Large N Scalar QCD_2 : LFQ and DLCQ (Talk at NTSE-2013, ISU,Ames, Iowa, May 13-17, 2013)

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⋆ Talk consists of 2 parts: LFQ and DLCQ

⋆ A Recap of Some Facts, Basics and Motivations

⋆ Gauge Symmetries, Gauge-Fixing and LFQ

⋆ Discrete Light-Cone Quantization (DLCQ)

⋆ Coherent State Formalism

Crucial References for the Model of Large N $sQCD_2$:
G. 't Hooft, NPB 75 (1974) 461
G.'t Hooft, G. Isidori, L. Maini, A.D. polosa and V. Riquer,
— A Theory of Scalar Mesons: PLB 662 (2008) 424
L. Maini, F. Piccinini, A.D. Polosa and V. Riquer,
— PRL 93 (2004) 212002 (hep-ph/0407017)
— B. Grinstein, R. Jora and A. D. Polosa, PLB 671 (2009) 440
— R. L. Jaffe, Phys. Rep. 409 (2005) 1

⋆ UK, DSK, JPV, PLB 708 (2012) 195

⋆ UK,DSK, JPV, SJB- under preparation (2013)

— Light scalar mesons are most likely the lightest particles with an exotic structure: –(and they cannot be classified as the standard $q\bar{q}$ mesons).

— Instanton physics plays a role in their decay dynamics based on the hypothesis that these particles are diquark-antidiquark mesons.

— Therefore, new ways of aggregation of quark matter could be established by the experimental/theoretical investigation of these particles.

— The idea of discussing exotic mesons and hadrons in terms of diquarks dates back to the pioneering papers:
R.L. Jaffe, F. Wilczek, PRL 91 (2003), p.232003 (arXiv:hep-ph/0307341)

— and extended to the scalar meson sector – (by G.'t Hooft et al. PLB 662 (2008))

— These Bound States of a spin zero diquark and an anti-diquark are often called Tetraquark states.

— Tetraquark Particles $(q_1 q_2)(q_3 q_4)$ or the $Q\bar{Q}$-States:
— Expt: and Theoretical Evidences:

— Expts: E. M. Aitala et al. PRL 86 (2001) 770
M. Ablikim et al. PLB 598 (2004) 149

— Theory: I. Caprini et al. hep-ph/0512364
S. Descotes et al. EPJC 48 (2006) 553
N.A. Tornqvist, M. Ross PRL 76 (1996) 1575;
J, Schechter et al, PRD 58 (1998) 054012
J.R. Pelaez MPLA 19 (2004) 2879
Luciano Maini-PoS HEP-2005 (2006) 105

— Series of Papers by Italian Group on Tatraquark (4-quark) states:

— Sub-Gev tetraparticles: $f_0(980), a_0(980), \kappa 800, \sigma(500)$

— Heavier tetraparticles $X(3872), X(3876), Y(2175), Z(4430),$

⋆⋆ Grinstein et al., have considered the 't Hooft Model:
- a Model of Large N scalar QCD in 2D ($sQCD_2$)

⋆ - Imp. Ansatz made by Grinstein et al. is that the Scalar Fields of Large-N $sQCD_2$ can be thought of as the Diquark fields and the corresponding Bethe Salpeter Eq (BSE) of this theory should then yield the spectrum of Tetraquark $Q\bar{Q}$ particles

⋆ They have considered the features of the bound state equation and have computed the discrete hadron mass spectrum in this theory

⋆ Scalar fields of this model represent spin zero diquarks and they estimate the minimum allowed mass for the first radial excitation of the lowest diquark-antidiquark scalar mesons

⋆ They even try to extend their considerations to the case of spin one diquarks

— G. 't. Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer (in the Paper: - A Theory of Scalar Mesons: PLB662 (2008)424)

— have shown as to how one could explain the decays of the light scalar mesons by assuming a dominant diquark-antidiquark $(Q\bar{Q})$ structure for the lightest scalar mesons, where the diquark (Q) is being taken to be a spin zero antitriplet color state.

— In the first approximation the nonet formed by $f_0(980)$, $a_0(980)$, $\kappa(900)$, $\sigma(500)$ is interpreted as the lowest $(Q\bar{Q})$ multiplet.

— and the decuplet of scalar mesons with masses above 1 GeV, formed by $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$, $K_0(1430)$,

(and possibly containing the lowest glueball), is interpreted as the lowest $Q\bar{Q}$ scalar multiplet (which is against the naive $q\bar{q}$ picture).

Grinstein, Jora and Polosa provide an evidence in support of this hypothesis by estimating the mass of the first radial excitation of the lowest sub-GeV $Q\bar{Q}$ scalar meson

Grinstein et al. perform the calculation in Large-N *sQCD* in 2D

$\star$ This is a planar, linearly confining theory which admits a Bethe Salpeter equation (BSE) describing the discrete spectrum of $Q\bar{Q}$ bound states

NB: In this theory no orbital angular momentum excitations are possible since no rotation operator can be introduced and the discrete spectrum describes radial excitations

⋆ — Imp. Ansatz made by Grinstein et al.

— is that the Scalar Fields of Large-N $sQCD_2$ can be thought of as the Diquark fields and the corresponding BSE of this theory should then yield the spectrum of Tetraquark $Q\bar{Q}$ particles

⋆ — Here, one does not expect that the mass spectra in $sQCD_2$ reproduce numerically the physical values of the masses of real pions and sigmas

— but one assumes that the regularities in the spectra of this kind of hadron-string models resemble those in the physical ones

— In the work of Grinstein et al., the gauge fields have been considered in the adjoint representation of $SU(N)$ and the scalar fields in the fundamental representation

— Further the theory is asymptotically free and linearly confining

⋆ Different aspects of this theory have been studied by several authors in various contexts

⋆ In a recent paper (Usha, Daya, James, PLB (2012)):

— We have studied LFQ of this theory (with a mass term for the complex scalar (diquark) field but without the Higgs potential) on the LF (i.e., on the hyperplanes defined by the ELCT:

$\tau = x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1) =$ constant

Fot the LFQ, we consider:

$\tau = x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1) =$ constant, as the LC time coordinate and

$\tau = x^- = \frac{1}{\sqrt{2}}(x^0 - x^1) =$ const. is longitudinal LC space coordinate

— PAM Dirac, Rev. Mod. Phys. 21 (1949) 392:
— S.J. Brodsky, H.C. Pauli, S.S. Pinsky Phys. Rep. 301 (1998) 299

$\star$ We consider the 2D model of large N scalar QCD in the presence of a Higgs potential (– studied by Grinstein, Jora and Polosa without Higgs potential but with a mass term for the complex scalar- diquark field ϕ)

$\star$ We absorve the mass term for the complex scalar (diquark) field ϕ in our Higgs potential

$\star$ The bosonized action of the theory that we propose to study is defined (suppresing the color indices and after ignoring the gluon self coupling term) by the action:

Our gauging prescription is: $\partial_\mu \to D_\mu = (\partial_\mu + i\rho A_\mu)$
(where the color indices are being suppresed)

Action of the theory (with the color indices being suppresed) is:

$$
\begin{aligned}
S &= \int \mathcal{L}(\phi, \phi^\dagger, A^\mu) d^2x \\
\mathcal{L} &= \Big[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \partial_\mu \phi^\dagger \partial^\mu \phi \\
&\quad + [i\rho(\phi A^\mu \partial_\mu \phi^\dagger - \phi^\dagger A_\mu \partial^\mu \phi) + \rho^2 \phi^\dagger \phi A_\mu A^\mu] - V(|\phi|^2) \Big] \\
V(|\phi|^2) &= V(\phi^\dagger\phi) = [\mu^2(\phi^\dagger\phi) + \frac{\lambda}{6}(\phi^\dagger\phi)^2] \ , \ |\phi|^2 = \phi^\dagger\phi \ , \ \phi_0 \neq 0 \\
F^{\mu\nu} &= (\partial^\mu A^\nu - \partial^\nu A^\mu) \ , \ \rho = \frac{g}{\sqrt{N}} \ , \ (-\mu^2 > 0 \ , \ \lambda > 0) \\
g^{\mu\nu} &= g_{\mu\nu} := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \mu, \ \nu = 0, \ 1 \ \ (IFQ) \\
g^{\mu\nu} &= g_{\mu\nu} := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \mu, \ \nu = +, \ - \ \ (LFQ)
\end{aligned} \tag{1}
$$

— 1st term in $\mathcal{L}$ represents the KE term of the gluon field (the color indices have been suppressed)

— 2nd term represents the KE term for the scalar (diquark) field

— 3rd represents the interaction term for the scalar (diquark) field with the gluon field (the color indices have again been suppressed)

— the last term represents the Higgs potential where the mass term for the scalar (diquark) field has been absorbed in the Higgs potential

— Here λ represents the self coupling of the scalar (diquark) field and $\lambda = 0$ reproduces the theory of Grinstein, Jora and Polosa

We choose $(-\mu^2 > 0, \lambda > 0)$ s.t. the potential remains a DWP, and also $|\phi_0| \neq 0$ here, so that the SSB could take place

⋆ Euler-Lagrange equations of motion of the theory (with $V \equiv V(|\phi|^2)$) are obtained as:

$$
\begin{aligned}
[\partial_\mu F^{\mu\nu} + i\rho(\phi\partial^\nu\phi^\dagger - \phi^\dagger\partial^\nu\phi) + 2\rho^2\phi^\dagger\phi A^\nu] &= 0 \\
\left[-\frac{\partial V}{\partial\phi} + \rho^2\phi^\dagger A_\mu A^\mu + i\rho A_\mu\partial^\mu\phi^\dagger + i\rho\partial_\mu(\phi^\dagger A^\mu) - \partial_\mu\partial^\mu\phi^\dagger\right] &= 0 \\
\left[-\frac{\partial V}{\partial\phi^\dagger} + \rho^2\phi A_\mu A^\mu - i\rho A_\mu\partial^\mu\phi - i\rho\partial_\mu(\phi A^\mu) - \partial_\mu\partial^\mu\phi\right] &= 0 \\
IFQ : \mu,\ \nu = 0, 1\ ;\ LFQ : \mu,\ \nu &= +, - \quad (2)
\end{aligned}
$$

The Light-Front Hamiltonian and Path Integral Quantization:

— The bosonized action of the theory (suppresing the color indices) in LF coordinates $x^{\pm} := (x^0 \pm x^1)/\sqrt{2}$ reads:

$$
\begin{aligned}
S &= \int \mathcal{L} dx^+ dx^- \\
\mathcal{L} &= \Big[\frac{1}{2}(\partial_+ A^+ - \partial_- A^-)^2 + (\partial_+ \phi^\dagger \partial_- \phi + \partial_- \phi^\dagger \partial_+ \phi) - V(\phi^\dagger \phi) \\
&\quad + i\rho A^+ (\phi \partial_+ \phi^\dagger - \phi^\dagger \partial_+ \phi) + i\rho A^- (\phi \partial_- \phi^\dagger - \phi^\dagger \partial_- \phi) \\
&\quad + 2\rho^2 \phi^\dagger \phi A^+ A^- \Big] \qquad (3)
\end{aligned}
$$

NB: Grinstein et al. have studied this action, after implementing the gauge-fixing condition (GFC) $A^+ \approx 0$ "strongly" in the above action However, we follow the standard Dirac quantization procedure (DQP) and we do not fix any gauge at this stage

We consider this GFC ($A^+ \approx 0$) only as one of the gauge constraints which becomes strongly equal to zero only on the reduced hypersurface of the constraints and remains non-zero in the rest of the phase space of the theory and we do not set it strongly equal to zero in the Lag. density (this is the conceptual difference of our present work with that of Grinstein et al.)

Also, we have introduced the Higgs potential in our present work and have absorbed the mass term for the scalar (diquark) field in our Higgs potential

Canonical momenta obtained from above action are:

$$\begin{aligned}
\pi &:= \frac{\partial \mathcal{L}}{\partial(\partial_+ \phi)} = (\partial_- \phi^\dagger - i\rho A^+ \phi^\dagger) \\
\pi^\dagger &:= \frac{\partial \mathcal{L}}{\partial(\partial_+ \phi^\dagger)} = (\partial_- \phi + i\rho A^+ \phi) \\
\Pi^+ &:= \frac{\partial \mathcal{L}}{\partial(\partial_+ A^-)} = 0 \\
\Pi^- &:= \frac{\partial \mathcal{L}}{\partial(\partial_+ A^+)} = (\partial_+ A^+ - \partial_- A^-)
\end{aligned} \tag{4}$$

Here $\pi, \pi^\dagger, \Pi^+$ and Π^- are the momenta canonically conjugate respectively to $\phi, \phi^\dagger, A^-$ and A^+

The above equations however, imply that the theory possesses three primary constraints:

$$
\begin{aligned}
\chi_1 &= \Pi^+ \approx 0 \\
\chi_2 &= [\pi - \partial_-\phi^\dagger + i\rho A^+\phi^\dagger] \approx 0 \\
\chi_3 &= [\pi^\dagger - \partial_-\phi - i\rho A^+\phi] \approx 0
\end{aligned}
\tag{5}
$$

Canonical Hamiltonian density in LFQ is:

$$
\begin{aligned}
\mathcal{H}_c &= \left[\pi\partial_+\phi + \pi^\dagger\partial_+\phi^\dagger + \Pi^+\partial_+A^- + \Pi^-\partial_+A^+ - \mathcal{L}\right] \\
&= \left[\frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial_-A^-) + V(\phi^\dagger\phi)\right. \\
&\qquad \left. + i\rho A^-(\phi^\dagger\partial_-\phi - \phi\partial_-\phi^\dagger) - 2\rho^2\phi^\dagger\phi A^+A^-\right]
\end{aligned}
\tag{6}
$$

After including the primary constraints χ_1, χ_2 and χ_3 in the canonical Hamiltonian density $\mathcal{H}_c$ with the help of the Lagrange multiplier fields u, v and w, the total Hamiltonian density $\mathcal{H}_T$ could be written as :

$$\mathcal{H}_T = \Big[(\Pi^+)u + (\pi - \partial_-\phi^\dagger + i\rho A^+\phi^\dagger)v + (\pi^\dagger - \partial_-\phi - i\rho A^+\phi)w + \frac{1}{2}(\Pi^-)^2 + \Pi^-\partial_-A^- + V(\phi^\dagger\phi) + i\rho A^-(\phi^\dagger\partial_-\phi - \phi\partial_-\phi^\dagger) - 2\rho^2\phi^\dagger\phi A^+A^-\Big] \quad (7)$$

HE's could be obtained from the total Hamiltonian: $H_T = \int \mathcal{H}_T dx^-$. Demanding that the PC: χ_1 be preserved in the course of time, one obtains the secondary Gauss-law constraint of the theory as:

$$\chi_4 = [\partial_-\Pi^- + i\rho(\phi\partial_-\phi^\dagger - \phi^\dagger\partial_-\phi) + 2\rho^2\phi^\dagger\phi A^+] \approx 0 \quad (8)$$

Preservation of χ_2, χ_3 and χ_4, for all times does not give rise to any further constraints.
The theory is thus seen to possess only 4 C's χ_i (with i = 1,2,3,4).
The constraints χ_2, χ_3 and χ_4 could however, be combined in to a single constraint:

$$\psi = [\partial_- \Pi^- + i\rho(\phi\pi - \phi^\dagger \pi^\dagger)] \approx 0 \tag{9}$$

with this modification, the new set of C's becomes:

$$\begin{aligned} \Omega_1 &= \chi_1 = \Pi^+ \approx 0 \\ \Omega_2 &= \psi = [\partial_- \Pi^- + i\rho(\phi\pi - \phi^\dagger \pi^\dagger)] \approx 0 \end{aligned} \tag{10}$$

Further, the matrix of PB's among the C's Ω_i, with $(i = 1, 2)$ is seen to be a singular matrix implying that the set of C's Ω_i is first-class and that the theory is GI

Vector gauge current of the theory $J^\mu \equiv (J^+, J^-)$ is given by:

$$
\begin{aligned}
J^+ &= \int dx^- \Big[-i\rho\beta\phi\partial_-\phi^\dagger + i\rho\beta\phi^\dagger\partial_-\phi \\
&\qquad + (\partial_-\beta)(\partial_+ A^+ - \partial_- A^-) - 2\rho^2\beta A^+\phi^\dagger\phi \Big] \\
J^- &= \int dx^- \Big[-i\rho\beta\phi\partial_+\phi^\dagger + i\rho\beta\phi^\dagger\partial_+\phi \\
&\qquad - (\partial_+\beta)(\partial_+ A^+ - \partial_- A^-) - 2\rho^2\beta A^-\phi^\dagger\phi \Big]
\end{aligned} \tag{11}
$$

Divergence of vector gauge current density of the theory could be easily seen to vanish satisfying the continuity equation: $\partial_\mu j^\mu = 0$ — implying that the theory possesses at the classical level, a LVGS.
— Action of the theory is indeed seen to be invariant under LVGT's:

$$\begin{aligned} \delta\phi &= -i\rho\beta\phi \ , \ \delta\phi^\dagger = i\rho\beta\phi^\dagger \ , \ \delta A^- = \partial_+\beta \ , \ \delta A^+ = \partial_-\beta \\ \delta\pi &= [\rho^2\beta\phi^\dagger A^+ + i\rho\beta\partial_-\phi^\dagger] \ , \ \delta\pi^\dagger = [\rho^2\beta\phi A^+ - i\rho\beta\partial_-\phi] \\ \delta u &= \delta v = \delta w = \delta\Pi^+ = \delta\Pi^- = \delta\Pi_u = \delta\Pi_v = \delta\Pi_w = 0 \end{aligned} \quad (12)$$

where $\beta \equiv \beta(x^+, x^-)$ is an arbitrary function of its arguments.

For DQ and HF we convert the set of I-CC's of the theory η_i into a set of II-CC's, — by imposing, arbitrarily, some additional constraints on the system called GFC's or the gauge-constraints.

We could now choose, e.g., the following set of GFC's:

$$\zeta_1 = A^+ \approx 0 \ , \ \zeta_2 = A^- \approx 0 \tag{13}$$

Here the gauge $A^+ \approx 0$ represents the LC time-axial or temporal gauge and the gauge $A^- \approx 0$ represents the LC coulomb gauge and both of these gauges are physically important gauges.

Corresponding to this gauge choice, the theory has the following set of constraints under which the quantization of the theory could, e.g., be studied:

$$\begin{aligned}
\xi_1 &= \Omega_1 = \chi_1 = \Pi^+ \approx 0 \\
\xi_2 &= \Omega_2 = \psi = [\partial_- \Pi^- + i\rho(\phi\pi - \phi^\dagger \pi^\dagger)] \approx 0 \\
\xi_3 &= \zeta_1 = A^+ \approx 0 \\
\xi_4 &= \zeta_2 = A^- \approx 0
\end{aligned} \tag{14}$$

Matrix $R_{\alpha\beta}$ of PB's among the set of C's ξ_i with $(i = 1, 2, 3, 4)$ is seen to be nonsingular with the determinant given by

$$\left[||det(R_{\alpha\beta})|| \right]^{\frac{1}{2}} = \left[[\delta'(x^- - y^-)][\delta(x^- - y^-)] \right] \tag{15}$$

Nonvanishing ELCT-CR's of the theory, under the GFC's: $A^+ = 0$ and $A^- = 0$ are:

$$[\phi(x^+, x^-) \; , \; \pi(x^+, y^-)] = i \; \delta(x^- - y^-) \tag{16}$$

$$[\phi^\dagger(x^+, x^-) \; , \; \pi^\dagger(x^+, y^-)] = i \; \delta(x^- - y^-) \tag{17}$$

$$[\phi(x^+, x^-) \; , \; \Pi^-(x^+, y^-)] = \frac{1}{2}\rho\phi \; \epsilon(x^- - y^-) \tag{18}$$

$$[\phi^\dagger(x^+, x^-) \; , \; \Pi^-(x^+, y^-)] = -\frac{1}{2}\rho\phi^\dagger \; \epsilon(x^- - y^-) \tag{19}$$

$$[\pi(x^+, x^-) \; , \; \Pi^-(x^+, y^-)] = \frac{1}{2} \; \rho \; \pi \; \epsilon(x^- - y^-) \tag{20}$$

$$[\pi^\dagger(x^+,x^-)\ ,\ \Pi^-(x^+,y^-)] = -\frac{1}{2}\rho\ \pi^\dagger\ \epsilon(x^- - y^-) \tag{21}$$

$$[\Pi^-(x^+,x^-)\ ,\ \phi(x^+,y^-)] = \frac{1}{2}\rho\phi\ \epsilon(x^- - y^-) \tag{22}$$

$$[\Pi^-(x^+,x^-)\ ,\ \phi^\dagger(x^+,y^-)] = -\frac{1}{2}\rho\phi^\dagger\ \epsilon(x^- - y^-) \tag{23}$$

$$[\Pi^-(x^+,x^-)\ ,\ \pi(x^+,y^-)] = -\frac{1}{2}\rho\pi\ \epsilon(x^- - y^-) \tag{24}$$

$$[\Pi^-(x^+,x^-)\ ,\ \pi^\dagger(x^+,y^-)] = \frac{1}{2}\rho\pi^\dagger\ \epsilon(x^- - y^-) \tag{25}$$

First-order Lagrangian density $\mathcal{L}_{I0}$ of the theory is:

$$\begin{aligned}
\mathcal{L}_{I0} \quad := \quad & \Big[\pi(\partial_+\phi) + \pi^\dagger(\partial_+\phi^\dagger) + \Pi^+(\partial_+A^-) + \Pi^-(\partial_+A^+) \\
& \quad + \Pi_u(\partial_+u) + \Pi_v(\partial_+v) + \Pi_w(\partial_+w) - \mathcal{H}_T\Big] \\
= \quad & \Big[\frac{1}{2}(\Pi^-)^2 + \partial_+\phi^\dagger\partial_-\phi + \partial_-\phi^\dagger\partial_+\phi \\
& \quad - i\rho A^-(\phi^\dagger\partial_-\phi - \phi\partial_-\phi^\dagger) \\
& \quad - i\rho A^+(\phi^\dagger\partial_+\phi - \phi\partial_+\phi^\dagger) \\
& \quad + 2\rho^2\phi^\dagger\phi A^+A^- - V(\phi^\dagger\phi)\Big] \qquad (26)
\end{aligned}$$

In PI formulation, transition to quantum theory is made by writing the vacuum to vacuum TA for the theory called the generating functional $Z[J_k]$

PI for our theory under the GFC's: $\zeta_1 = A^+ \approx 0$ and $\zeta_2 = A^- \approx 0$, in the presence of the external sources J_k is:

$$\begin{aligned} Z[J_k] \;=\; & \int [d\mu] \exp\Big[i \int d^2x \Big[J_k \Phi^k + \pi \partial_+ \phi + \pi^\dagger \partial_+ \phi^\dagger + \Pi^+ \partial_+ A^- \\ & + \Pi^- \partial_+ A^+ + \Pi_u \partial_+ u + \Pi_v \partial_+ v + \Pi_w \partial_+ w - \mathcal{H}_T \Big]\Big] \end{aligned} \quad (27)$$

Phase space variables of the theory are: $\Phi^k \equiv (\phi, \phi^\dagger, A^-, A^+, u, v, w)$ with the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\pi, \pi^\dagger, \Pi^+, \Pi^-, \Pi_u, \Pi_v, \Pi_w)$

The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under the above gauge-fixing is obtained as :

$$\begin{aligned}
[d\mu] \;&=\; [\delta^{'}(x^{-}-y^{-})\delta(x^{-}-y^{-})][d\phi][d\phi^{\dagger}][dA^{+}][dA^{-}] \\
&\quad [du][dv][dw][d\pi][d\pi^{\dagger}][d\Pi^{-}][d\Pi^{+}] \\
&\quad [d\Pi_{u}][d\Pi_{v}][d\Pi_{w}]\delta[\Pi^{+}\approx 0]\delta[A^{-}\approx 0] \\
&\quad \delta[(\partial_{-}\Pi^{-}+i\rho(\phi\pi-\phi^{\dagger}\pi^{\dagger}))\approx 0]\delta[A^{+}\approx 0] \qquad (28)
\end{aligned}$$

LF Hamiltonian and PIQ of the theory under the set of GFC's: $A^{+}\approx 0$ and $A^{-}\approx 0$ is now complete

Further, the reduced Hamiltonian density of the theory expressed on the reduced hypersurface of the constraints (obtained by implementing the constraints of the theory strongly)
after taking a specific value for the Higgs potential $V(\phi^\dagger\phi)$ (with $\phi_0 \neq 0$) reads:

$$\begin{aligned} \mathcal{H}_R &= \left[\frac{1}{2}(\Pi^-)^2 + \mu^2(\phi^\dagger\phi) + \frac{\lambda}{6}(\phi^\dagger\phi)^2\right] \\ V(\phi^\dagger\phi) &= \left[\mu^2(\phi^\dagger\phi) + \frac{\lambda}{6}(\phi^\dagger\phi)^2\right] \\ \Pi^- &:= (\partial_+ A^+ - \partial_- A^-) \end{aligned} \tag{29}$$

Here $(-\mu^2)$ is positive and λ is positive

So the Higgs potential remains a DWP and the SSB can take place

⋆⋆⋆ DLCQ and Coherent State Formalism — Some Crucial References: Literature on DLCQ is rather Vast

— Hans Pauli, Stan Brodsky and Co-workers:
PRD32 (1985) 1993; PRD32(1985)2001; and Chain of Papers

— Rozowsky and Thorn: PRL 85 (2000) 1614

— James Vary and his Co-workers -in a Series of Papers:
(Dipankar Chakrabarti et al. PLB582 (2004); PLB617 (2005));
(A.Harindernath, James Vary PRD 36 (1987); PRD 37 (1988) 1064; PRD 37 (1988) 1076); James Vary et al., PRD 71(2005); PRD 69 (2004); hep-th/0106248; NPB-PS 161 (2006); PRC 81 (2010)

— have studied the DLCQ and the Coherent State Formalism (CSF) of the 2D ϕ^4 theory and 2D ϕ^3 theory, using the PBC and APBC (they have also studied DLCQ of several field theories)

— and we use it as our guide for the present work

We now study the DLCQ of the present theory with $(-L \leq x^- \leq +L)$ using

(A) anti-periodic Boundary Conditions (APBC) and
(B) periodic boundary conditions (PBC) – after excluding the ZM

The mode expansions (at $x^+ = 0$) with $(\frac{k_n^+}{2} = \frac{n\pi}{L})$ have the form :

$$\phi(x^-) = \sum_n \left[\frac{1}{\sqrt{4\pi n}} \left[a_n e^{-i(\frac{n\pi}{L})x^-} + b_n^\dagger e^{i(\frac{n\pi}{L})x^-} \right] \right]$$
$$\phi^\dagger(x^-) = \sum_n \left[\frac{1}{\sqrt{4\pi n}} \left[b_n e^{-i(\frac{n\pi}{L})x^-} + a_n^\dagger e^{i(\frac{n\pi}{L})x^-} \right] \right]$$
$$A^\mu(x^-) = \sum_n \left[\frac{\epsilon^\mu}{\sqrt{4\pi n}} \left[d_n e^{-i(\frac{n\pi}{L})x^-} + d_n^\dagger e^{i(\frac{n\pi}{L})x^-} \right] \right] \tag{30}$$

Here (A): $n = \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2}, ...,$ for APBC's and
(B): $n = 1,\ 2,\ 3, ...,$ for PBC's (after excluding the ZM ($for\ n = 0$))
(sumation over n is implied)

For the polarization vector ϵ^μ we take $\epsilon^+ = 0$ and $\epsilon^- = 1$ in our mode expansion for $A^\mu(x^-)$

The energy-momentum tensor $T^{\mu\nu}$ is:

$$T^{\mu\nu} = \int \frac{1}{2} dx^- \mathcal{T}^{\mu\nu} \tag{31}$$

where $\mathcal{T}^{\mu\nu}$ is the energy-momentum tensor density of the theory.

$$\mathcal{T}^{\mu\nu} := \sum_k \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_k)} \partial^\nu \phi_k - \mathcal{L} g^{\mu\nu} \right] \tag{32}$$

The longitudinal momentum operator of the theory is:

$$P^{+} = \int \frac{1}{2} dx^{-} \mathcal{P}^{+} = \int \frac{1}{2} dx^{-} \mathcal{T}^{++} := \frac{2\pi}{L} K \tag{33}$$

where L defines the compact domain $-L < x^{-} < +L$ and

Hamiltonian operator of the of the theory
(with $\mathcal{P}^{-} = \mathcal{T}^{+-} = \mathcal{H}_c$) is:

$$P^{-} = \int \frac{1}{2} dx^{-} \mathcal{P}^{-} = \int \frac{1}{2} dx^{-} \mathcal{T}^{+-} = \int \frac{1}{2} dx^{-} \mathcal{H}_c := \frac{L}{2\pi} H \tag{34}$$

The normal ordered value for K is obtained as:

$$\begin{aligned} K &= \frac{L}{2\pi} \mathcal{P}^{+} = \frac{L}{2\pi} \mathcal{T}^{++} \\ &= \frac{L}{2\pi} \left[8 \frac{1}{4\pi} \frac{\pi}{L} \frac{\pi}{L} \frac{L}{\pi} 2\pi \right] \left[n(a_n^{\dagger} a_n + b_n^{\dagger} b_n + 1) \right] \end{aligned} \tag{35}$$

Normal ordered Hamiltonian ($H = \int \frac{1}{2} dx^- \mathcal{H}_c$) is:

$$H = \left[H_0^m + H_0^{em} + H_I^\rho + H_I^\lambda \right] \tag{36}$$

with

$$\begin{aligned} H_0^m &= \left[\frac{1}{2} \frac{\mu^2}{2} \frac{L}{\pi} \frac{2\pi}{4\pi} \right] \left[2 \frac{1}{2n} (a_n^\dagger a_n + b_n^\dagger b_n + 1) \right] \\ H_0^{em} &= \left[-\frac{1}{2} \frac{1}{4\pi} \frac{L}{\pi} (2\pi) \frac{\pi}{L} \frac{\pi}{L} \right] \left[2n (d_n^\dagger d_n) \right] \end{aligned} \tag{37}$$

and

$$
\begin{aligned}
H_l^{\rho} &= \left[\frac{1}{2}\frac{1}{2}\frac{1}{4\pi}\frac{1}{\sqrt{4\pi}}\frac{i\pi}{L}\frac{L}{\pi}\frac{2\pi}{1}\right]\left[\frac{ni\rho}{\sqrt{lmn}}\right]\left[H_l^{\rho 1} - H_l^{\rho 2}\right] \\
H_l^{\rho 1} &= \Big[d_l^{\dagger} a_n^{\dagger} a_m \delta_{m,n+l} - d_l^{\dagger} a_m^{\dagger} a_n \delta_{n,l+m} + d_l^{\dagger} b_n^{\dagger} b_m \delta_{m,n+l} \\
&\qquad - d_l^{\dagger} b_m^{\dagger} b_n \delta_{n,l+m}\Big] \\
H_l^{\rho 2} &= \Big[d_n^{\dagger} a_m d_l \delta_{n,l+m} - a_m^{\dagger} a_n d_l \delta_{m,n+l} + b_n^{\dagger} b_m d_l \delta_{n,l+m} \\
&\qquad - b_m^{\dagger} b_n d_l \delta_{m,n+l}\Big] \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
H_I^{\lambda} &= \left[\frac{1}{2}\frac{L}{\pi}\frac{1}{4\pi}\frac{1}{4\pi}\frac{2\pi}{1}\right]\left[\frac{\lambda}{6}\frac{1}{\sqrt{klmn}}\right]\left[H_I^{\lambda 1}+H_I^{\lambda 2}+H_I^{\lambda 3}\right] \\
H_I^{\lambda 1} &= \Big[a_k^{\dagger}a_m^{\dagger}b_n^{\dagger}a_l\delta_{l,k+m+n}+a_k^{\dagger}a_m^{\dagger}b_l^{\dagger}a_n\delta_{n,k+l+m} \\
&\quad + a_k^{\dagger}b_l^{\dagger}b_n^{\dagger}b_m\delta_{m,k+l+n}+a_m^{\dagger}b_n^{\dagger}b_l^{\dagger}b_k\delta_{k,l+m+n}\Big] \\
H_I^{\lambda 2} &= \Big[a_k^{\dagger}a_m^{\dagger}a_l a_n\delta_{k+m,l+n}+b_l^{\dagger}b_n^{\dagger}b_k b_m\delta_{l+n,k+m}+a_m^{\dagger}b_n^{\dagger}b_k a_l\delta_{m+n,k+l} \\
&\quad + a_k^{\dagger}b_l^{\dagger}b_m a_n\delta_{k+l,m+n}+a_k^{\dagger}b_l^{\dagger}b_m a_n\delta_{k+l,m+n}+b_l^{\dagger}a_m^{\dagger}b_k a_n\delta_{l+m,k+n}\Big] \\
H_I^{\lambda 3} &= \Big[a_k^{\dagger}a_l a_n b_m\delta_{k,l+m+n}+a_m^{\dagger}a_l a_n b_k\delta_{m,l+n+k} \\
&\quad + b_l^{\dagger}b_k b_m a_n\delta_{l,k+m+n}+b_n^{\dagger}b_k b_m a_l\delta_{n,k+l+m}\Big] \qquad (39
\end{aligned}
$$

Here

(A): $k,\ l,\ m,\ n\ = \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2}, ...,$ for APBC's, and

(B): $k,\ l,\ m,\ n\ = 1,\ 2,\ 3, ...,$ for PBC's (– ZM Excluded)

and the summation over $k,\ l,\ m,\ n$ is implied

We define the coherent states $|\alpha_n >$, $|\beta_n >$ and $|\gamma_n >$ as follows:

$$|\alpha_n > \ = \ exp(-\frac{1}{2}|\alpha_n|^2) \sum_n \frac{\alpha^n}{\sqrt{n!}}|n > \quad (40)$$

$$|\beta_n > \ = \ exp(-\frac{1}{2}|\beta_n|^2) \sum_n \frac{\beta^n}{\sqrt{n!}}|n > \quad (41)$$

$$|\gamma_n > \ = \ exp(-\frac{1}{2}|\gamma_n|^2) \sum_n \frac{\gamma^n}{\sqrt{n!}}|n > \quad (42)$$

$$\alpha_n \neq \alpha_n^* \ , \ \beta_n \neq \beta_n^* \ , \ \gamma_n \neq \gamma_n^* \ , \ n = 0, 1, 2, ...\infty \quad (43)$$

These coherent states $|\alpha_n >$, $|\beta_n >$ and $|\gamma_n >$ are the eigen states of the annihilitation operators a_n , b_n and d_n respectively:

$$a_n|\alpha_n > = \alpha_n|\alpha_n > \quad , \quad < \alpha_n|\alpha_n >= 1 \tag{44}$$
$$b_n|\beta_n > = \beta_n|\beta_n > \quad , \quad < \beta_n|\beta_n >= 1 \tag{45}$$
$$d_n|\gamma_n > = \gamma_n|\gamma_n > \quad , \quad < \gamma_n|\gamma_n >= 1 \tag{46}$$

Further, one can also show that:

$$< \alpha_n|a_n|\alpha_n > = \alpha_n \quad , \quad < \alpha_n|a_n^\dagger|\alpha_n >= \alpha_n^* \tag{47}$$
$$< \beta_n|b_n|\beta_n > = \beta_n \quad , \quad < \beta_n|b_n^\dagger|\beta_n >= \beta_n^* \tag{48}$$
$$< \gamma_n|d_n|\gamma_n > = \gamma_n \quad , \quad < \gamma_n|d_n^\dagger|\gamma_n >= \gamma_n^* \tag{49}$$

Further, we construct the state $|\psi_n>$ as:

$$|\psi_n> \;=\; |\alpha_n, \beta_n, \gamma_n> \tag{50}$$

which implies

$$< \psi_n|\phi(x^-)|\psi_n> \;=\; \frac{1}{\sqrt{4\pi}} f(x^-) \tag{51}$$

$$< \psi_n|\phi^\dagger(x^-)|\psi_n> \;=\; \frac{1}{\sqrt{4\pi}} f^*(x^-) \tag{52}$$

$$< \psi_n|A^-(x^-)|\psi_n> \;=\; \frac{1}{\sqrt{4\pi}} f_A(x^-) \tag{53}$$

with

$$f(x^-) = \sum_m \left[\frac{1}{\sqrt{m}} \left[\alpha_m e^{-i(\frac{m\pi}{L})x^-} + \beta_m^* e^{i(\frac{m\pi}{L})x^-} \right] \right] , \quad m = 1, 2, 3, \ldots \quad (54)$$

$$f^*(x^-) = \sum_m \left[\frac{1}{\sqrt{m}} \left[\beta_m e^{-i(\frac{m\pi}{L})x^-} + \alpha_m^* e^{i(\frac{m\pi}{L})x^-} \right] \right] , \quad m = 1, 2, 3, \ldots \quad (55)$$

$$f_A(x^-) = \sum_m \left[\frac{1}{\sqrt{m}} \left[\gamma_m e^{-i(\frac{m\pi}{L})x^-} + \gamma_m^* e^{i(\frac{m\pi}{L})x^-} \right] \right] , \quad m = 1, 2, 3, \ldots \quad (56)$$

One can now easily show that

$$< \phi^\dagger \phi > \;:=\; < \psi_n|\phi^\dagger\phi|\psi_n > \\ = \frac{1}{4\pi} f^* f \;=\; \frac{1}{4\pi}|f|^2 \tag{57}$$

and

$$< \partial_- A^- > \;:=\; < \psi_n|\partial_- A^-|\psi_n > \\ = \frac{1}{\sqrt{4\pi}}\partial_- f_A = \frac{1}{\sqrt{4\pi}} f_A^{'} \;=\; \frac{1}{\sqrt{4\pi}}\tilde{f} \\ \tilde{f} \;:=\; f_A^{'} \;=\; \partial_- A^- \tag{58}$$

We also consider the expectation value of the Hamiltonian:

$$h \;:=\; < H > \;:=\; < \psi_n|H|\psi_n > \tag{59}$$

We also consider the expectation value of the Hamiltonian:

$$h \ := \ < H > \ := \ < \psi_n | H | \psi_n > \tag{60}$$

For minimizing the above expectaction value of the Hamiltonian, the conditions for the stationary points (defined by $(f_0, \tilde{f}_0)$) are determined by:

$$\left.\frac{\partial h}{\partial f}\right|_{(f_0,\tilde{f}_0)} = 0 \quad , \quad \left.\frac{\partial h}{\partial \tilde{f}}\right|_{(f_0,\tilde{f}_0)} = 0 \tag{61}$$

giving

$$f_0 = 0 \quad , \quad f_0^2 = \left[\frac{-12\pi\mu^2}{\lambda}\right] \quad , \quad \tilde{f}_0 = 0 \tag{62}$$

The stationary points are:

$$(f_0 \ , \ \tilde{f}_0) \equiv (0,0) \ \ , \ \ (f_0 \ , \ \tilde{f}_0) \equiv \left(\theta \ \ , \ \ 0\right) \tag{63}$$

with

$$f_{min} = f_0 = \pm\,\theta = \pm\sqrt{\frac{-12\pi\mu^2}{\lambda}} \quad , \quad (-\mu^2) > 0 \tag{64}$$

Further we set

$$f(x^-) \;=\; +\,\theta = +\sqrt{\frac{-12\pi\mu^2}{\lambda}} \;,\quad 0 < x^- < L \tag{65}$$

$$f(x^-) \;=\; -\,\theta = -\sqrt{\frac{-12\pi\mu^2}{\lambda}} \;,\quad -L < x^- < 0 \tag{66}$$

The Fourier expansion for $f = f(x^-)$ is found to be

$$f(x^-) = \sum_m \left[\frac{4\theta}{(2m+1)\pi} sin\left(\frac{(2m+1)\pi x^-}{L}\right)\right] \quad , \quad m = 0, 1, 2, ..., \tag{67}$$

and $\tilde{f} = \partial_- f_A$ does not have any Fourier expansion.

Vacuum Energy Density and Soliton (kink anti-kink) Mass

Classical vacuum energy density (VED) of the theory is:

$$VED = < \psi_n | \int \frac{1}{2} dx^- \left[\mu^2 \phi^\dagger \phi + \frac{1}{2} (\partial_- A^-)^2 \right] | \psi_n > \tag{68}$$

with

$$< \phi^\dagger \phi > = \frac{f_0^2}{4\pi} \quad , \quad < \partial_- A^- > = \frac{\tilde{f}_0}{\sqrt{4\pi}} \quad , \quad \tilde{f}_0 := < \partial_- f_A > \tag{69}$$

where

$$f_0 = \pm \, \theta = \pm \sqrt{\frac{-12\pi\mu^2}{\lambda}} \quad , \quad \tilde{f}_0 = 0 \tag{70}$$

Finally the VED is obtained as

$$VED = \left[\frac{-12\pi\mu^2}{\lambda} \cdot \frac{\mu^2}{8\pi}\right] = \left[\frac{-6\pi\mu^4}{4\pi\lambda}\right] \quad (71)$$

$$< 0 \quad , \quad (-\mu^2) > 0 \quad (72)$$

NB: The VED is Negative because $(-\mu^2) > 0$

The soliton (kink anti-kink) mass can be extracted from the numerical results of matrix diagonalization

(— work is in progress)

⋆ Some Observations and Open Questions:
(most of them follow from Stan Brodsky and remain to be achieved.....!!!)

— One would now like to compare e.g., the transition matrix elements of the normal ordered Hamiltonian for:
(A) APBC and (B) PBC after excluding the ZM

This should be applicable to the case of bound state problems for the confined theories: — where colored fields are always within the finite domain of bound states which do not have ZM's

— One would like to add a constant field ϕ_0 to the mode expansions and then again calculate the normal ordered Hamiltonian and redo the exercise once again to see what happens.....???

— this should be applicable to the case of an unconfined theory where the ZM can be interpreted as an external constant field, much like a Stark or Zeeman field in atomic physics

— It is important to emphasize here that the $k_n^+ = \frac{n\pi}{L} = 0$,   ZM requires uniform support of a field over all x^-.

— and one could possibly handle it by defining the simple ZM's and global ZM's etc as studied by Pauli et.al. in ZPC.

— another challenging task is to compute a convenient orthonormal basis for the higher Fock sectors and then compute the needed transition matrix elements of the LF Hamiltonian

One could perhaps handle it by defining the momentum fraction $x = \frac{n}{K}$ (where $K=$ field momentum) so that x lies between 0 and 1

— this might be helpful in finding the maximum value of n and therefore we could study the corresponding higher Fock states.....!!

— We need more enlightenment from all sides........???

The Lively and Versatile

James Vary

May You Live Thousand Years,

Happy B'day to You...............!!!!!

Daya

— Thanks for your kind attention............!!!!!

Photos13/DSCI0157.JPG