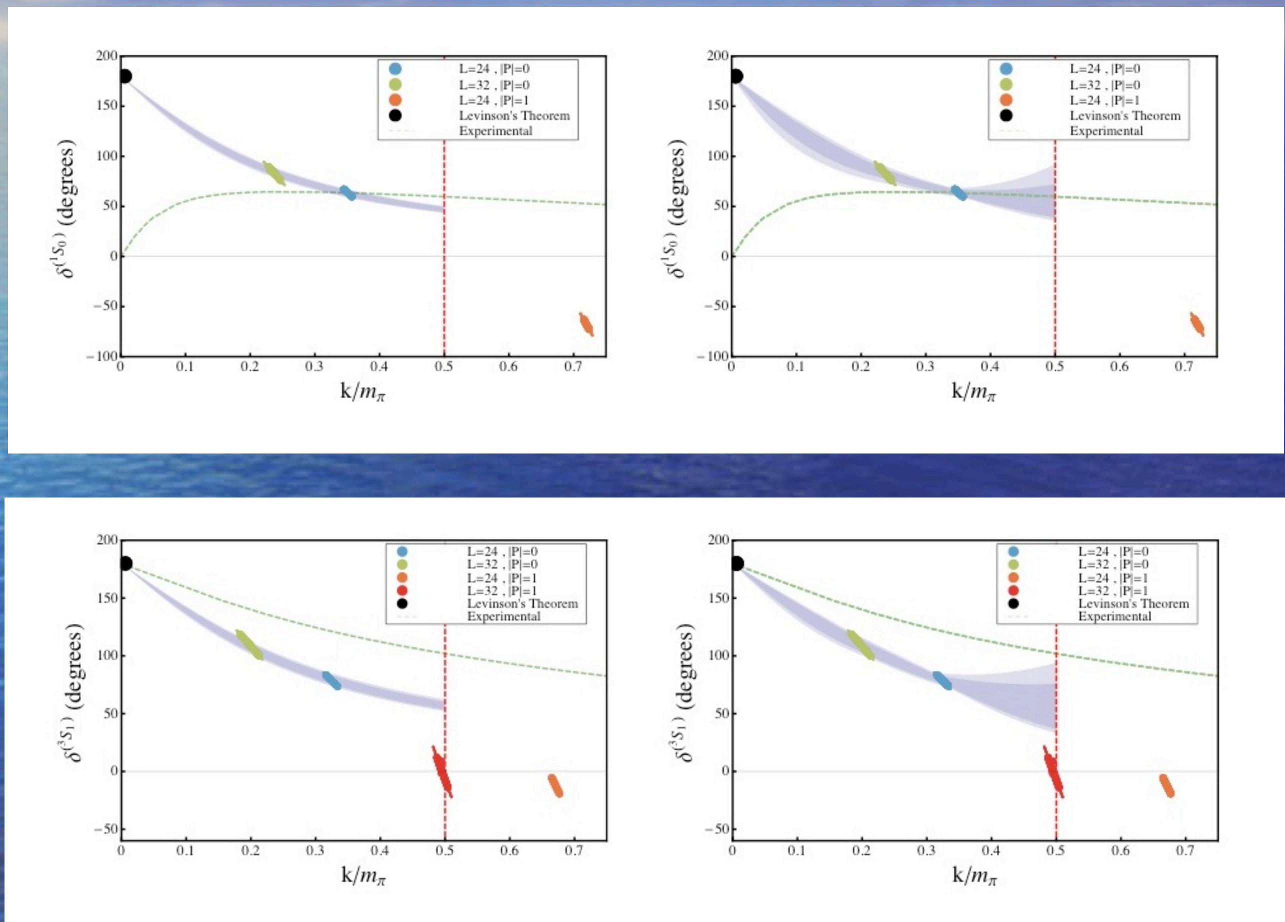
  Meson-theory based, very accurate.
- Phenomenological three-nucleon forces (3NFs):
  - Urbana (1995)
  - Tucson-Melbourne (1975-1999)
  - Illinois (2001-2010)
  - CD-Bonn + Δ (Deltuva, Sauer, 2003)

Phenomenology, cont'd

- Non-local INOY potentials ("inside non-local, outside Yukawa"), Doleschall (2000-2004). The 2NF reproduces triton and alpha energy.
- JISP potentials (J-matrix Inverse Scattering Potential), non-local interaction in the form of a matrix in oscillator space in each partial wave; reproduces not only the NN data also light nuclei up to A=16 ("JISP16") due to variations of the off-shell behavior. No 3NF needed. (Cf. talk by Pieter Maris)

First principal approaches to nuclear forces

- **Lattice QCD**
  - NPLQCD Collaboration, M. Savage et al.



- **Lattice QCD, cont'd**
  - HAL QCD Collaboration, T. Hatsuda et al.

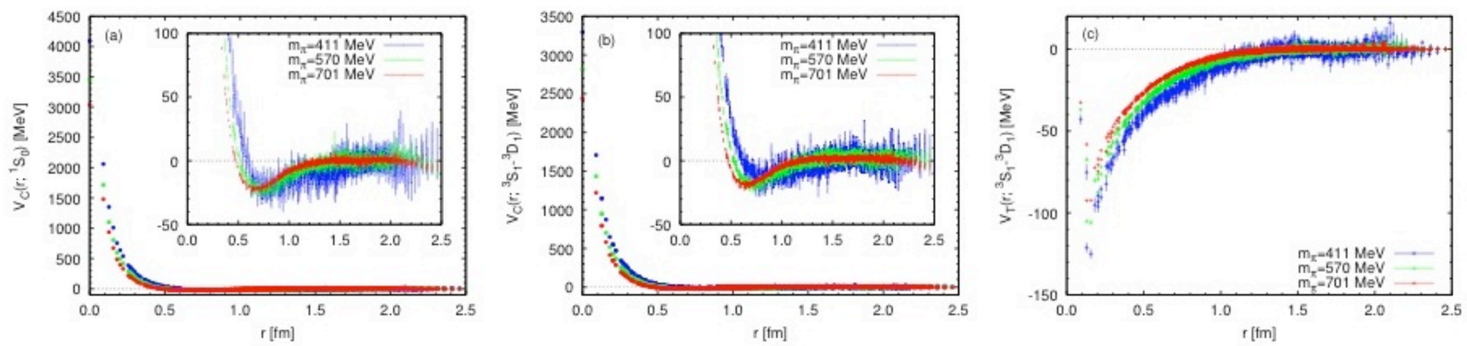


Figure 2: Quark mass dependence of the LO potentials in (2+1)-flavor QCD. (a) The central potential in the spin-singlet channel, (b) the central potential in the spin-triplet channel, and (c) the tensor potential in the spin-triplet channel [22].

• Lattice QCD, cont'd
- HAL QCD Collaboration, T. Hatsuda et al.

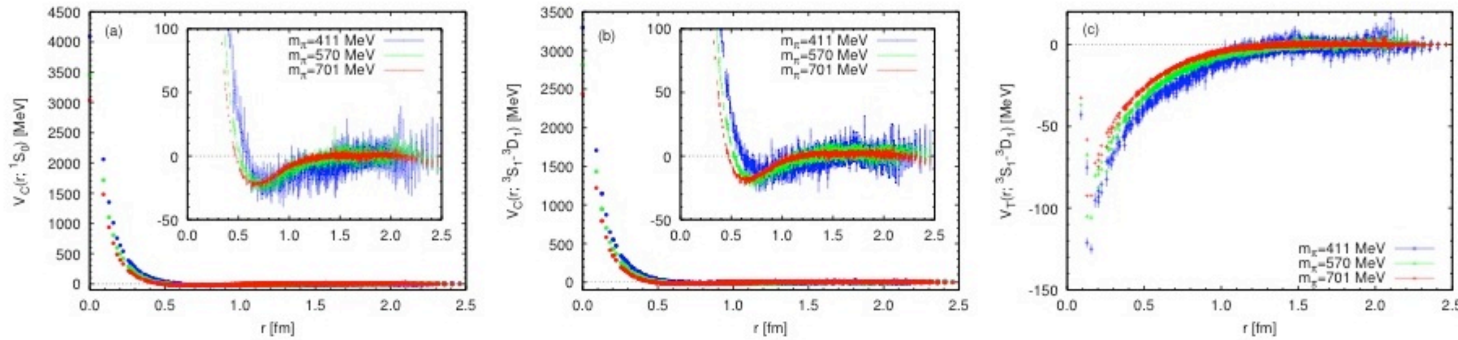


Figure 2: (potential in
the spin-si   r potential
in the spin-

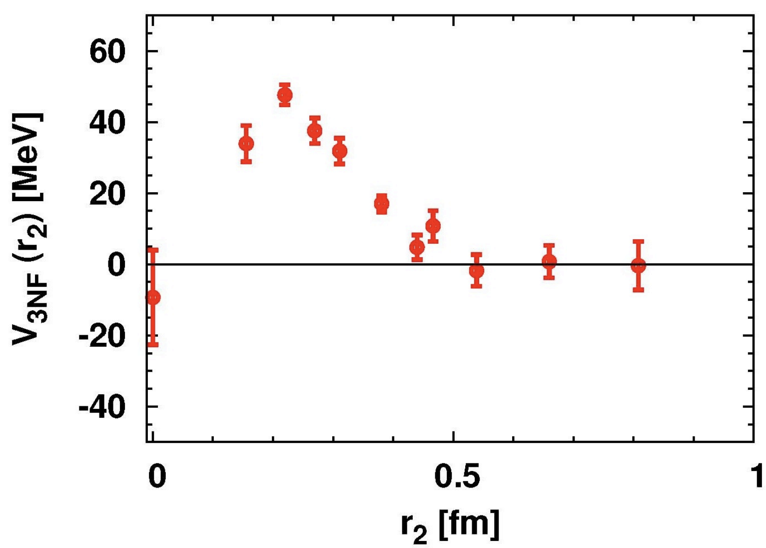


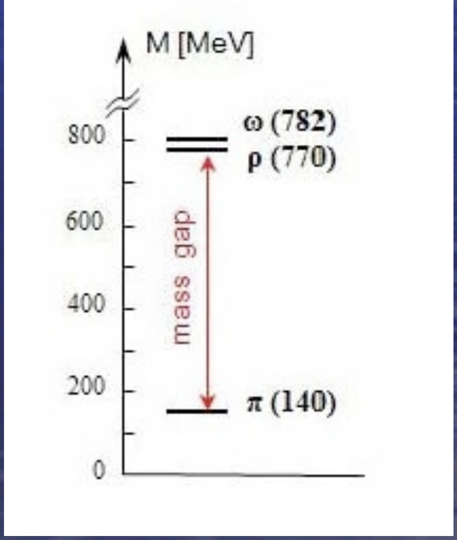
Fig. 3. The effective scalar-isoscalar 3NF in the triton channel with the linear setup obtained at $(t - t_0)/a = 8$.

First Principal approaches, cont'd

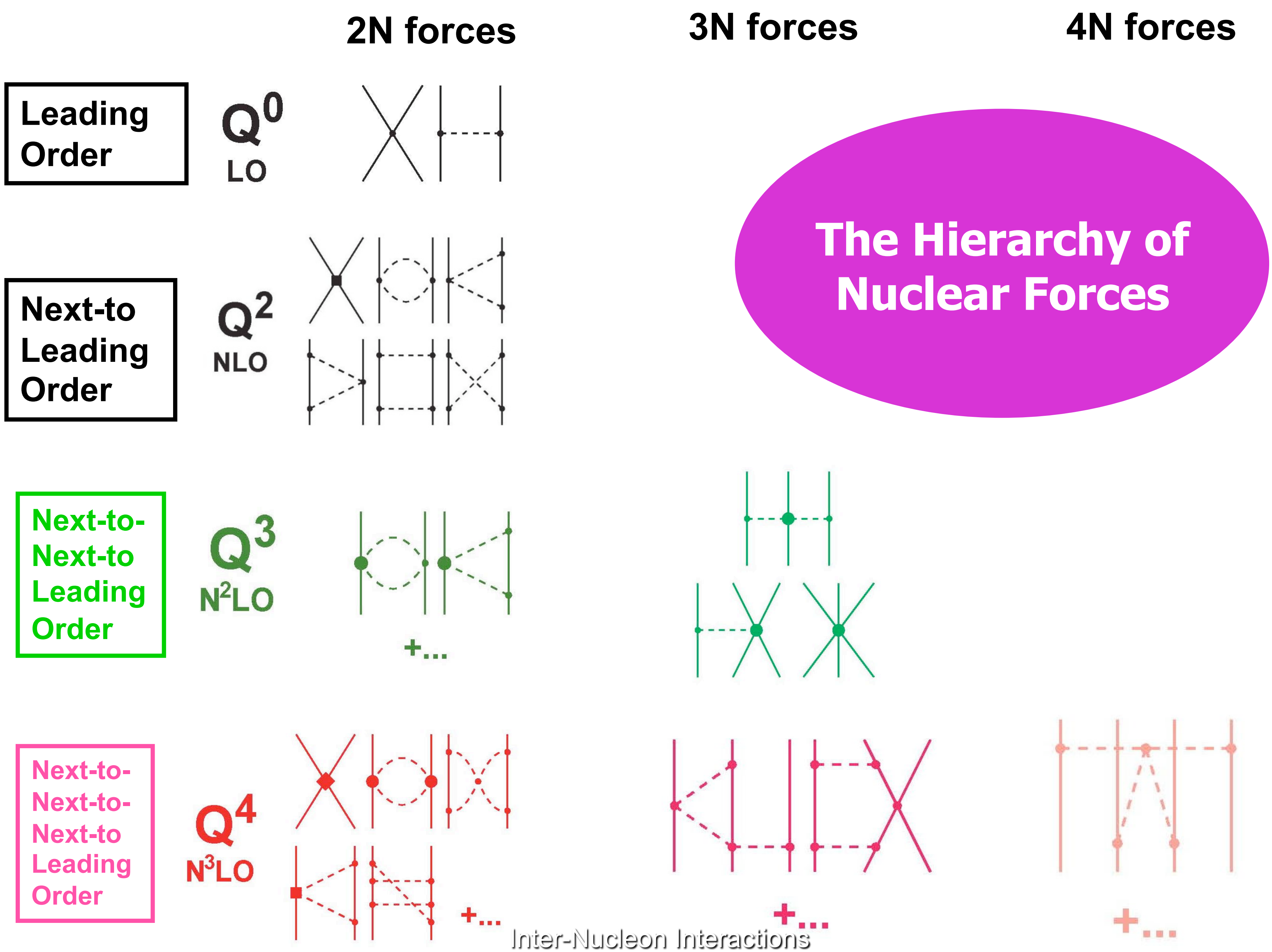
- Effective Field Theory (EFT) approaches (two- and many-body forces)
  - pion-less
  - pion-full
    - * Δ-less
    - * Δ-full

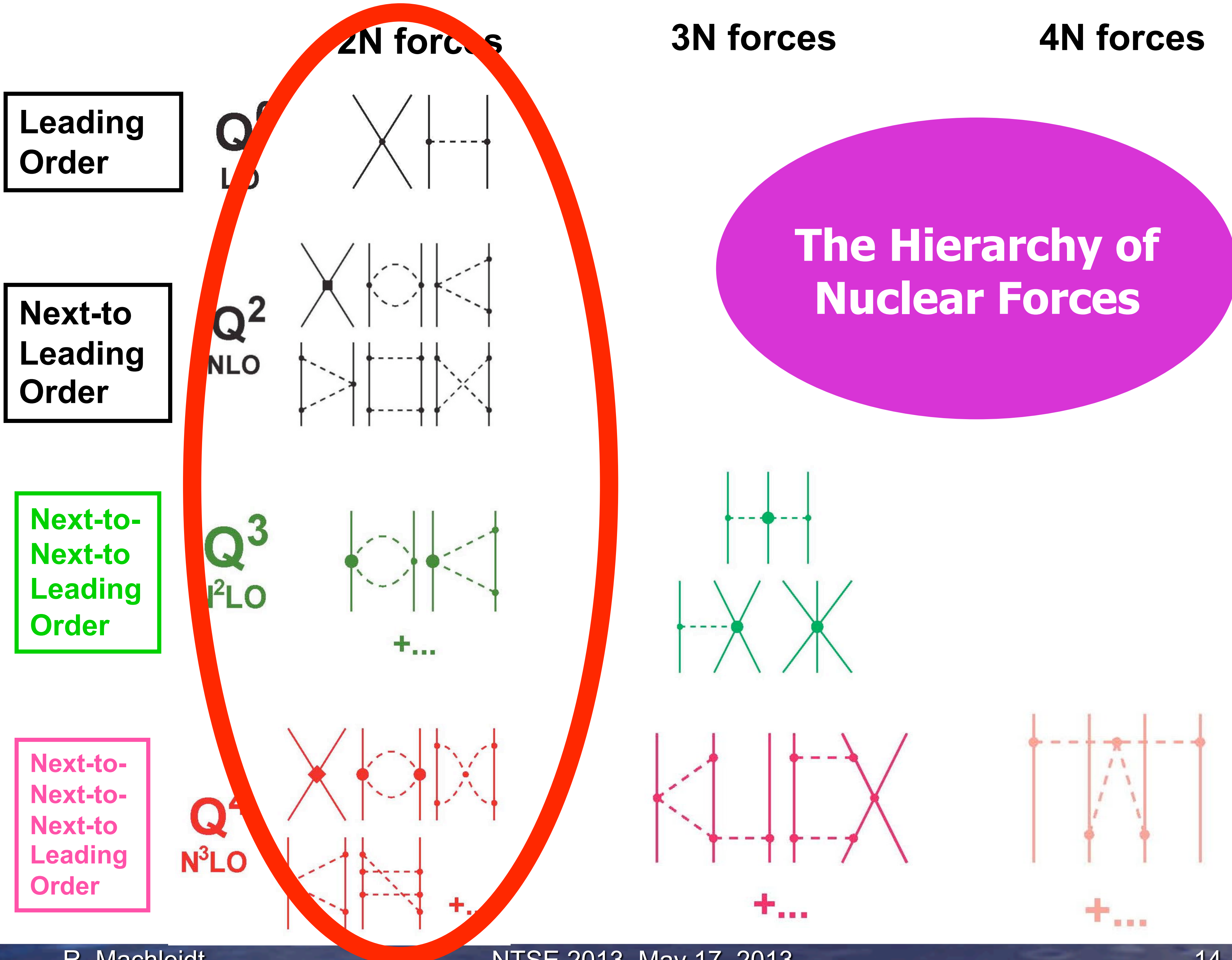
The chiral EFT approach

- QCD at low energy is strong.
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual forces (similar to van der Waals forces)
- Separation of scales



- **Calls for an EFT:**
  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$;
  pions and nucleons are relevant d.o.f.
- **Low-momentum expansion: $(Q/\Lambda_\chi)^\nu$**
  with ν bounded from below.
- **Most general Lagrangian consistent with all symmetries of low-energy QCD, particularly, chiral symmetry which is spontaneously broken.**
- **Weakly interacting Goldstone bosons = pions.**
- **π-π and π-N perturbatively**
- **NN has bound states:**
  (i) NN potential perturbatively
  (ii) apply nonpert. in LS equation.
  (Weinberg)


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...
The Hierarchy of Nuclear Forces


3N forces
4N forces
Leading Order
Next-to Leading Order
Next-to-Next-to Leading Order
Next-to-Next-to-Next-to Leading Order
Q²
NLO
Q³
N³LO
+...
+...
+...
The Hierarchy of Nuclear Forces

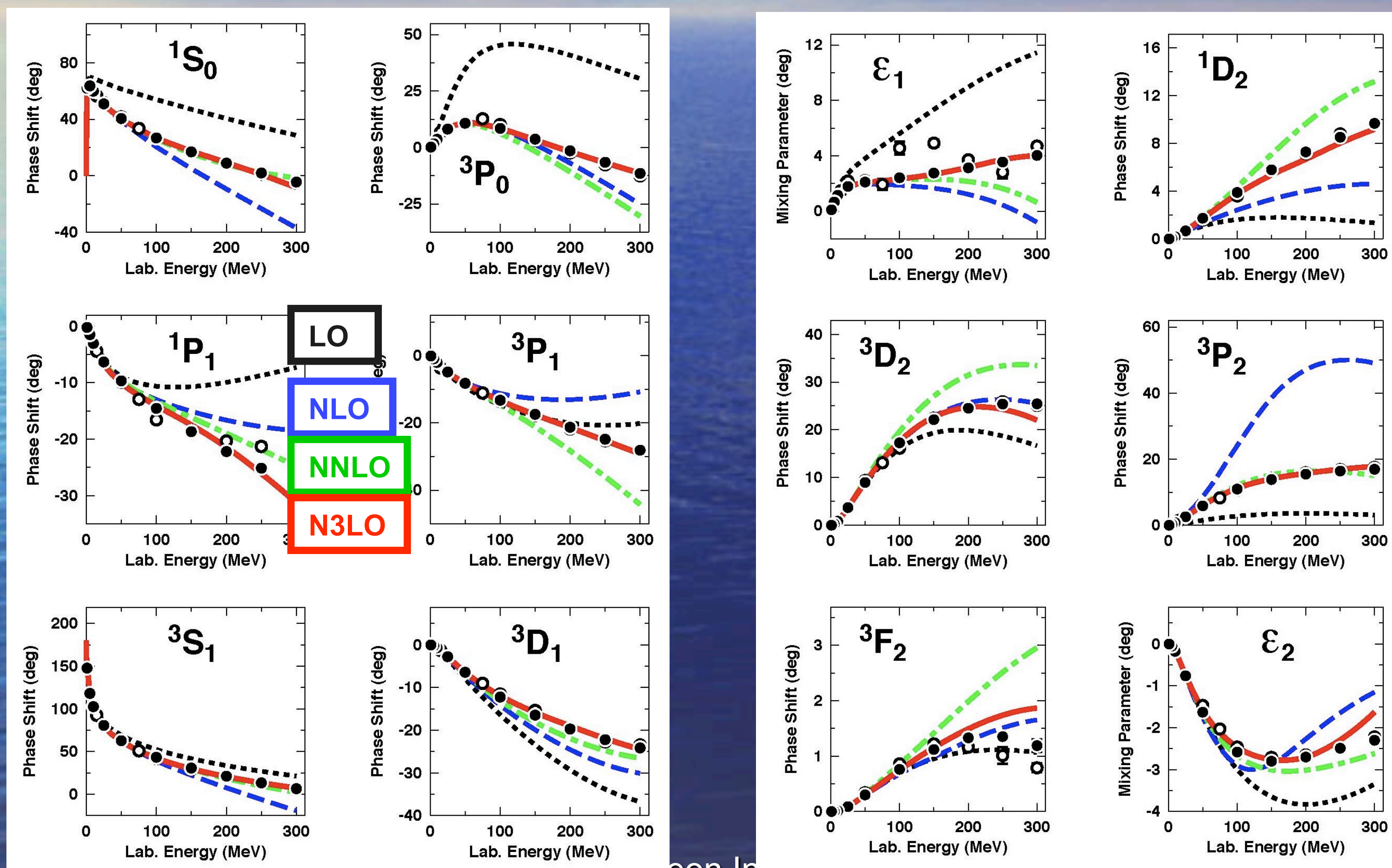
NN phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).

Green dash-dotted line: NNLO Potential, and

blue dashed line: NLO Potential

by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).



χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
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Optimized Chiral Nucleon-Nucleon Interaction at Next-to-Next-to-Leading Order

A. Ekström,[1,2] G. Baardsen,[1] C. Forssén,[3] G. Hagen,[4,5] M. Hjorth-Jensen,[1,2,6] G. R. Jansen,[4,5] R. Machleidt,[7] W. Nazarewicz,[5,4,8] T. Papenbrock,[5,4] J. Sarich,[9] and S. M. Wild[9]

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.00	4.5	0.95
100–190	501	1.08	1.87	100	1.10
190–290	843	1.15	6.09	180	1.11
0–290	2402	1.10	2.95	86	1.04

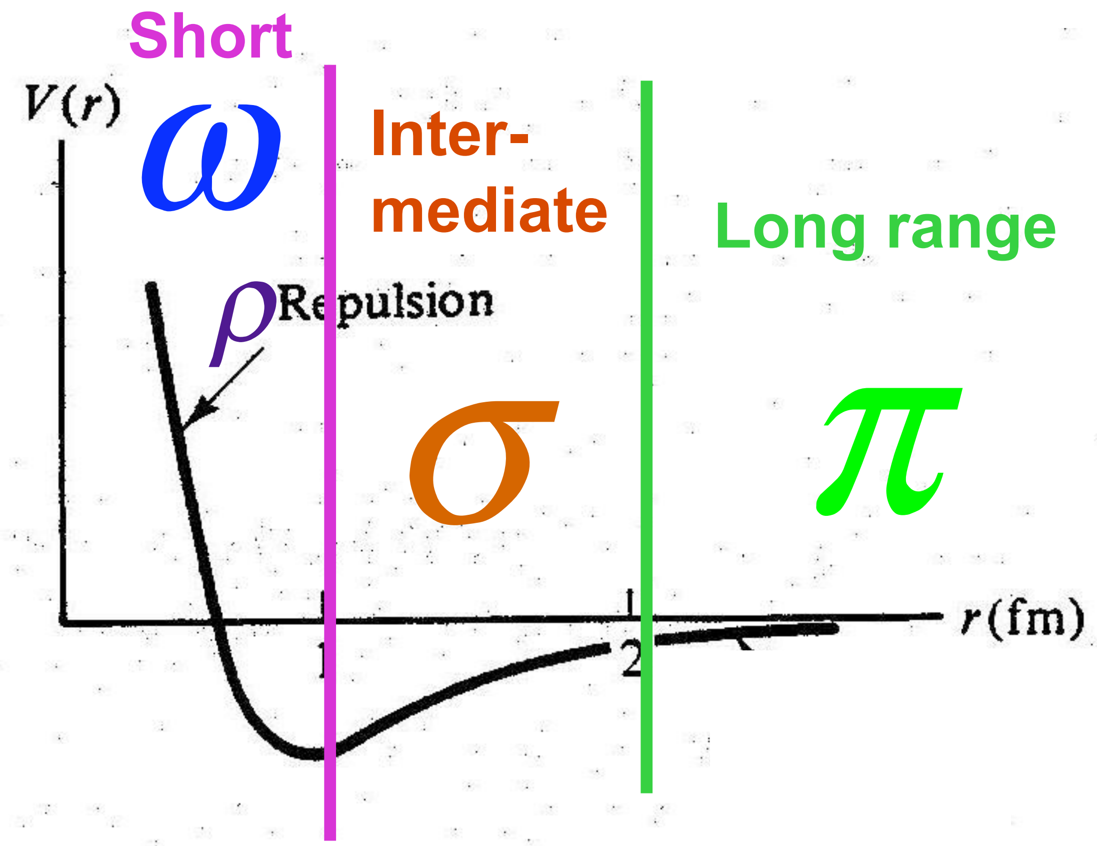
N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

How does this compare to conventional meson theory?

Main differences

- **Chiral perturbation theory** (ChPT) is an expansion in terms of small momenta.
- **Meson theory** is an expansion in terms of ranges (masses).

The nuclear force in the meson picture

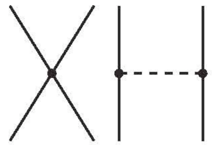


2N forces

Leading Order

Q^0

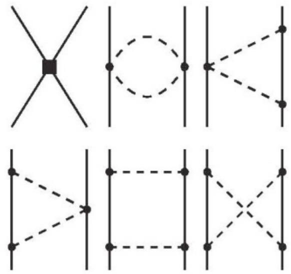
LO



Next-to Leading Order

Q^2

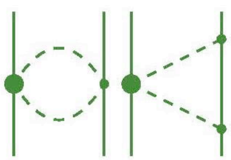
NLO



Next-to-Next-to Leading Order

Q^3

N^2LO

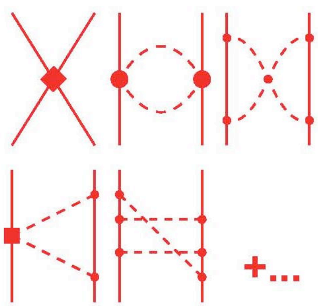


+...

Next-to-Next-to-Next-to Leading Order

Q^4

N^3LO



+...

<u>Question:</u> When everything is so equivalent to conventional meson theory, why not continue to use conventional meson theory?

- **Chiral EFT claims to be a theory, while "meson theory" is a model.**
- **Chiral EFT has a clear connection to QCD, while the QCD-connection of the meson model is more hand-woven.**
- **In ChPT, there is an organizational scheme ("power counting") that allows to estimate the size of the various contributions and the uncertainty at a given order (i.e., the size of the contributions we left out).**
- **Two- and many-body force contributions are generated on an equal footing in ChPT.**

So, chiral EFT wants to be a theory. How true is that?

If EFT wants to be a theory, it better be renormalizable.

The problem in all field theories are divergent loop integrals.

The method to deal with them in field theories:

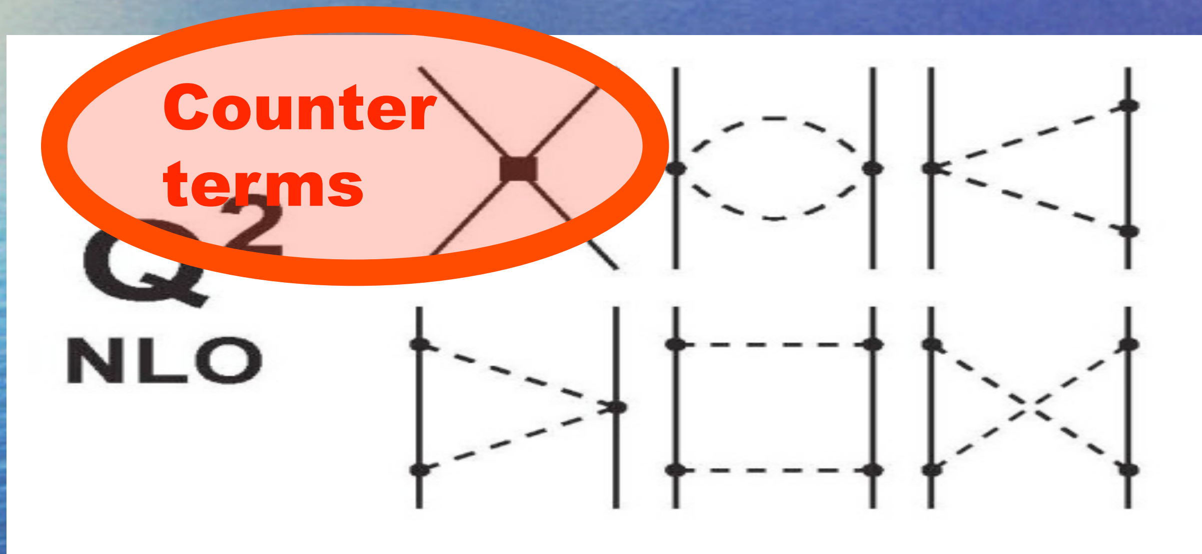
1. **Regularize the integral (e.g. apply a "cutoff") to make it finite.**
2. **Remove the cutoff dependence by Renormalization ("counter terms").**

For calculating pi-pi and pi-N reactions no problem.

However, the NN case is tougher, because it involves two kinds of (divergent) loop integrals.

The first kind:

- "NN Potential":
  irreducible diagrams calculated perturbatively up to a fixed order. Example:



➢ **perturbative renormalization**
(order by order)

The first kind:

- "NN Potential":

  irreducible diagrams calculated perturbatively up to a fixed order. Example:

This is fine. No problems.

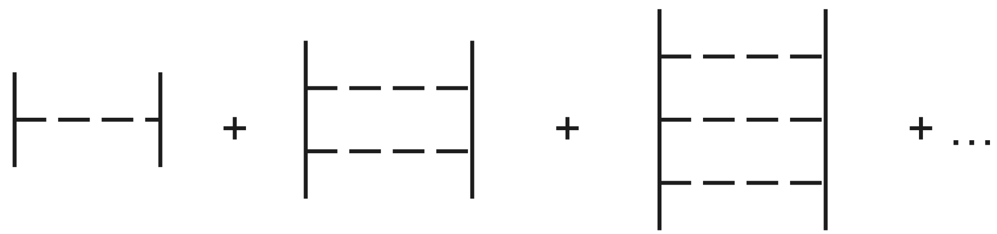
➢ **perturbative renormalization**
(order by order)

The second kind:

- Application of the NN Pot. in the Schrodinger or Lippmann-Schwinger (LS) equation: non-perturbative summation of ladder diagrams (infinite sum):

$$T(\vec{p}\,', \vec{p}) = V(\vec{p}\,', \vec{p}) + \int d^3p''\, V(\vec{p}\,', \vec{p}\,'')\, \frac{M_N}{p^2 - p''^2 + i\epsilon}\, T(\vec{p}\,'', \vec{p}) \,,$$

In diagrams: $T =$



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- Divergent integral.
- Regularize it:

$$V(\vec{p}\,',\vec{p}) \longmapsto V(\vec{p}\,',\vec{p})\, e^{-(p'/\Lambda)^{2n}}\, e^{-(p/\Lambda)^{2n}} \,.$$

- Cutoff dependent results.
- Renormalize to get rid of the cutoff dependence:

➢**Non-perturbative renormalization**

Goal: Find "cutoff independence" for a certain finite range below 1 GeV.

Very recently, a systematic investigation of this kind has been conducted by us at NLO and NNLO using Weinberg Counting, i.e.

2 contacts in each S-wave (used to adjust scatt. length and eff. range),

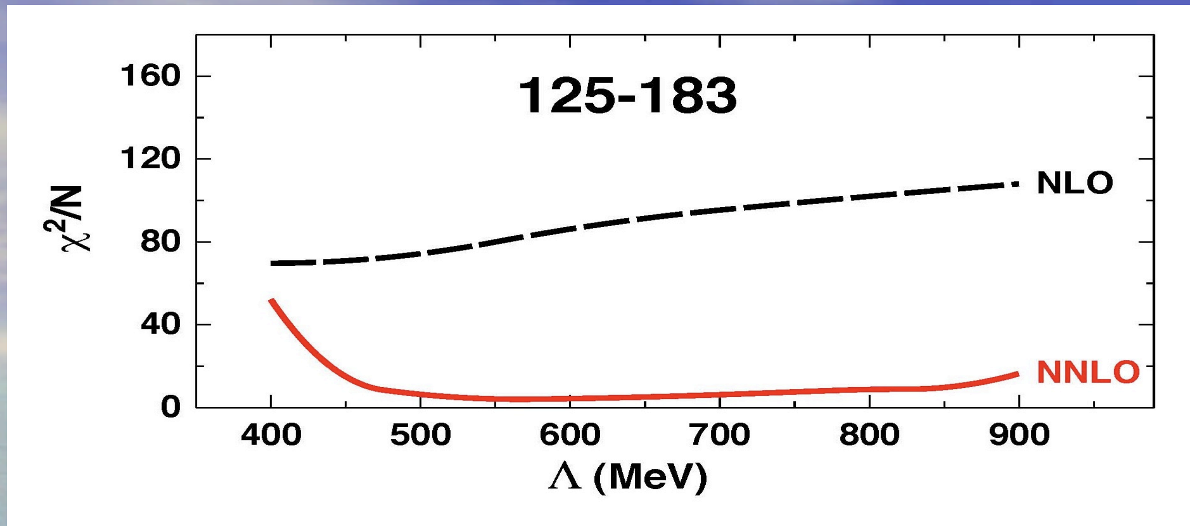
1 contact in each P-wave (used to adjust phase shift at low energy).

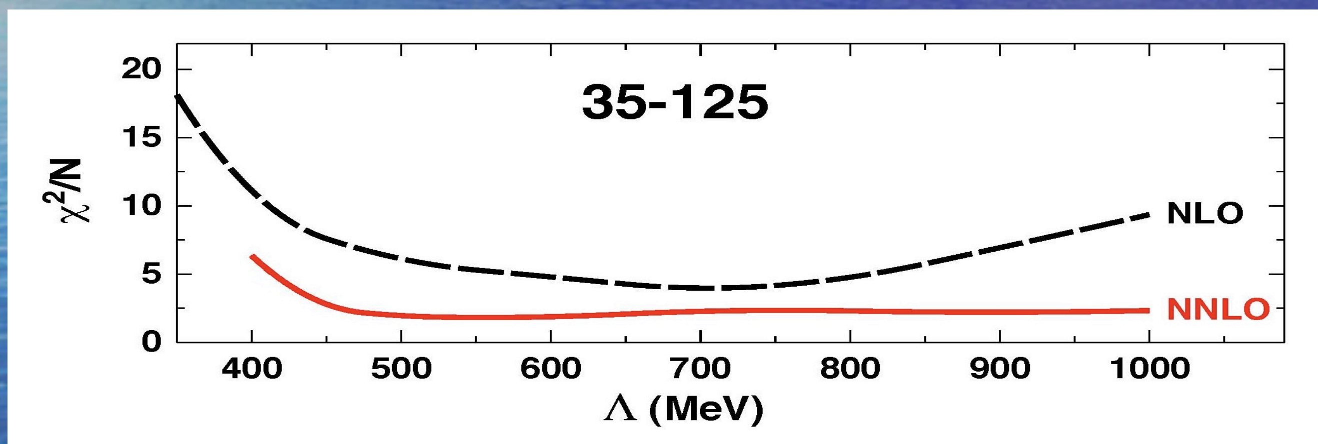
Note that the real thing are DATA (not phase shifts), e.g., NN cross sections, etc. Therefore better: Look for cutoff independence in the description of the data.

Notice, however, that there are many data (about 6000 NN Data below 350 MeV). Therefore, it makes no sense to look at single data sets (observables). Instead, one should calculate

$$\chi^2 = \sum_{i=1}^{i=N} \frac{\left(z_i^{theory} - z_i^{\exp}\right)^2}{\left(\Delta z_i^{\exp}\right)^2}$$

with N the number of NN data in a certain energy range.





The plateaus improve with increasing order.

Renormalization Summary

Non-perturbative reno using finite cutoffs ≤ $\Lambda_\chi \approx 1$ GeV.
For this, we have shown:

Cutoff independence for a certain finite range below 1 GeV (shown for NLO and NNLO).

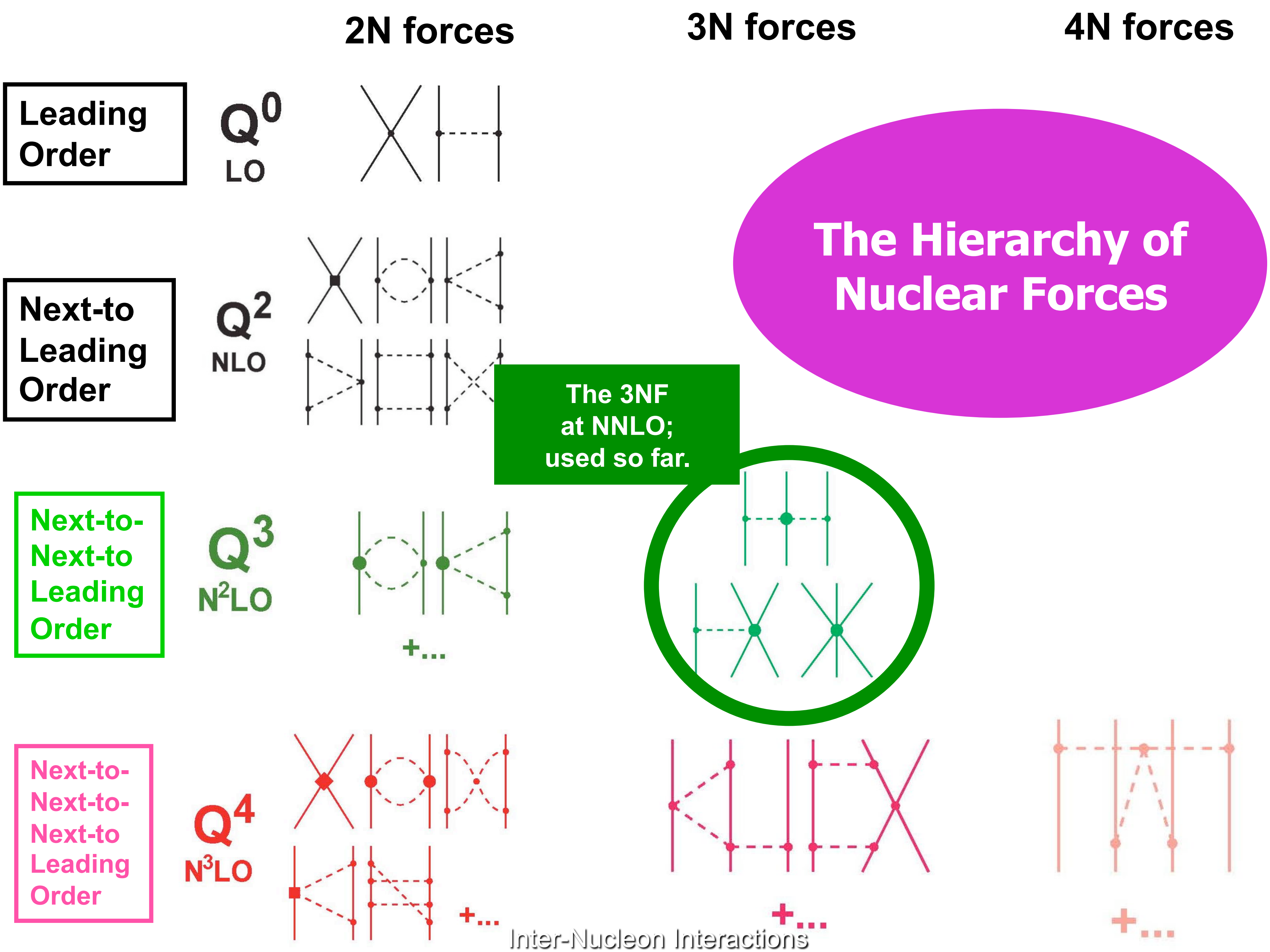
Order-by-order improvement of the predictions.

This is what you want to see in an EFT!

On another topic:

Chiral three-nucleon forces (3NF)

The Hierarchy of Nuclear Forces



Chiral 3N Force

Δ-less

Now, showing only 3NF diagrams.

LO
$(Q/\Lambda_\chi)^0$

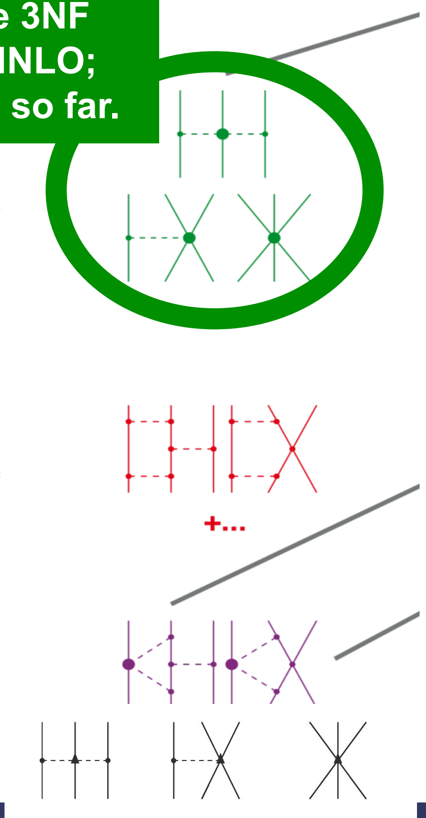
NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$

N³LO
$(Q/\Lambda_\chi)^4$

N⁴LO
$(Q/\Lambda_\chi)^5$



Analyzing Power Ay

Calculations by the Pisa Group

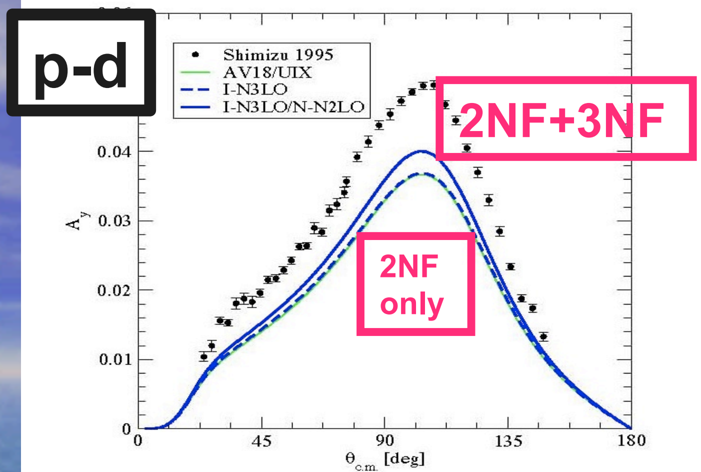


Fig. 9. $p-d$ observable A_y at $E_p = 3$ MeV calculated with the I-N3LO (blue dashed line), I-N3LO/N-N2LO (blue solid line) AV18/UIX (thin green solid line) interaction mod MeV. The experimental data are from Ref

The Ay puzzle is NOT solved by the 3NF at NNLO.

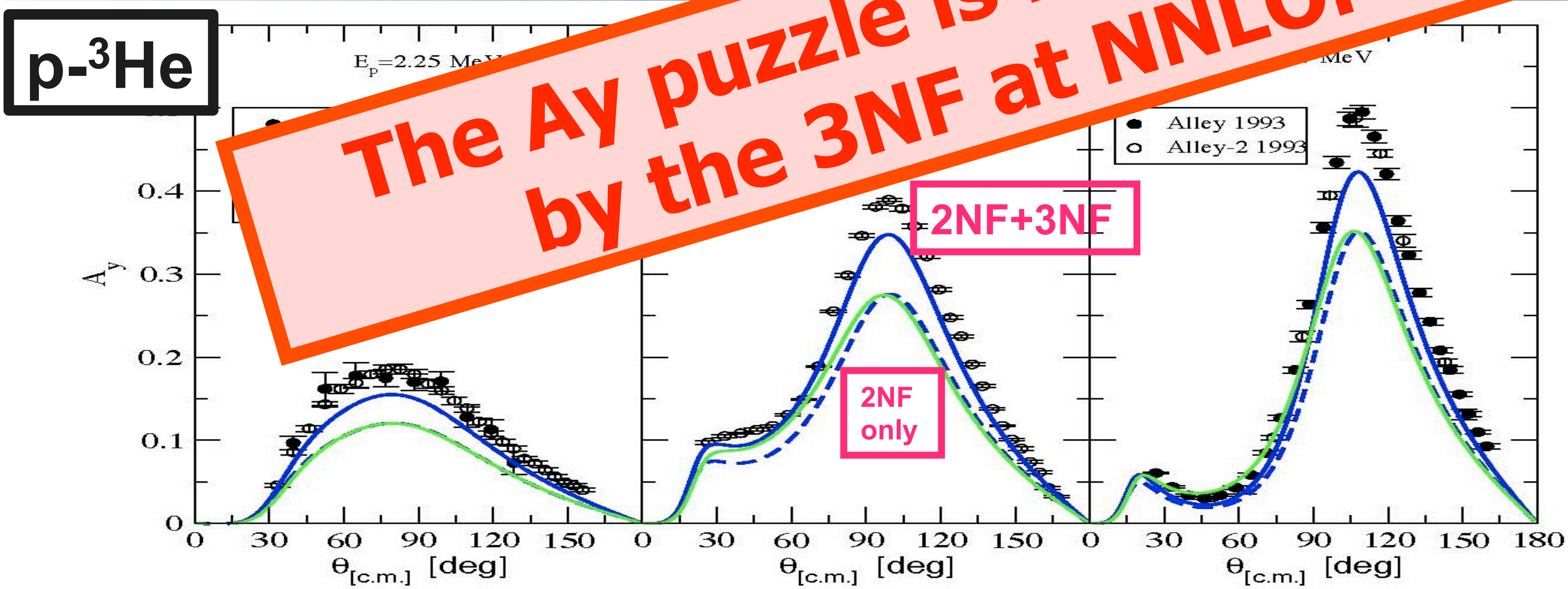


Fig. 6. $p-{}^3$He A_y observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].

And so,
we need 3NFs beyond NNLO, because …

- The 2NF is N3LO;
  consistency requires that all contributions are included up to the same order.

- There are unresolved problems in 3N and 4N scattering, and nuclear structure.

Chiral 3N Force

Back to the drawing board.

Δ-less

LO $(Q/\Lambda_\chi)^0$

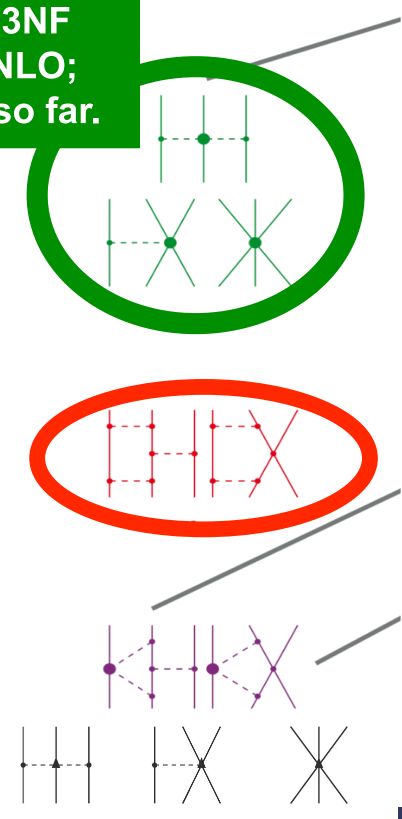
NLO $(Q/\Lambda_\chi)^2$

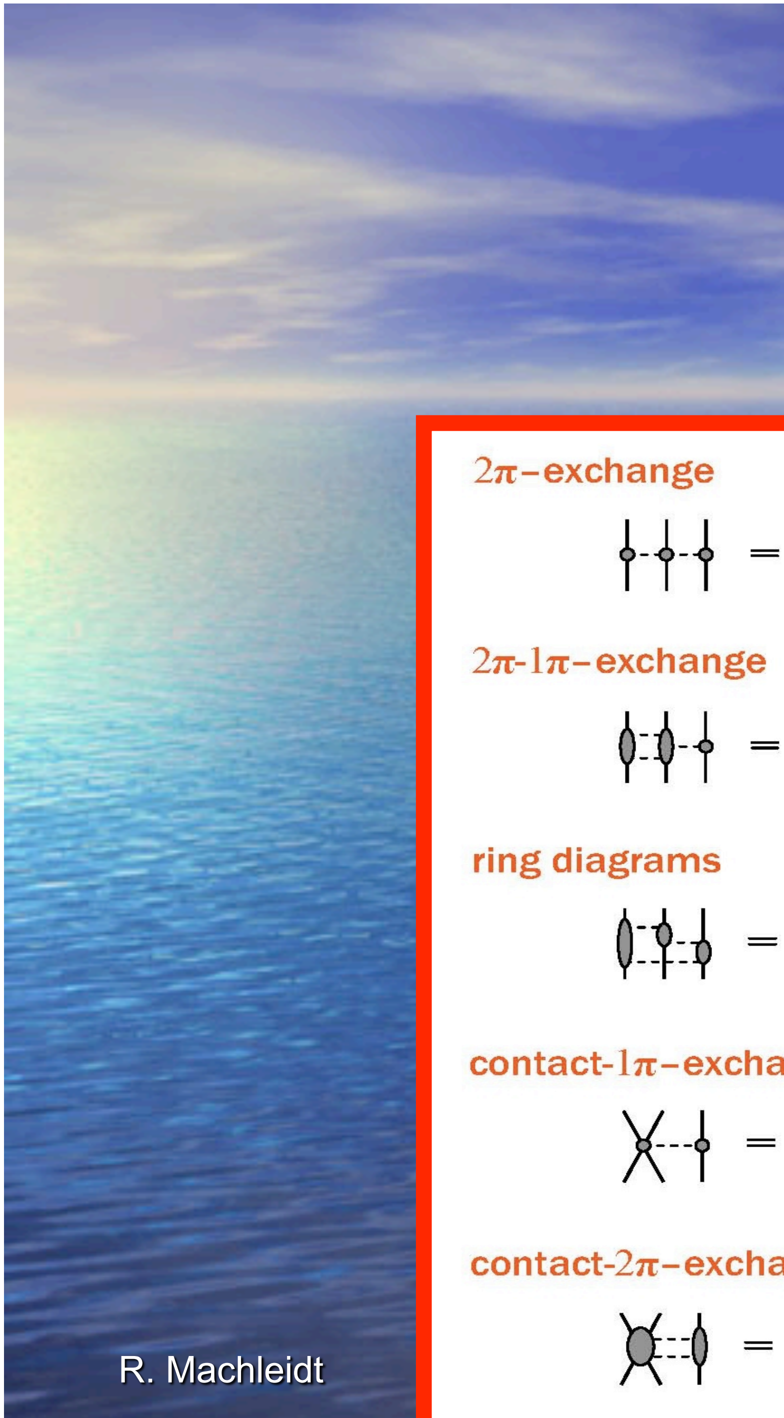
The 3NF at NNLO; used so far.

NNLO $(Q/\Lambda_\chi)^3$

N^{3}LO $(Q/\Lambda_\chi)^4$

N^{4}LO $(Q/\Lambda_\chi)^5$





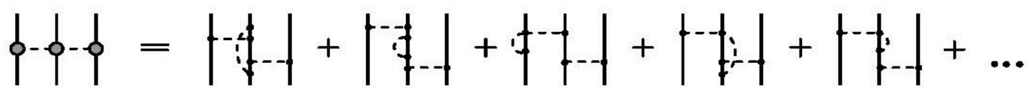
Chiral 3N Force

Δ-less

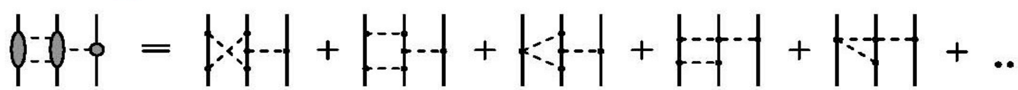
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

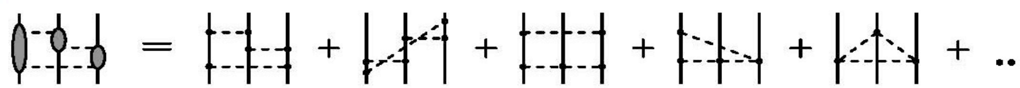
2π−exchange



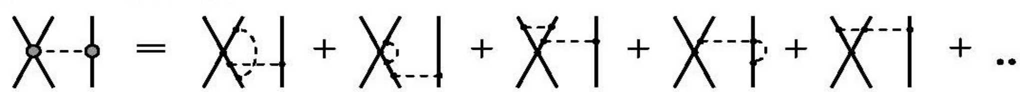
2π-1π−exchange



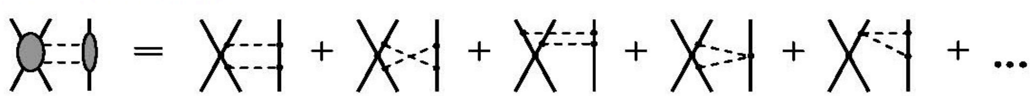
ring diagrams



contact-1π−exchange



contact-2π−exchange

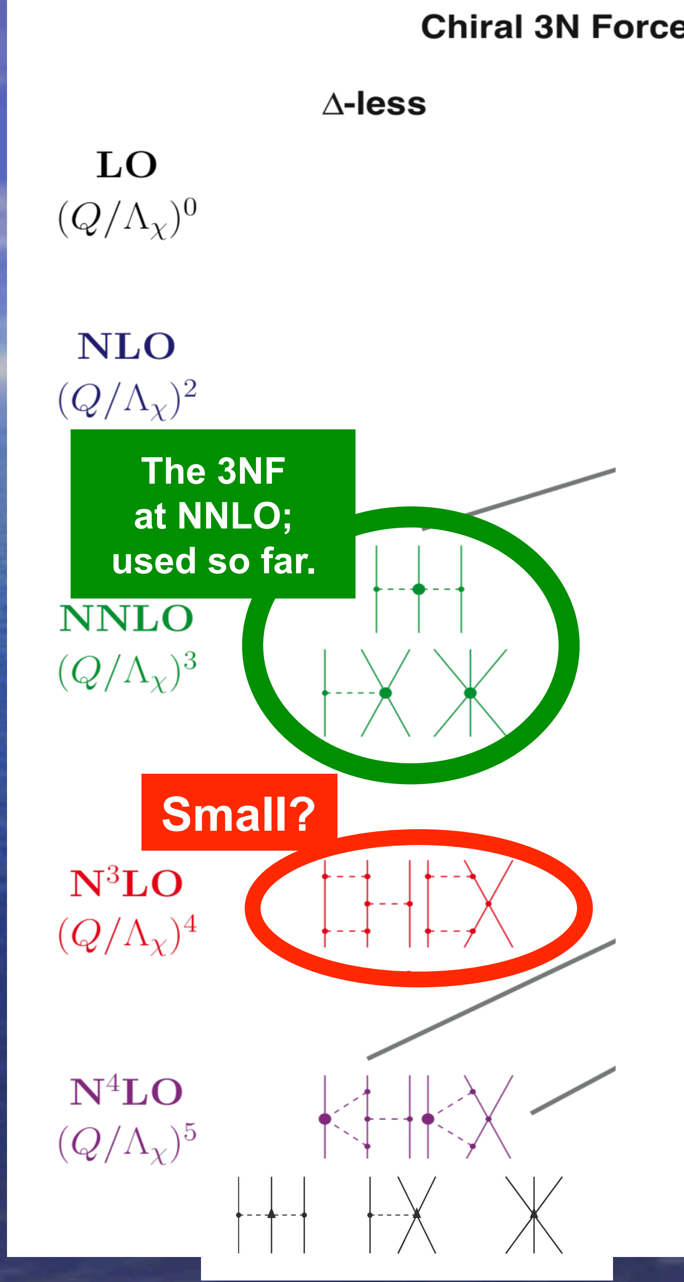


Apps of N3LO 3NF:

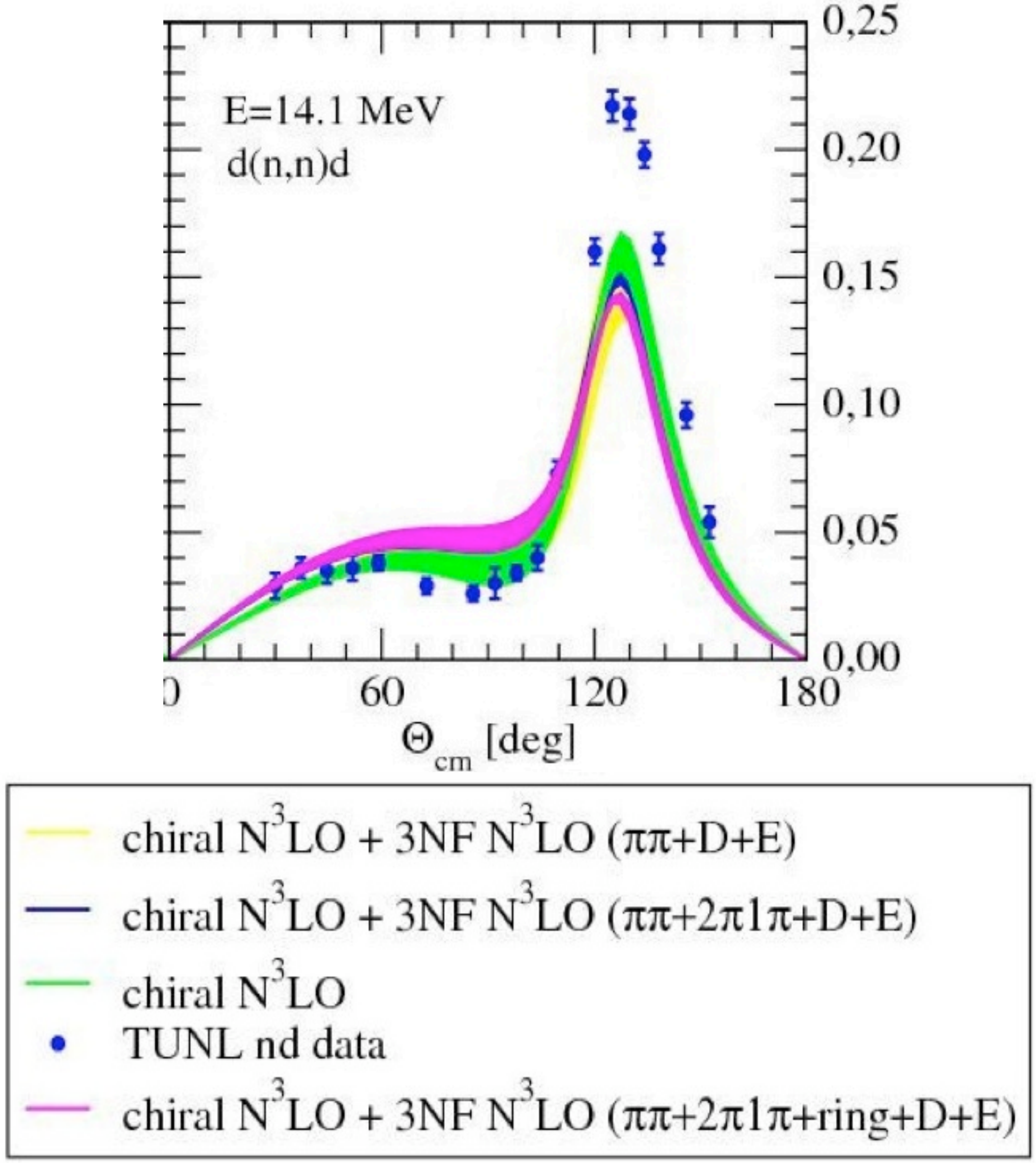
Triton: Skibinski et al., PRC 84, 054005 (2011).
Not conclusive.

Neutron matter:
Hebeler, Schwenk and co-workers, PRL 110, 032504 (2013).
Not small!(?)

N-d scattering (Ay):
Witala et al.
Small!



N-d A_y calculations by Witala et al.



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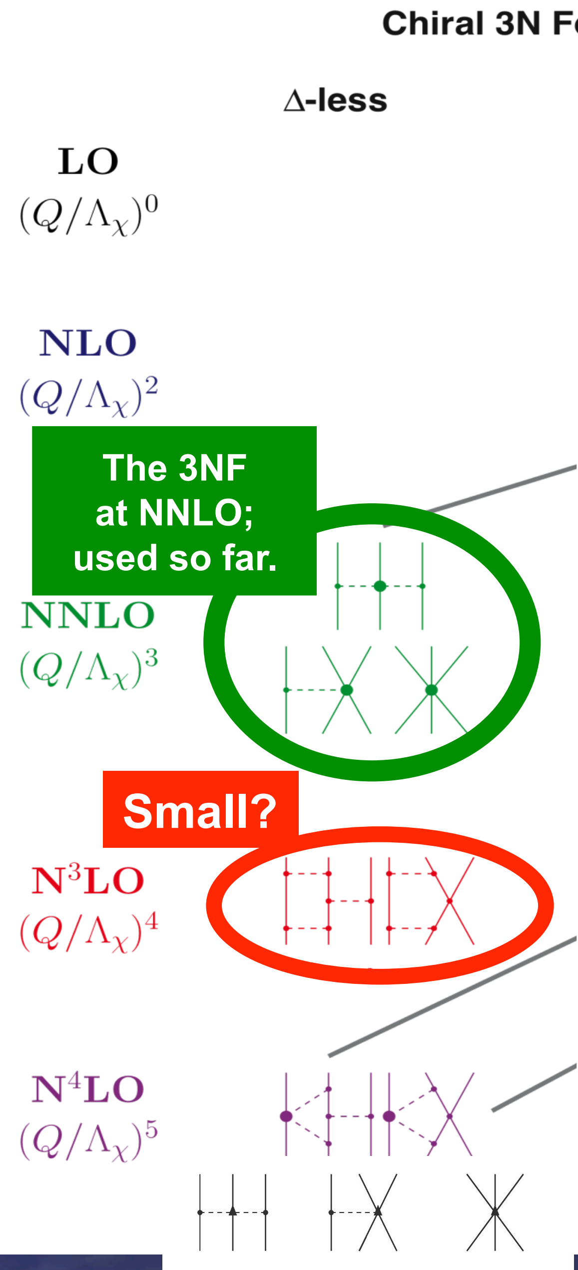
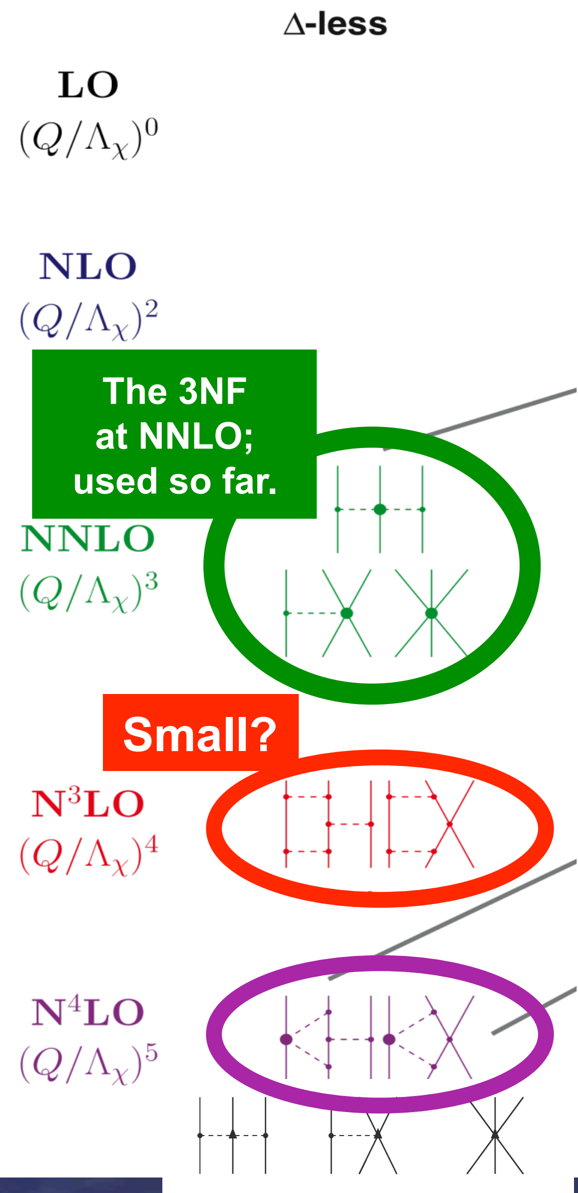


Chiral 3N Force



Chiral 3N Force

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LO
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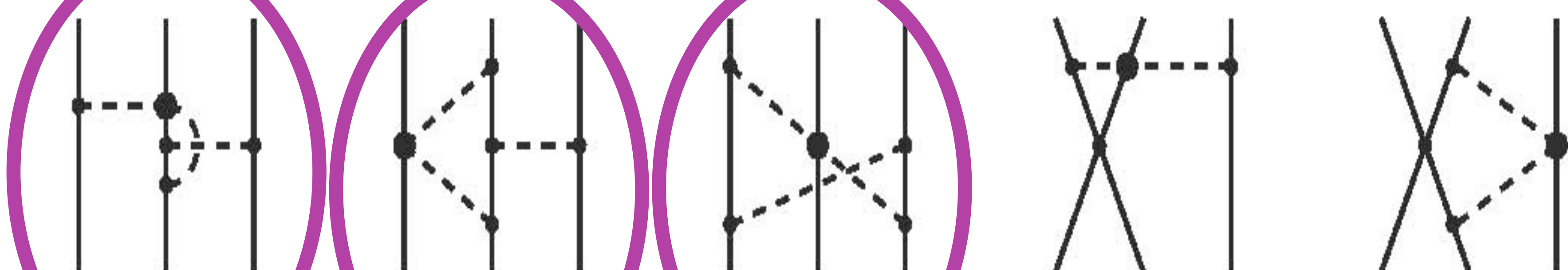
The 3NF
at NNLO;
used so far.

NNLO
$(Q/\Lambda_\chi)^3$

Small?

1-loop graphs: 5 topologies



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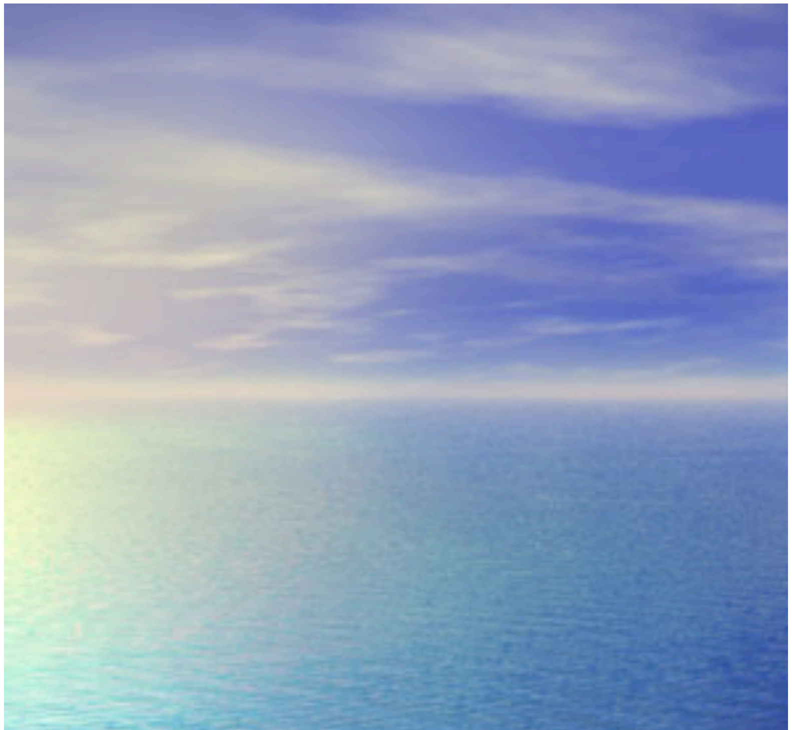

Chiral 3N Force

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The 3NF at NNLO; used so far.

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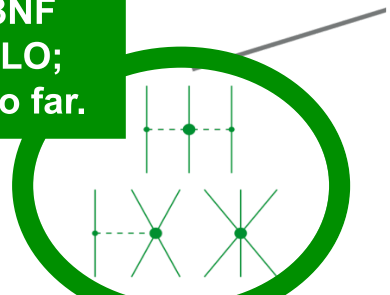

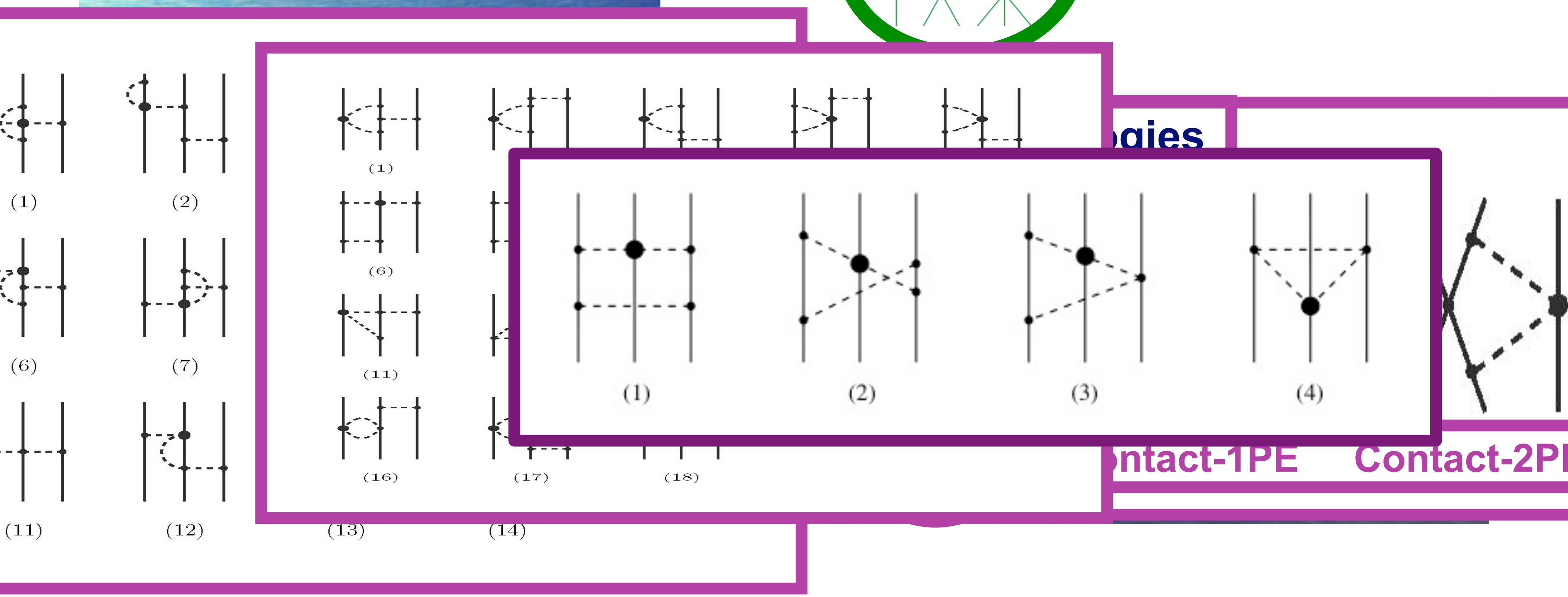

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N^{3}LO
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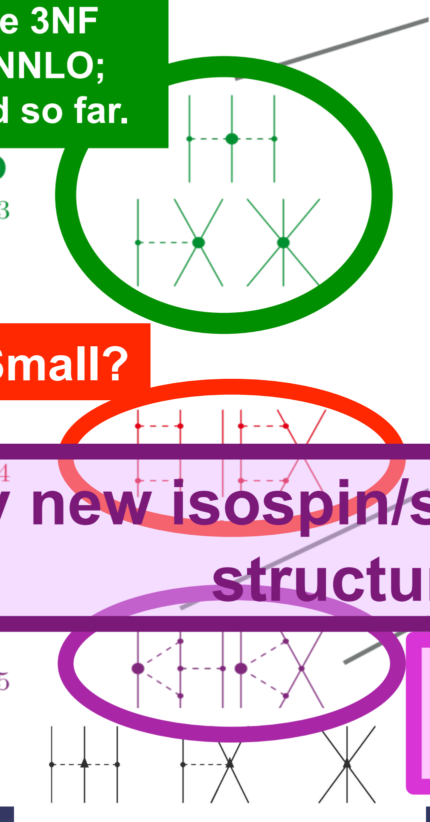


The 3NF at NNLO; used so far.

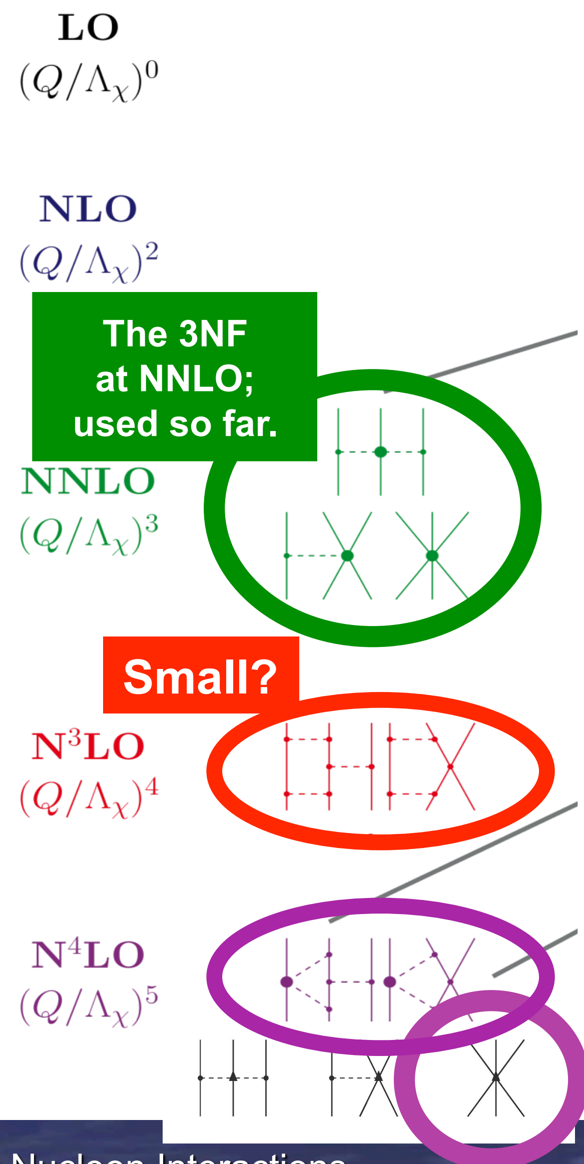
Small?

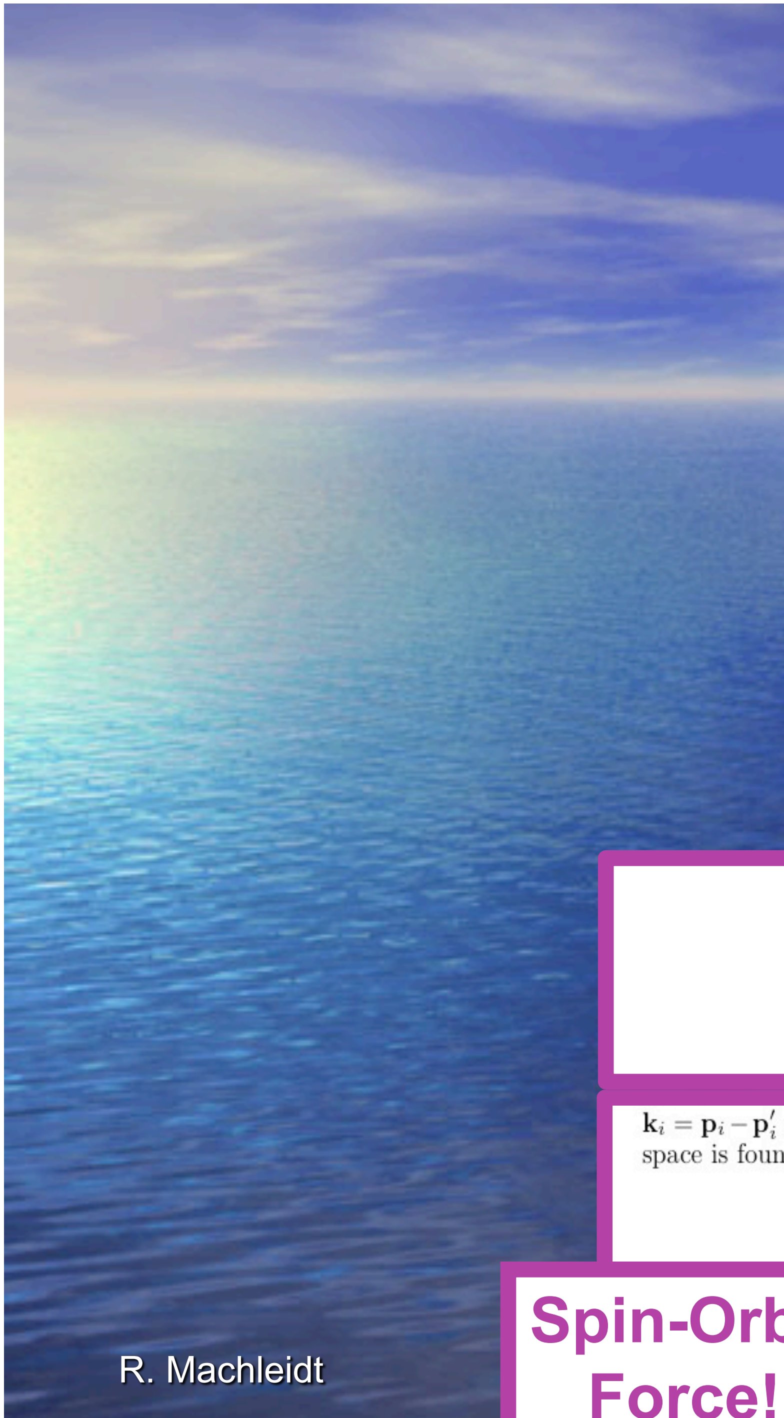
Many new isospin/spin/momentum structures.

Large?!

Chiral 3N Force

Δ-less





Chiral 3N Force

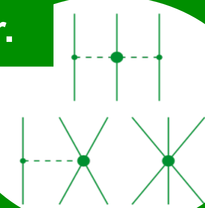
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

The 3NF at NNLO; used so far.



Small?

3NF contacts at N4LO

Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

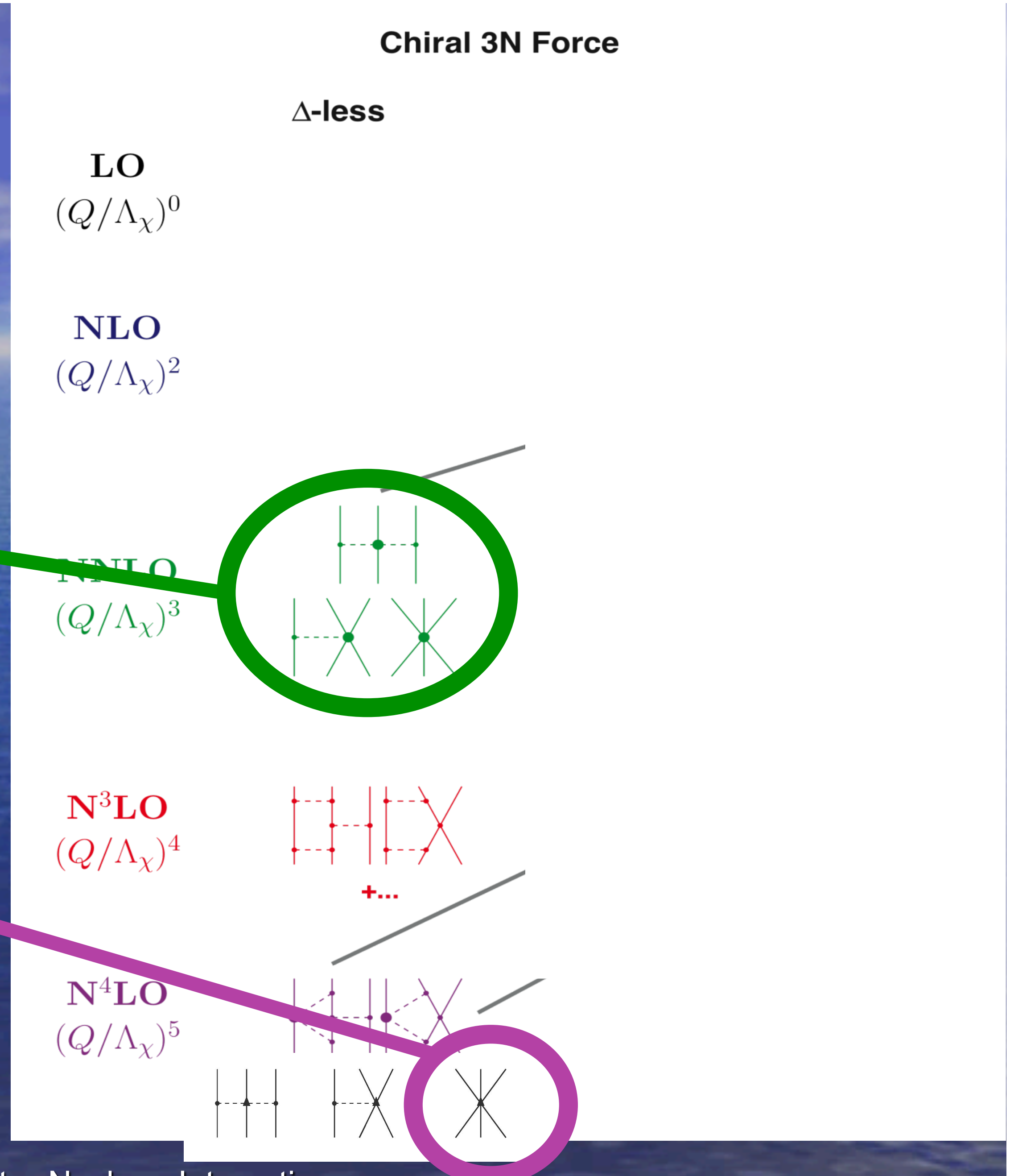
$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i, the potential in momentum space is found to be

$$V = \sum_{i \neq j \neq k} \Big[-E_1 \mathbf{k}_i^2 - E_2 \mathbf{k}_i^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_3 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - E_4 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$
$$- E_5 (3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2) - E_6 (3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$
$$+ \frac{i}{2} E_7 \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) + \frac{i}{2} E_8 \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k$$
$$- E_9 \mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - E_{10} \mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \Big], \quad (15)$$

Spin-Orbit Force!

A realistic, investigational approach:

- use Δ-less
- include NNLO 3NF
- skip N3LO 3NF
- at N4LO start with contact 3NF, use one term at a time, e.g. spin-orbit
- that may already solve some of your problems.



… and then there Is also the Δ-full theory …

Chiral 3N Force

Δ-less

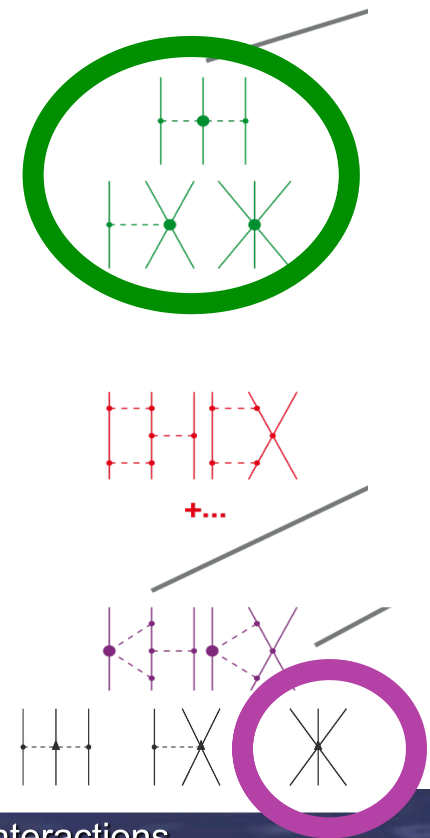
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

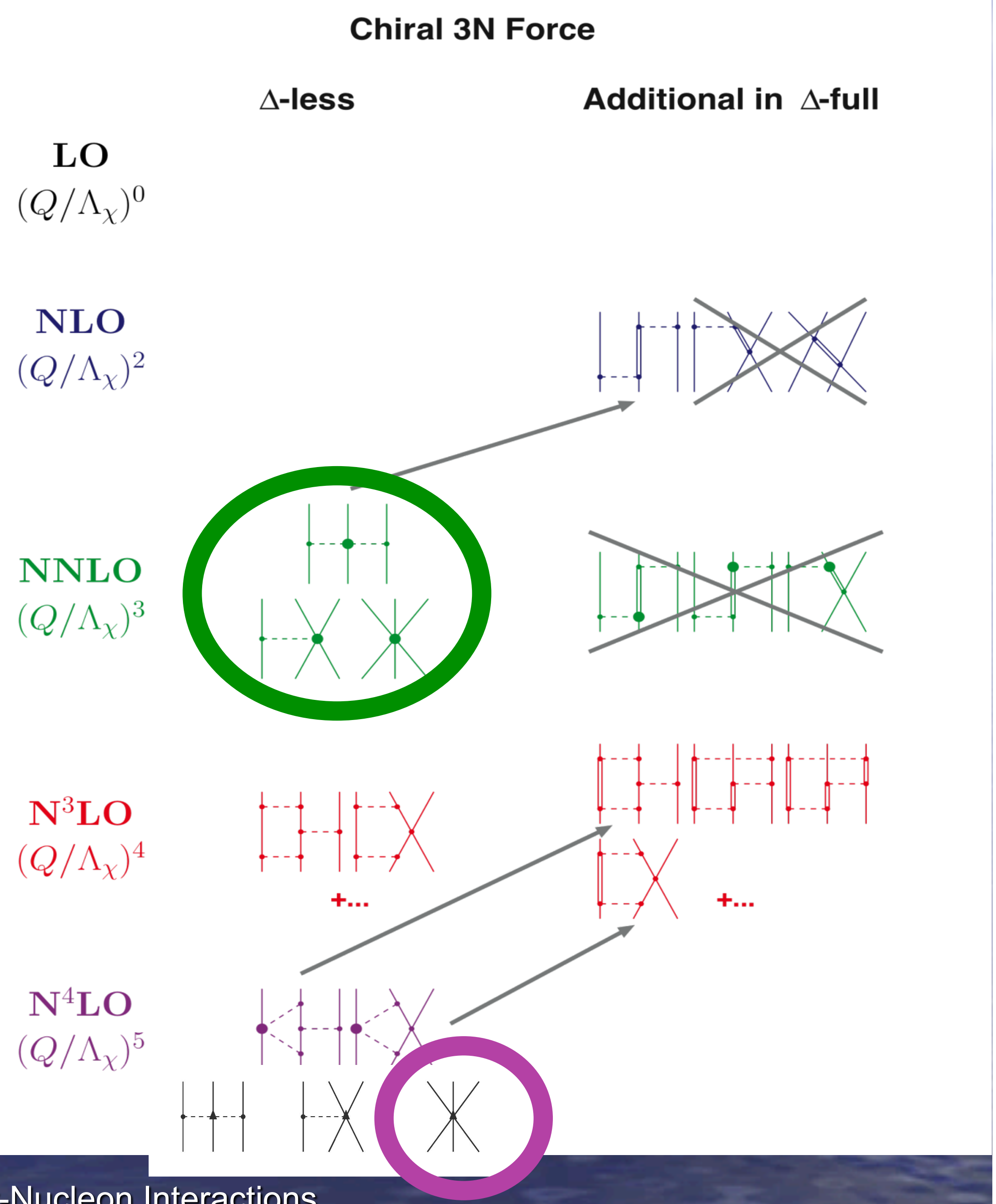
N^3LO
$(Q/\Lambda_\chi)^4$

N^4LO
$(Q/\Lambda_\chi)^5$



… and then there
Is also the
Δ-full theory …

… but we have no
time left for that.



Conclusions

- The research on the nuclear force continues to be exciting and divers.
- Presently, the chiral EFT approach appears to be the most promising one.
- But there are still some not so subtle “subtleties” to be taken care of:
- The renormalization of the chiral 2NF
- Sub-leading 3NFs

The 3NF issue

- The 3NF at NNLO is insufficient.
- The 3NF at N3LO (in the Δ-less theory) may be weak.
- However, large 3NFs with many new structures to be expected at N4LO (of Δ-less). Construction is under way.
- Order by order convergence of the chiral 3NF may be questionable.
- There will be many new 3NFs in the near future. Too many?
- Practitioners: Don't panic! For a while just do what you can do---on an investigational basis.

Happy Birthday, James!

… and continue to always

listen

To Hildegard!