High-Resolution Probes of Low-Resolution Nuclei

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"Nuclear Theory in the Supercomputing Era"

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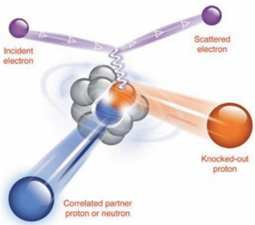
Congratulations, James! (and thanks)

Context:

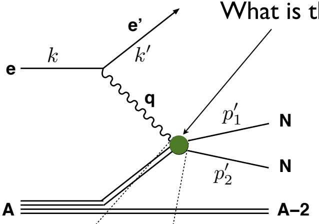
- "New applications of renormalization group methods in nuclear physics," rjf, K. Hebeler, arXiv:1305.3800 (just posted review article)

Prelude: analyzing high resolution experiments



Subedi et al., Science 320, 1476 (2008)

What is this vertex?



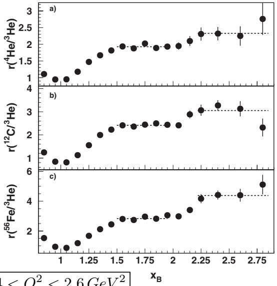
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2m_N \nu}$$

Higinbotham, arXiv:1010.4433



$1.4 < Q^2 < 2.6\, GeV^2$

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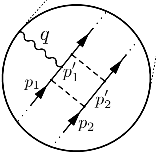

SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

- How can nuclear renormalization group technology be used here?

Why the RG is a good thing [S. Weinberg for F. Low Festschrift]

"The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."

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- Improving perturbation theory; e.g., in QCD calculations
  - Mismatch of energy scales can generate large logarithms
  - RG: shift between couplings and loop integrals to reduce logs
  - Nuclear: decouple high- and low-momentum modes
- Identifying universality in critical phenomena
  - RG: filter out short-distance degrees of freedom
  - Nuclear: evolve toward universal interactions (How?)

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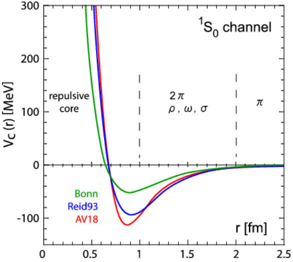
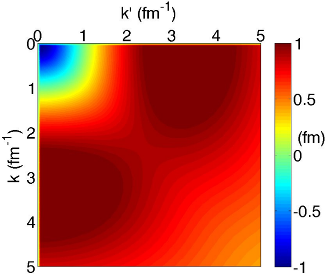
  - RG: filter out short-distance degrees of freedom
  - Nuclear: evolve toward universal interactions (How?)
- Nuclear: simplifying calculations of structure/reactions
  - RG gains can violate conservation of difficulty!
  - Use RG scale (resolution) dependence as a *probe* or *tool*

Lowering the resolution for nuclear structure



AV18, Bonn, Reid93

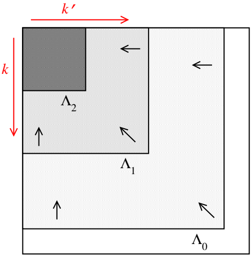


$\langle k | V_{\mathrm{AV18}} | k' \rangle$

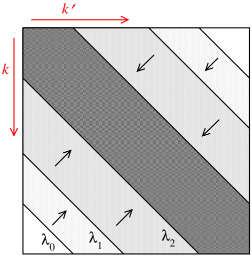
Coupling of low-k/high-k modes: non-perturbative, strong correlations, . . .

Remedy: Use RG to decouple modes $\Longrightarrow$ low resolution

Lowering the resolution for nuclear structure



"$V_{\mathrm{low}\,k}$"



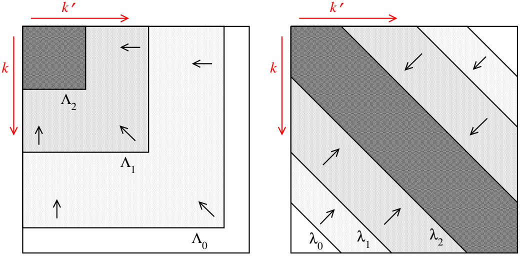
Similarity RG

- $V_{\mathrm{low}\,k}$: lower cutoff Λ_i in k, k' via $dT(k,k';k^2)/d\Lambda = 0$
- SRG: drive H toward diagonal with flow equation $(\lambda = 1/s^{1/4})$

$$dH_s/ds = [[G_s, H_s], H_s]$$

Continuous unitary transforms (cf. running couplings)

Lowering the resolution for nuclear structure

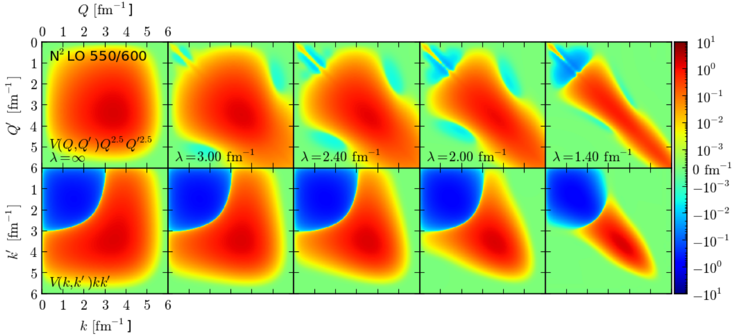


"$V_{\text{low }k}$" Similarity RG

- Evolving three-body force is an essential part (four-body?)
- Three methods now:
  - harmonic oscillator basis
  - momentum pw [K. Hebeler]
  - hyperspherical harmonics momentum [K. Wendt]

NNN

NN



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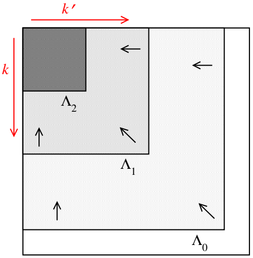

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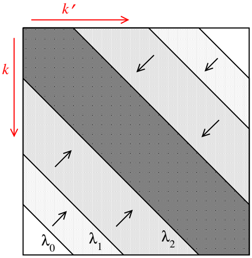
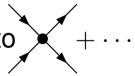


Similarity RG

Where does the physics of the decoupled high-momentum modes go at low momentum?

Claim: goes to



NNN

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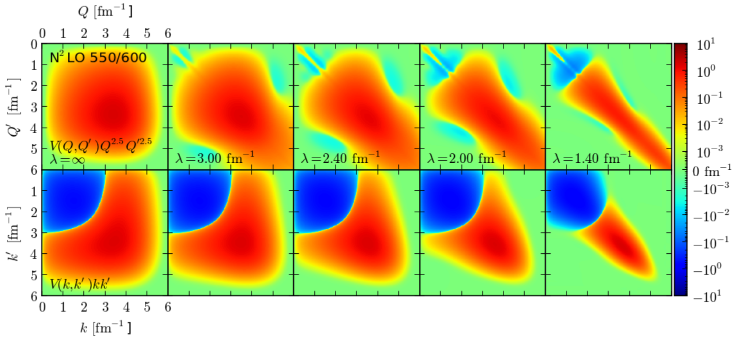
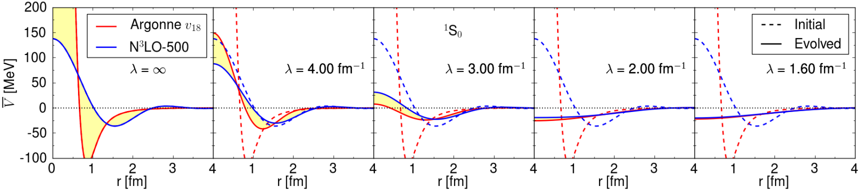
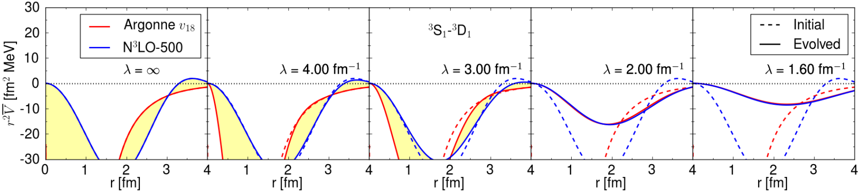


Local projections [K. Wendt et al., PRC 86, 014003 (2012)]

- $V_{\mathrm{low}\,k}$ or SRG unitary transformations to soften interactions
- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]



- Tensor part (S-D mixing) [graphs from K. Wendt]



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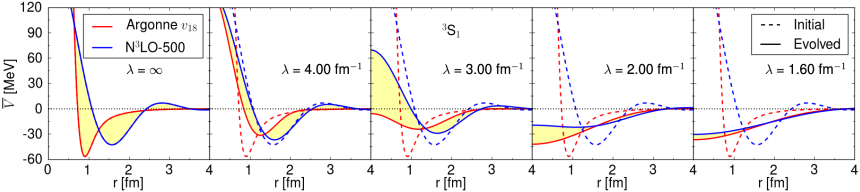

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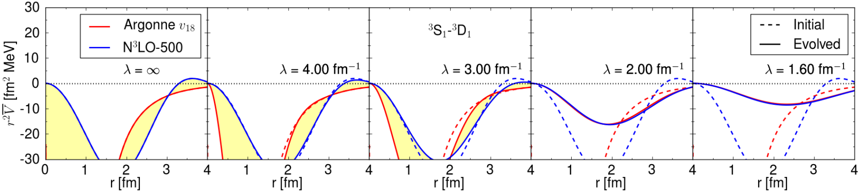
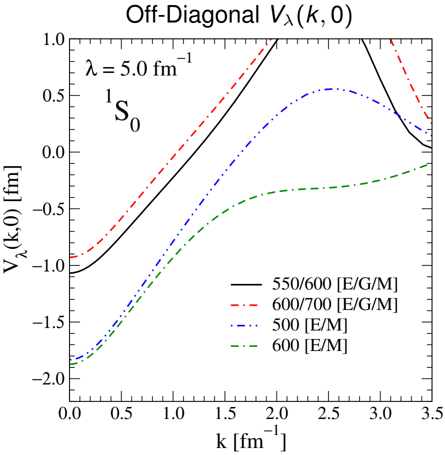
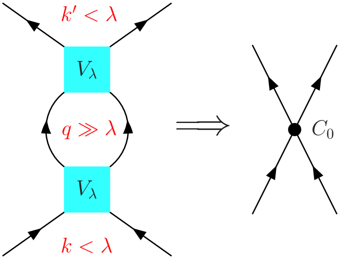


Approach to universality (fate of high-q physics)

Run NN to lower λ via SRG $\Longrightarrow$ $\approx$Universal low-k V_{NN}





$q \gg \lambda$ (or λ) intermediate states
$\Longrightarrow$ replace with contact term:
$$C_0\delta^3(\mathbf{x}-\mathbf{x}')$$
[cf. $\mathcal{L}_{\rm eft} = \cdots + \frac{1}{2}C_0(\psi^\dagger\psi)^2 + \cdots$]

- Similar pattern with phenomenological potentials (e.g., AV18)

Preview: $\Delta V_\lambda(k,k') = \int U_\lambda(k,q)V_\lambda(q,q')U^\dagger_\lambda(q',k')$ for $k,k' < \lambda,\ q,q' \gg \lambda$

$$\xrightarrow{U_\lambda \to K\cdot Q} K(k)[\textstyle\int Q(q)V_\lambda(q,q')Q(q')]K(k')$$ with $K(k) \approx 1$!

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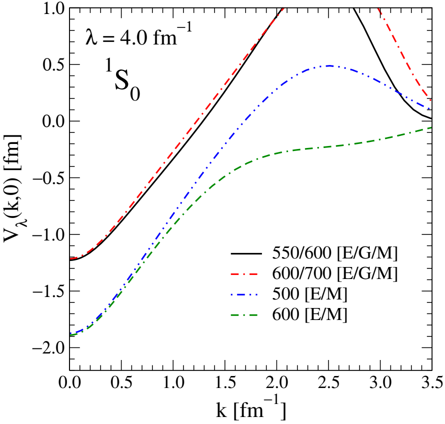
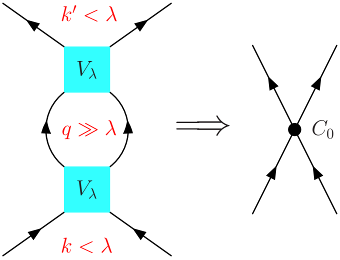
Off-Diagonal $V_\lambda(k,0)$





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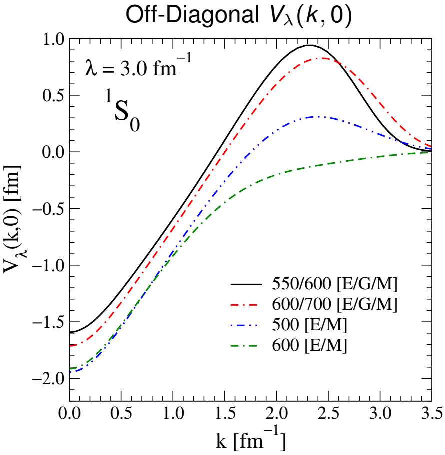

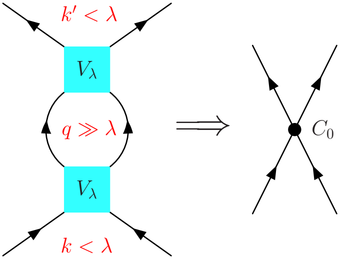

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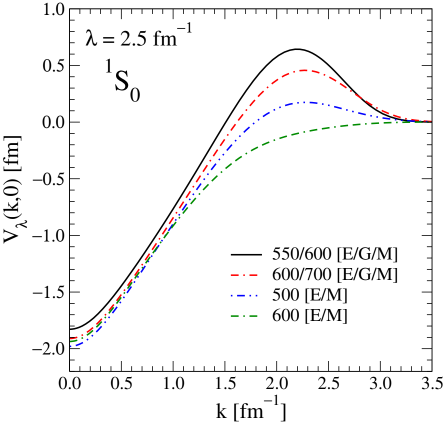

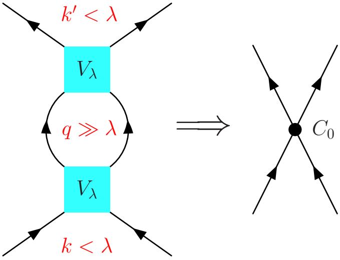

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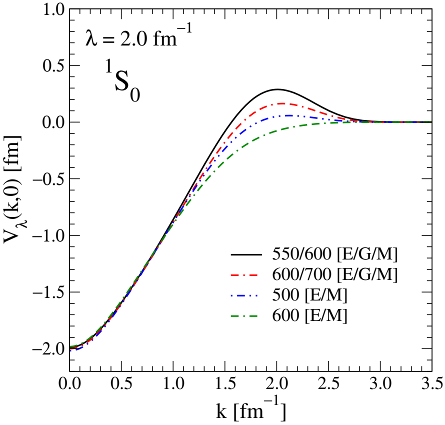

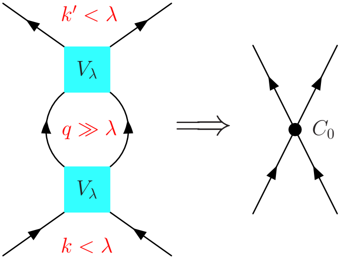

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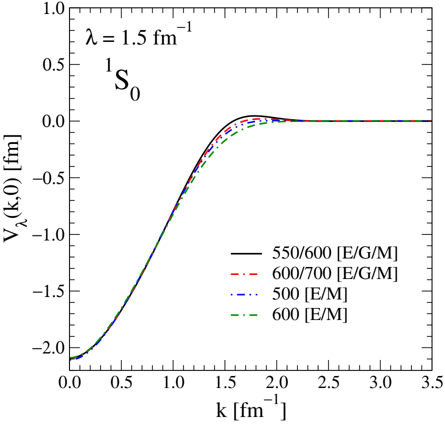

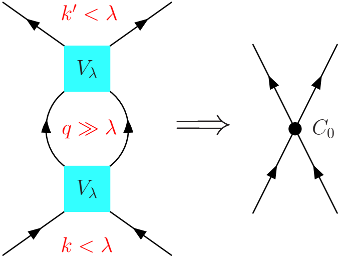

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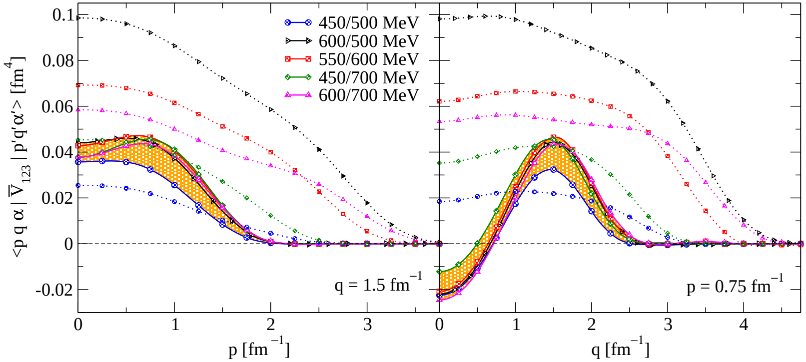
3NF universality?

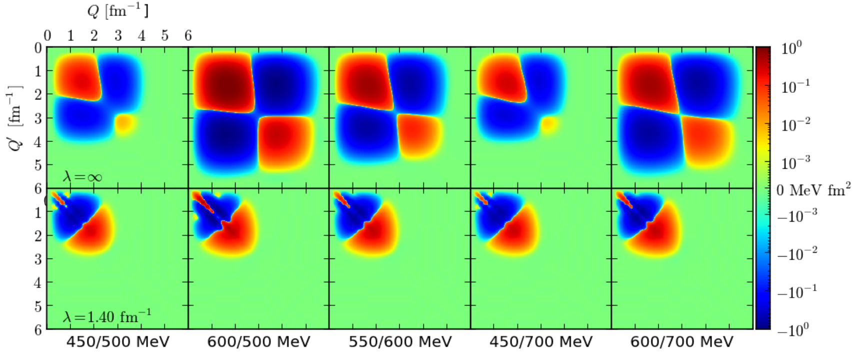
- Evolve chiral NNLO EFT potentials in momentum plane wave basis to $\lambda = 1.5\,\mathrm{fm}^{-1}$ [K. Hebeler, Phys. Rev. C85 (2012) 021002]
- In one 3-body partial wave, fix one Jacobi momentum (p, q) and plot vs. the other one:



- Collapse of curves includes non-trivial structure

3NF universality?

- Evolve in discretized momentum-space hyperspherical harmonics basis to $\lambda = 1.4\,\mathrm{fm}^{-1}$ [K. Wendt, arXiv:1304.1431]
- Contour plot of *integrand* for 3NF expectation value in triton

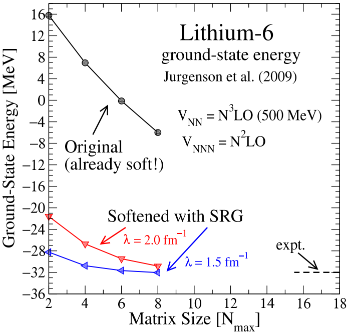


- Local projections of 3NF also show flow toward universal form

Nuclear structure natural with *low* momentum scale

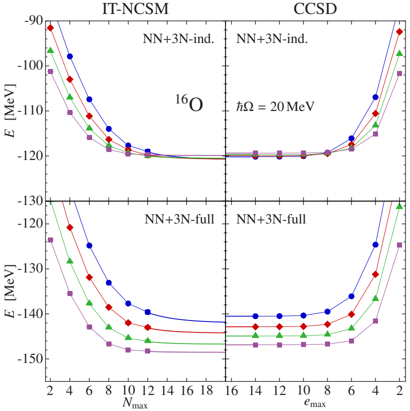
Softened potentials (SRG, $V_{\text{low }k}$, UCOM, ...) enhance convergence

- Convergence for no-core shell model (NCSM):



- (Already) soft chiral EFT potential and evolved (softened) SRG potentials, including NNN

- Softening allows importance truncation (IT) and converged coupled cluster (CCSD)



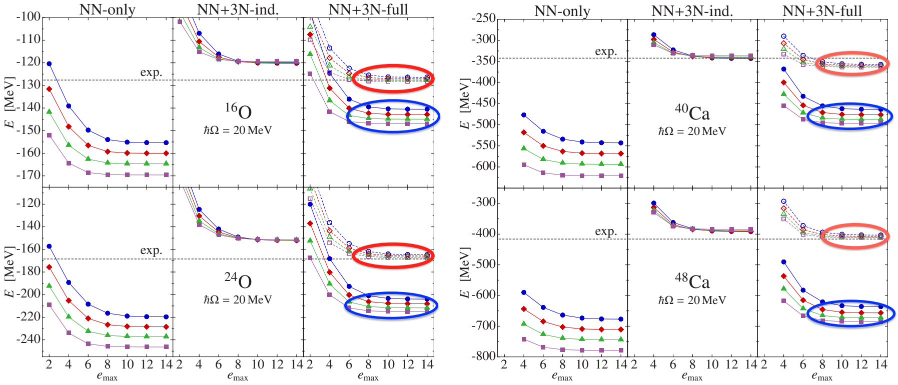
[Roth et al., PRL 109, 052501 (2012)]

- Also enables ab initio nuclear reactions with NCSM/RGM [Navratil et al.]

Nuclear structure natural with *low* momentum scale

R. Roth et al. SRG-evolved N^3LO with NNN [PRL 109, 052501 (2012)]

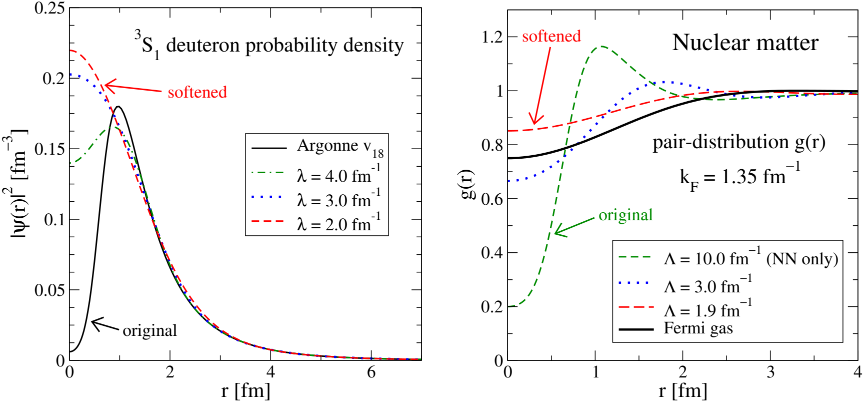
- Coupled cluster with interactions $H(\lambda)$: λ is a decoupling scale
  - *Only* when NNN-induced added to NN-only $\Longrightarrow$ λ independent
  - With initial NNN: predictions from fit only to $A = 3$ properties
- Open questions: red (400 MeV) works, blue (500 MeV) doesn't!



- Same predictions for λ's! (issues about NNN resolved by 4N?)

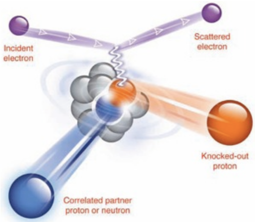
Nuclear structure natural with *low* momentum scale

But lowering resolution reduces short-range correlations (SRCs)!



- Continuously transformed potential $\Longrightarrow$ variable SRCs in wfs!
- Therefore, it is clear that SRCs are very resolution dependent
- But what does this mean for high resolution experiments?

Consider high resolution (large Q^2) e^- scattering



Subedi et al., Science 320, 1476 (2008)

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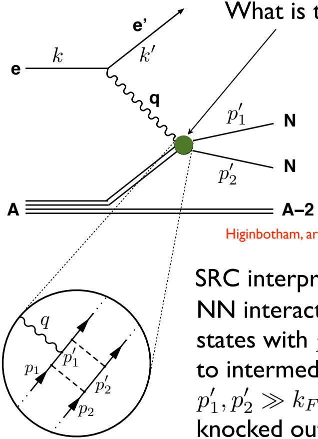

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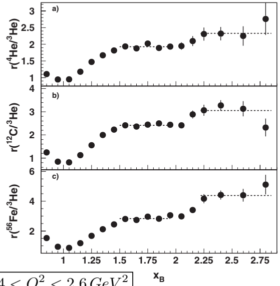

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Egiyan et al. PRL 96, 1082501 (2006)

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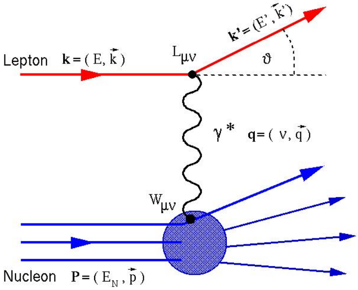
How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

- How does the resolution of the nuclear states come in?

Parton distributions as paradigm [C. Keppel]

DIS Kinematics



$L_{\mu\nu}$: lepton tensor

$W_{\mu\nu}$: hadron tensor

a virtual photon of four-momentum ***q*** *is able to resolve structures of the order ℏ/√q²*

- Four-momentum transfer:

$$q^2 = (E-E')^2 - (\vec{k}-\vec{k}')\cdot(\vec{k}-\vec{k}') =$$
$$= m_e^2 + m_{e'}^2 - 2(EE' - |\vec{k}||\vec{k}'|\cos\theta) =$$
$$\boxed{\approx \; -4EE'\sin^2\tfrac{\theta}{2} \equiv -Q^2}$$

- Mott Cross Section ($\hbar c = 1$):

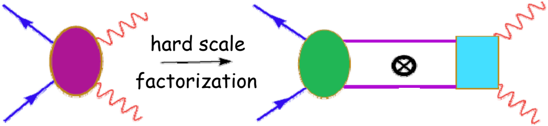
$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4\alpha^2 E'^2}{Q^4}\cos^2\frac{\theta}{2}\cdot\frac{E'}{E}$$
$$= \frac{4\alpha^2 E'^2}{16E^2E'^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}\cdot\frac{1}{1+\frac{E}{M}(1-\cos\theta)}$$
$$= \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\cdot\frac{1}{1+\frac{E}{M}(2\sin^2\frac{\theta}{2})}$$

Electron scattering of a spinless point particle

Parton distributions as paradigm [Marco Stratman]

Factorization schemes

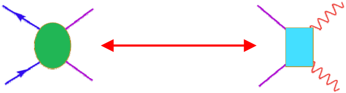
pictorial representation of factorization:



e.g. $F_2(x,Q^2) \sim \sum_a f_a(x,\mu_f) \otimes \hat{F}_2^a(x,\frac{Q}{\mu_f})$

the separation between long- and short-distance physics is not unique

long-distance parton density



short-distance Wilson coefficient

1. **choice of** μ_f: defines borderline between long-/short-distance
2. **choice of scheme**: re-shuffling finite pieces

Parton distributions as paradigm [Marco Stratman]

Deep-inelastic scattering (DIS) according to pQCD

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

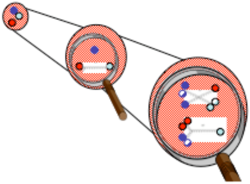
both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

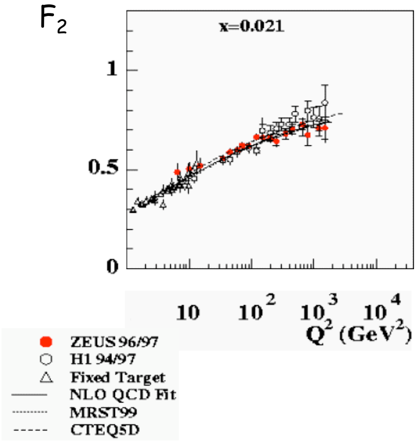
$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi, \mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi} \underbrace{\left[P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]}_{\text{short-distance "Wilson coefficient"}}\right]$$

yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

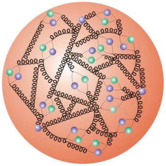
choice of the factorization scheme

Parton distributions as paradigm [C. Keppel]



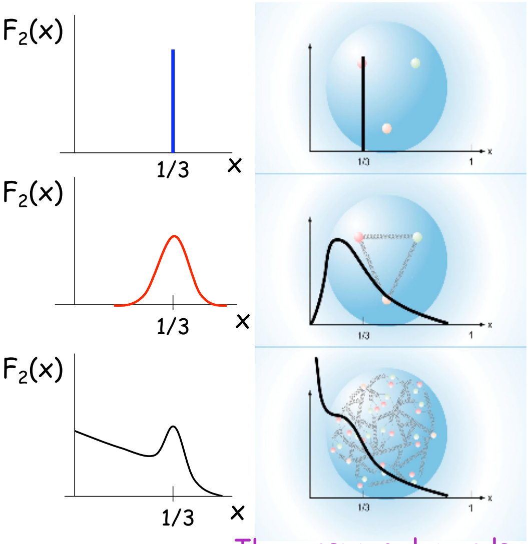


Higher the resolution (i.e. higher the Q^2) more low x partons we "see".



So what do we expect F_2 as a function of x at a fixed Q^2 to look like?

Parton distributions as paradigm [C. Keppel]



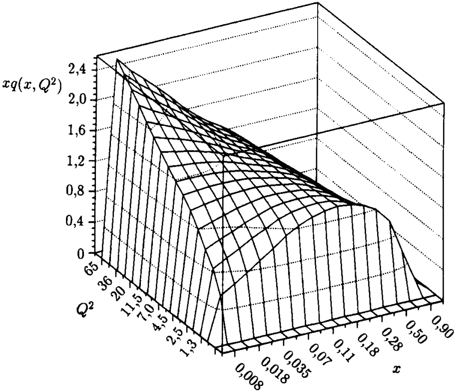
Three quarks with 1/3 of total proton momentum each.

Three quarks with some momentum smearing.

The three quarks radiate partons at low x.

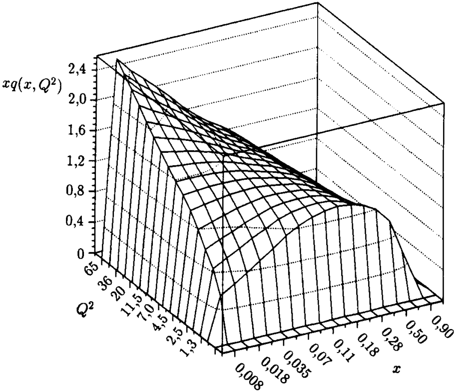
….The answer depends on the Q^2!

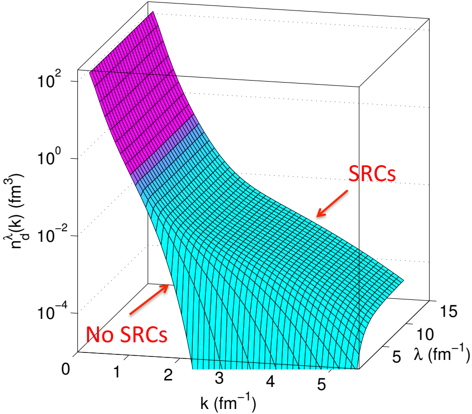
Parton vs. nuclear momentum distributions



- The quark distribution $q(x, Q^2)$ is scale *and* scheme dependent
- $x\ q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

Parton vs. nuclear momentum distributions





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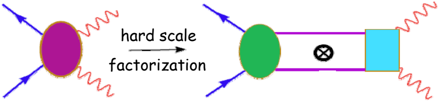
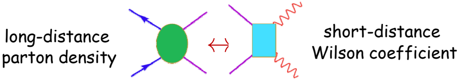
- Deuteron momentum distribution is scale *and* scheme dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\text{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

Factorization: high-E QCD vs. low-E nuclear



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Q^2 running of $f_a(x, Q^2)$ comes from choosing μ_f to optimize extraction from experiment

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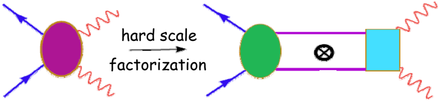

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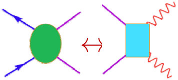

short-distance Wilson coefficient

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- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_f - J_i| \le j \le J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What does it mean to be *consistent* between structure and reaction models? Treat separately? (No!)
- How does scale/scheme dependence come in?
- What are the trade-offs? (Does simpler structure always mean much more complicated reaction?)

Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | \widehat{O} | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) U \widehat{O} U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

Note: matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

$$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

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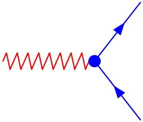
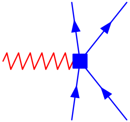
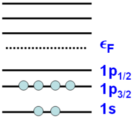
- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states.
  So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- [Field theory version: the equivalence principle says that only on-shell quantities can be measured. Field redefinitions change off-shell dependence only. E.g., see rjf, Hammer, PLB **531**, 203 (2002).]
- RG unitary transformations change the decoupling scale $\Longrightarrow$ change the factorization scale. Use to characterize and explore scale and scheme and process dependence!

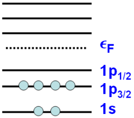
All pieces mix with unitary transformation

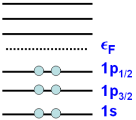
- A one-body current becomes many-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger =$  $+\,\alpha$  $+\cdots$

- New wf correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+\cdots \Longrightarrow Z$ 

$+\,\alpha$ 

$+\cdots$

  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger|\Psi_n^{A-1}\rangle$
  - Thus *spectroscopic factors* are scale dependent

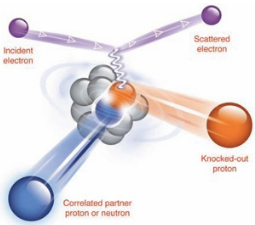
- Final state interactions (FSI) are also modified by $\widehat{U}$

- Bottom line: the cross section is unchanged *only* if all pieces are included, *with the same U*: $H(\lambda)$, current operator, FSI, ...

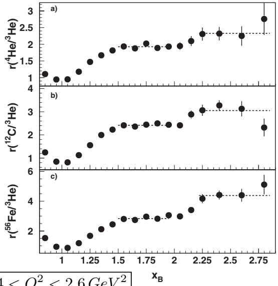
How should one choose a scale/scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - Low-k potential: improve CI or MBPT convergence, or to make microscopic connection to shell model or …
  - (Near-) local potential: quantum Monte Carlo methods work
- Better interpretation or intuition $\Longrightarrow$ predictability
  - SRC phenomenology?
- Cleanest extraction from experiment
  - Can one “optimize” validity of impulse approximation?
  - Ideally extract at one scale, evolve to others using RG
- Plan: use range of scales to test calculations and physics
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguities (e.g., in spectroscopic factors)
  - Match Hamiltonians and operators (EFT) and then use RG

Evolving the SRC explanation of nuclear scaling



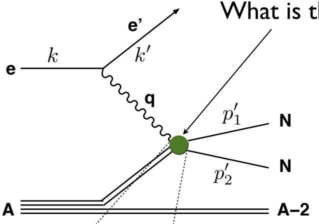
Subedi et al., Science 320, 1476 (2008)



$1.4 < Q^2 < 2.6\, GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

What is this vertex?



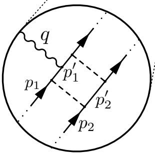
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2 m_N \nu}$$

Higinbotham, arXiv:1010.4433



SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

How to explain cross sections in terms of low-momentum interactions?

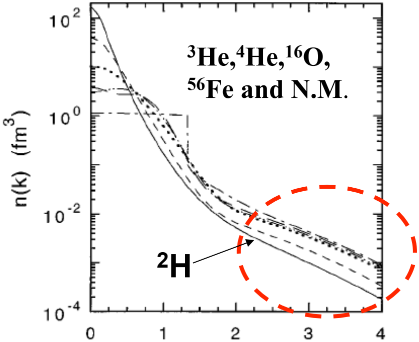
Vertex depends on the resolution!

- Instead of inclusive cross section, look at momentum distributions

Deuteron-like scaling at high *k* used to explain plateaus

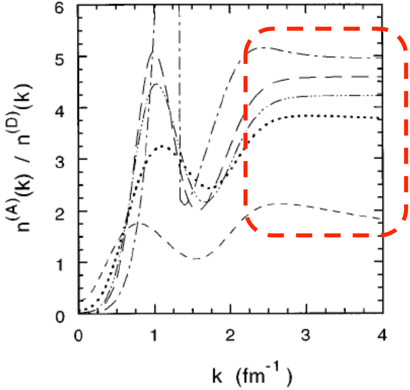
C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)

Momentum Distributions n(k)



n(k) at high Momentum regions are similar to it of the Deuteron

Ratio to the Deuteron



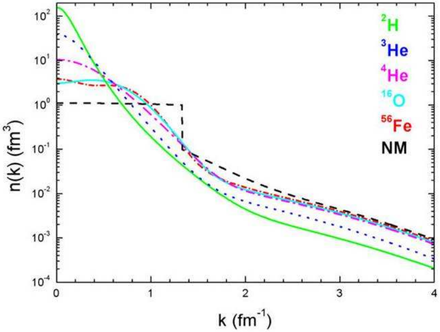
Almost Flat!

High resolution: Dominance of V_{NN} and SRCs (Frankfurt et al.)

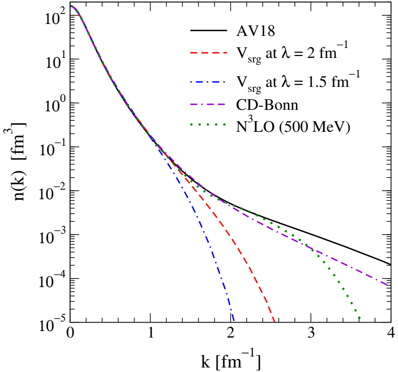
Lower resolution $\Longrightarrow$ lower separation scale $\Longrightarrow$ fall-off depends on V_λ

Changing the factorization scale with RG evolution

- Conventional analysis has (implied) high momentum scale
  - Based on potentials like AV18 and one-body current operator



[From C. Ciofi degli Atti and S. Simula]



- With RG evolution, probability of high momentum decreases, but

$$n(k) \equiv \langle A | a^\dagger_{\mathbf{k}} a_{\mathbf{k}} | A \rangle = \left(\langle A | \widehat{U}^\dagger \right) \widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger \left(\widehat{U} | \Psi_n \rangle \right) = \langle \widetilde{A} | \widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger | \widetilde{A} \rangle$$

is unchanged! $|\widetilde{A}\rangle$ is easier to calculate, but is operator too hard?

Operator flow in practice [e.g., see arXiv:1008.1569]

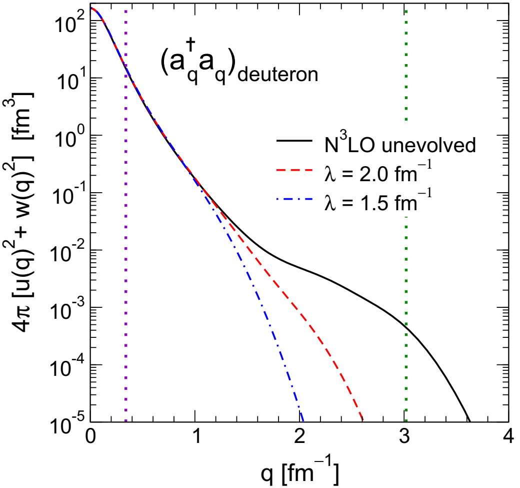
- Evolution with s of any operator O is given by:

  $$O_s = U_s O U_s^\dagger$$

  so O_s evolves via

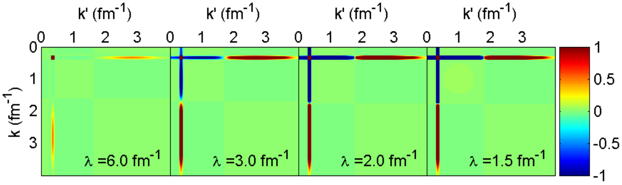
  $$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$ or solve dU_s/ds flow
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 0.34$ and $3.0\,\mathrm{fm}^{-1}$ in deuteron

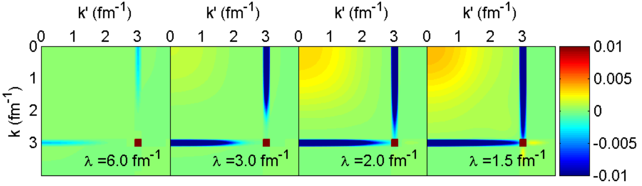


High and low momentum operators in deuteron

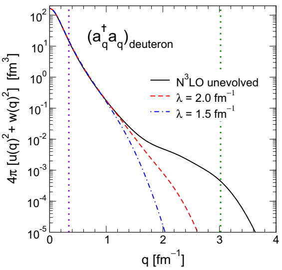
- **Integrand** of $(U a_q^\dagger a_q U^\dagger)$ for $q = 0.34\,\mathrm{fm}^{-1}$

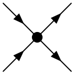


- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$



- Momentum distribution



- One-body operator does not evolve (for “standard” SRG)
- Induced two-body operator ≈ regularized delta function: 

High and low momentum operators in deuteron

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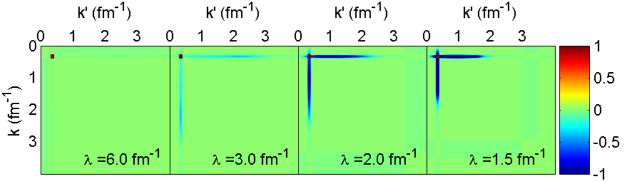

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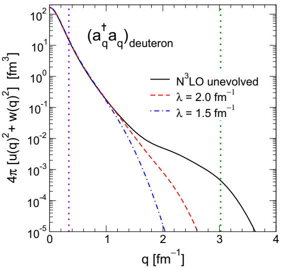

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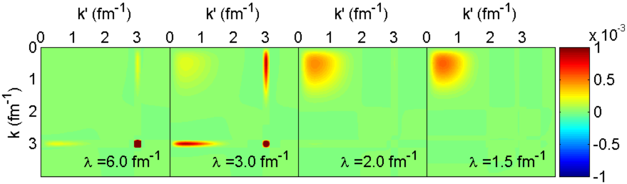


- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2\rangle$, $\langle Q_d\rangle$, $\langle 1/r\rangle$ $\left\langle \frac{1}{r}\right\rangle$, $\langle G_C\rangle$, ...

U-factorization with SRG [Anderson et al., Bogner and Roscher]

- U-factorization: $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
- Cf., operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp\, p^2\, Z(\lambda)\, \Psi_\alpha^\lambda(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

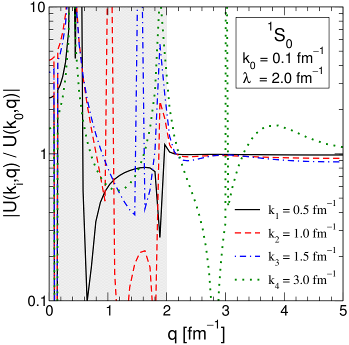
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda p^2 dp\, Z(\lambda)\Psi_\alpha^\lambda(p)\Big] \gamma^\lambda(q) + \cdots$$

- Test of U-factorization:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i) Q_\lambda(q)}{K_\lambda(k_0) Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \dfrac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$
$\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



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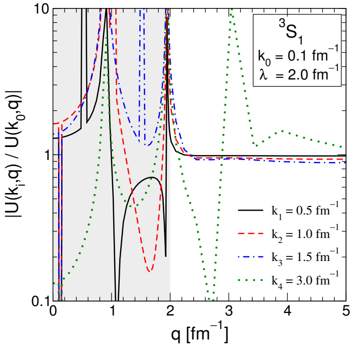
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle \to \left[\sum_\alpha^{\alpha_{low}} \langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda p^2 dp\, Z(\lambda)\Psi_\alpha^\lambda(p)\right]\gamma^\lambda(q) + \cdots$$

- Test of U-factorization:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \frac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!

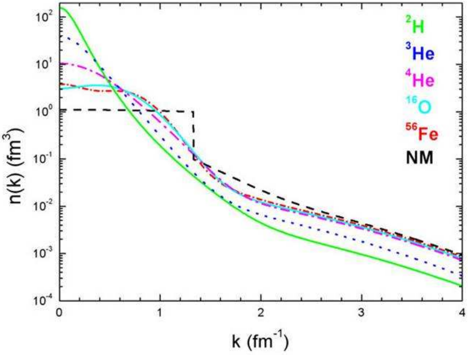


Nuclear scaling from factorization (schematic!)

- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$

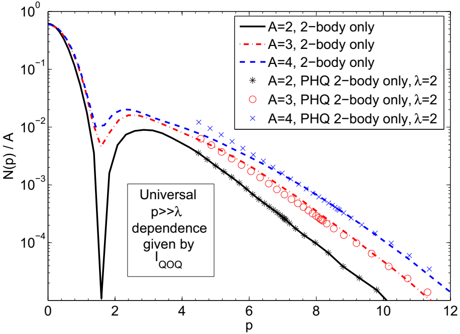
$$\frac{n_A(q)}{n_d(q)} = \frac{\langle \widetilde{A} | \widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger | \widetilde{A} \rangle}{\langle \widetilde{d} | \widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger | \widetilde{d} \rangle} = \frac{\langle \widetilde{A} | \int U_\lambda(k',q') \delta_{q'q} U^\dagger_\lambda(q,k) | \widetilde{A} \rangle}{\langle \widetilde{d} | \int U_\lambda(k',q') \delta_{q'q} U^\dagger_\lambda(q,k) | \widetilde{d} \rangle}$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

Test case: A bosons in toy 1D model



[Anderson et al., arXiv:1008.1569]

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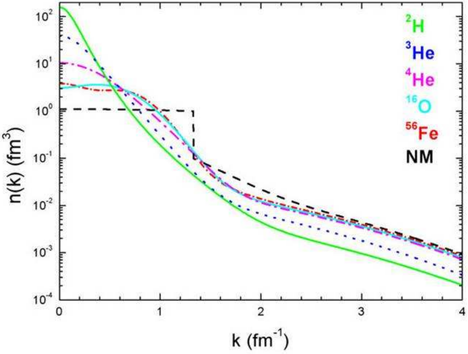

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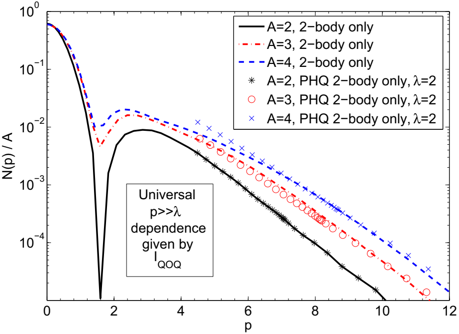

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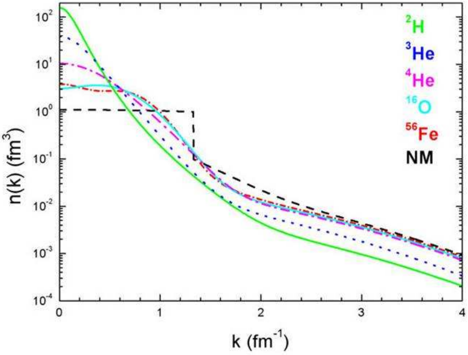
$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

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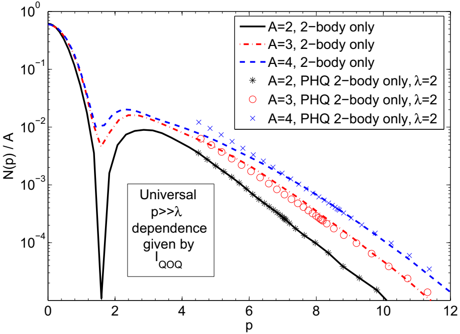


[Anderson et al., arXiv:1008.1569]

Proof of principle works. Realistic calculations in progress ...

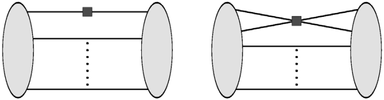
Dependence of EMC effect on A is long-distance physics!

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2 \, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

"The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics."

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators $\Longrightarrow$ parton dist. moments



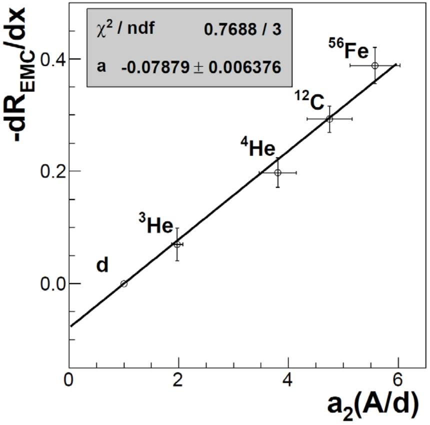
$$\Longrightarrow \langle x^n \rangle_q v^{\mu_0} \cdots v^{\mu_n} N^\dagger N [1 + \alpha_n N^\dagger N] + \cdots$$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle / A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited more?]

Scaling and EMC correlation via low resolution

- SRG factorization, e.g.,
  $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$
  when $k < \lambda$ and $q \gg \lambda$
  - Dependence on high-q independent of A $\Longrightarrow$ universal [cf. Neff et al.]
  - A dependence from low-momentum matrix elements $\Longrightarrow$ calculate!
- EMC from EFT using OPE:
  - Isolate A dependence, which factorizes from x
  - EMC A dependence from long-distance matrix elements

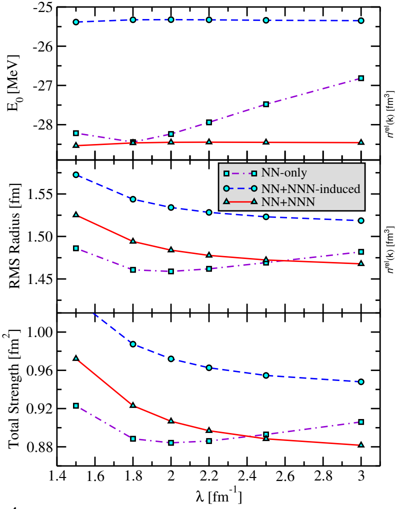


L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

If the same leading operators dominate, then does linear A dependence of ratios follow immediately?

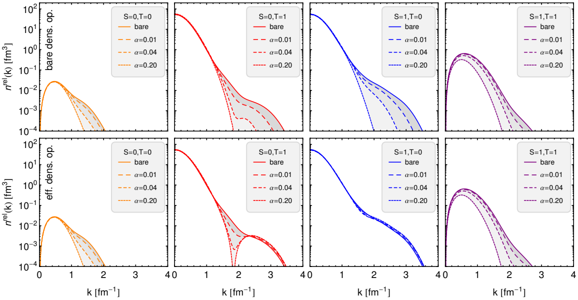
Need to do quantitative calculations to explore!

Where should we truncate many-body operators?



^{4}He photo-absorption [Schuster et al.]

- Induced two-body needed for radius, total dipole strength

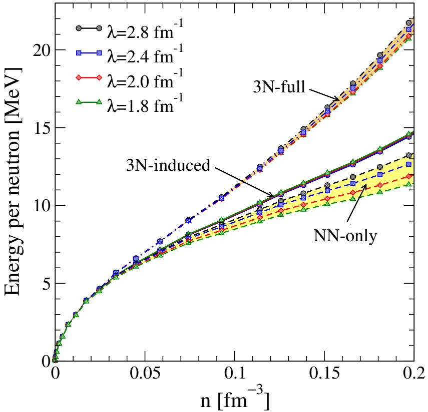


^{4}He relative momentum densities [T. Neff]

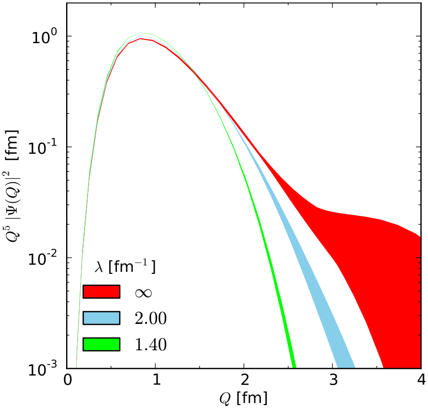
- Induced two-body contributions on bottom $\Longrightarrow$ effect depends on S, T
- Three-body needed at higher k
- Power counting insight needed!

Apply new technologies to operator evolution

Two examples (also many on-going developments by other groups):



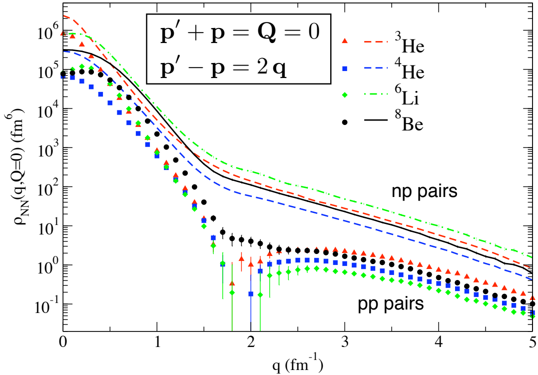
Neutron matter [K. Hebeler, rjf]



^{3}H hyper-momentum wf^2 [K. Wendt]

- SRG evolution in plane-wave momentum basis
- SRG evolution in hyperspherical harmonics momentum basis

MBPT for evolved operators (in uniform system)

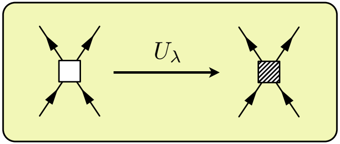


Schiavilla et al. PRL 98, 132501 (2007)

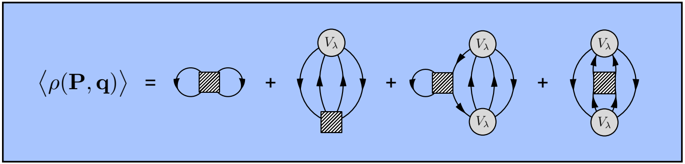
- Benchmark for finite nuclei: calculation at high resolution
- Nuclear scaling and ratio of np/pp pairs

MBPT for evolved operators (in uniform system)

RG transformation of pair density operator (induced many-body terms neglected):

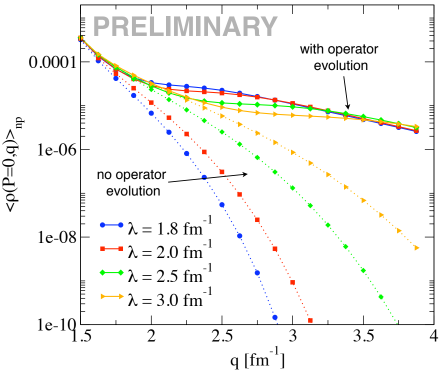


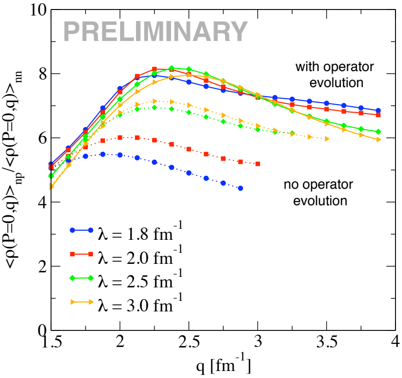
simple calculation of pair density at low resolution in nuclear matter:



- MBPT with leading induced operators only [Anderson, Hebeler]

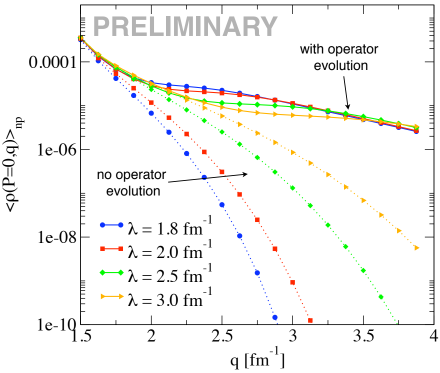
MBPT for evolved operators (in uniform system)

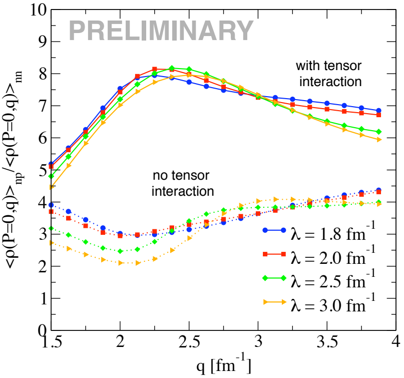




- MBPT with leading induced operators only [Anderson, Hebeler]
  - left: operator evolution restores initial $\langle\rho(P=0,q)\rangle_{np}$
  - right: ratio of *np* to *nn* $\Longrightarrow$ role of tensor

MBPT for evolved operators (in uniform system)





- MBPT with leading induced operators only [Anderson, Hebeler]
  - left: operator evolution restores initial $\langle\rho(P=0,q)\rangle_{np}$
  - right: ratio of *np* to *nn* $\Longrightarrow$ role of tensor
- Proof of principle. Next: add three-body and higher-order MBPT

Final comments and questions

- Summary (and follow-up) points
  - Lower resolution $\Longrightarrow$ more natural nuclear structure
  - While scale and scheme-dependent observables can be (to good approximation) unambiguous for *some* systems, they are often (generally?) not for nuclei. Physics changes!
  - Scale/scheme includes *consistent* Hamiltonian and operators. Be careful of treating experimental analysis in pieces!
- Questions for which RG/EFT perspective + tools can help
  - Can we have controlled factorization at low energies?
  - How should one choose a scale/scheme?
  - What *is* the scheme-dependence of SF's and other quantities?
  - What are the roles of short-range/long-range correlations?
  - How do we match Hamiltonians and operators?
  - When is the assumption of one-body operators viable?
  - ... and many more. Calculations are in progress!

Thanks: collaborators and others at low resolution

- Darmstadt: K. Hebeler, R. Roth, A. Schwenk
- ANL: L. Platter
- Iowa State: P. Maris, J. Vary
- Michigan State: S. Bogner, A. Ekström
- LLNL: E. Jurgenson, N. Schunck
- Ohio State: H. Hergert, S. More, R. Perry, K. Wendt
- ORNL / UofT: G. Hagen, M. Kortelainen, W. Nazarewicz, T. Papenbrock
- TRIUMF: S. Bacca, P. Navratil
- UNC: E. Anderson, J. Drut
- Warsaw: S. Glazek
- many others in UNEDF and NUCLEI

Some on-going calculations to address basic issues

- More general treatment of factorization [S. Bogner et al.]
- Deuteron electrodisintegration [S. More et al.]
  - No issues with three-body operators
  - Do full calculation with final state interactions (FSI)
  - Evolve with SRG, observe FSI/operator/wf contributions
- MBPT for operators: relative momentum distributions
- Quantitative scaling factors [E. Anderson, K. Hebeler]
  - Few-body directly; LDA from infinite mattter MBPT
- Many-body operators [E. Anderson, E. Jurgenson, K. Wendt]
  - Technology for evolution and embedding
  - Power counting investigations
- Variation of spectroscopic factors, single-particle quantities
  - T. Duguet, rjf, and G. Hagen