

International Conference on  
Nuclear Theory in the Supercomputing Era - 2013

# Monte Carlo shell model towards ab initio calculations

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# Collaborators

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  - Tooru Yoshida (CNS)
- JAEA
  - Yutaka Utsuno
- Iowa State U
  - James P. Vary
  - Pieter Maris

# Current status of ab initio approaches

- Major challenge of nuclear physics
  - Understand the nuclear structure from *ab-initio* calculations in non-relativistic quantum many-body system w/ realistic nuclear forces (potentials)
  - *ab-initio* approaches: GFMC, NCSM ( $A \sim 12-14$ ), CC (closed subshell +/- 1,2), SCGF theory, IM-SRG, Lattice EFT, ...

➔ demand for extensive computational resources

- ✓ *ab-initio*(-like) SM approaches (which attempt to go) beyond standard methods
  - IT-NCSM, IT-CI: R. Roth (TU Darmstadt), P. Navratil (TRIUMF), ...
  - SA-NCSM: T. Dytrych, K.D. Sviratcheva, J.P. Draayer, C. Bahri, J.P. Vary, ...  
(Louisiana State U, Iowa State U)
  - No-Core Monte Carlo Shell Model (MCSM)

## “Ab initio” in low-energy nuclear structure physics

- Solve the non-relativistic Schroedinger eq.  
and obtain the eigenvalues and eigenvectors.

$$H|\Psi\rangle = E|\Psi\rangle$$

$$H = T + V_{\text{NN}} + V_{\text{3N}} + \cdots + V_{\text{Coulomb}}$$

- **Ab initio**: All nucleons are active, and Hamiltonian consists of realistic NN (+ 3N) potentials.
- Two main sources of errors:
  - **Nuclear forces** (interactions btw/among nucleons),  
in principle, they should be obtained (directly) by QCD.
  - **Finite # of basis space** (the choice of basis function and truncation),  
we have to extrapolate to infinite basis dimensions

# Shell model (Configuration Interaction, CI)

- Eigenvalue problem of large sparse Hamiltonian matrix

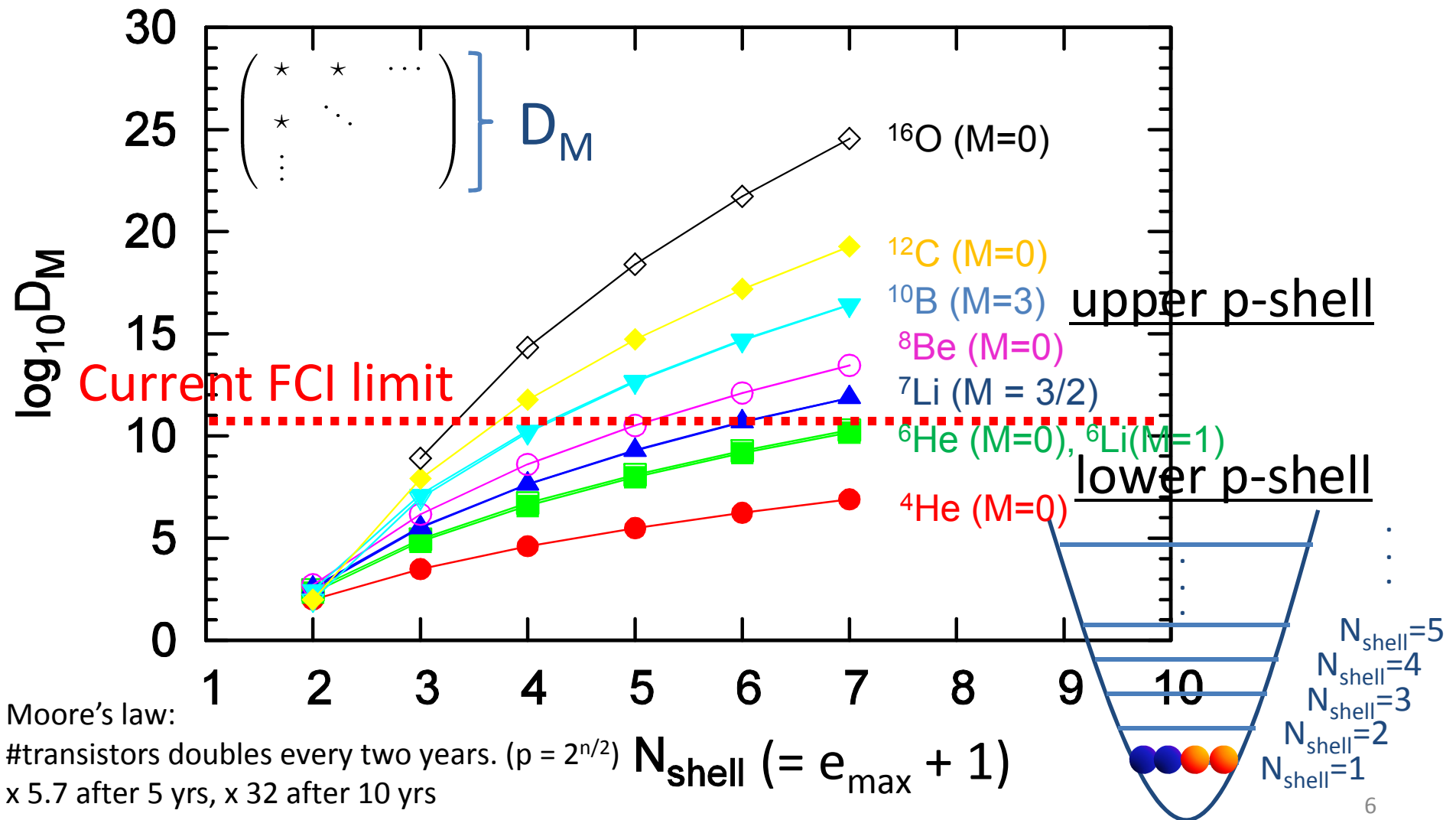
$$H|\Psi\rangle = E|\Psi\rangle$$

$$\underbrace{\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & \cdots \\ H_{21} & H_{22} & H_{23} & H_{24} & & \\ H_{31} & H_{32} & H_{33} & & & \\ H_{41} & H_{43} & & \ddots & & \\ H_{51} & & & & & \\ \vdots & & & & & \end{pmatrix}}_{\sim \mathcal{O}(10^{10})} \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \\ \Psi_5 \\ \vdots \end{pmatrix} = \begin{pmatrix} E_1 & & & & 0 \\ & E_2 & & & \\ & & E_3 & & \\ & & & \ddots & \\ 0 & & & & \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \\ \Psi_5 \\ \vdots \end{pmatrix}$$

Large sparse matrix (in M-scheme)

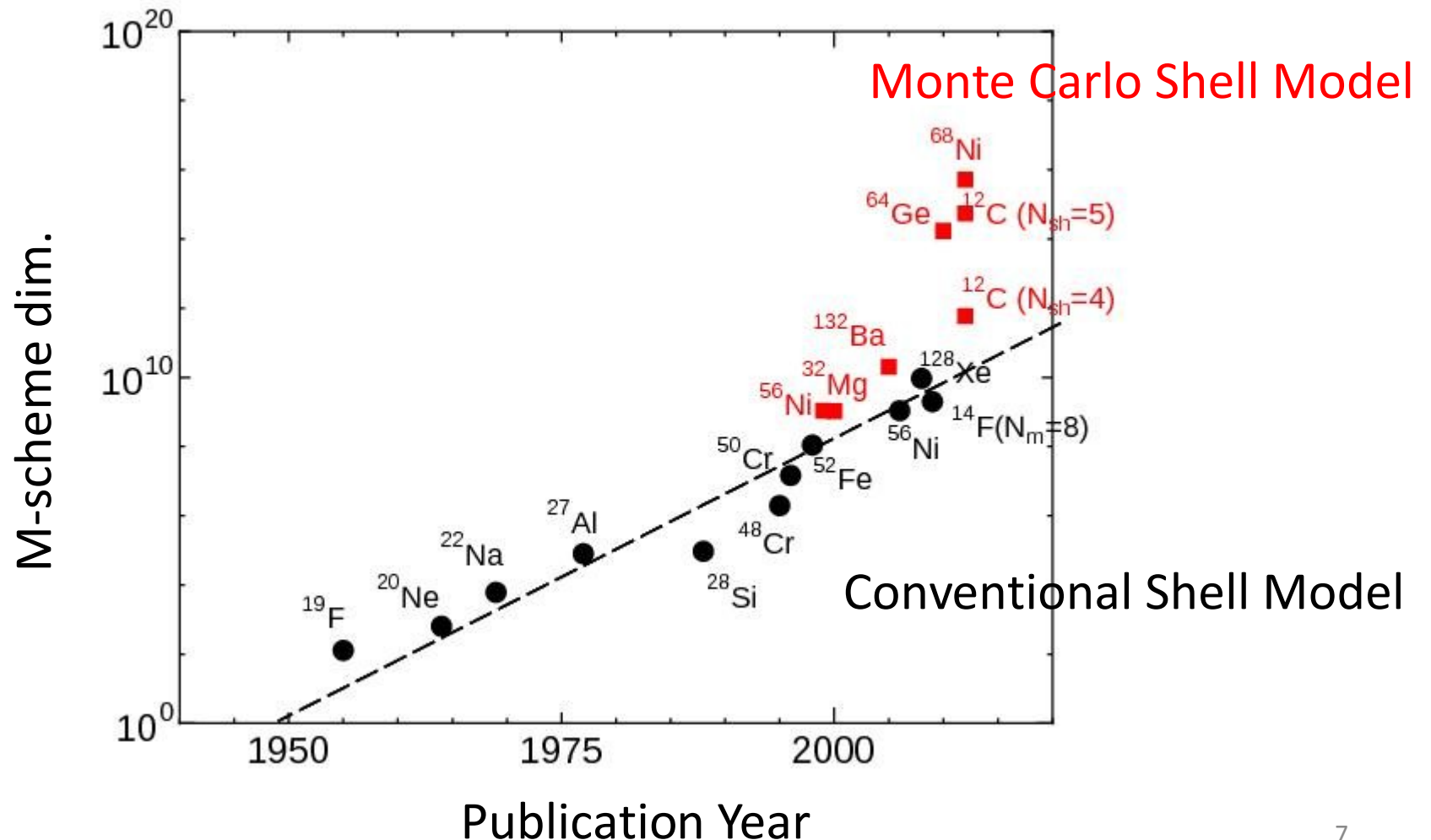
$$\left\{ \begin{array}{l} |\Psi_1\rangle = a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma}^{\dagger} \cdots |-\rangle \\ |\Psi_2\rangle = a_{\alpha'}^{\dagger} a_{\beta'}^{\dagger} a_{\gamma'}^{\dagger} \cdots |-\rangle \\ |\Psi_3\rangle = \cdots \\ \vdots \end{array} \right.$$

# M-scheme dimension of the p-shell nuclei



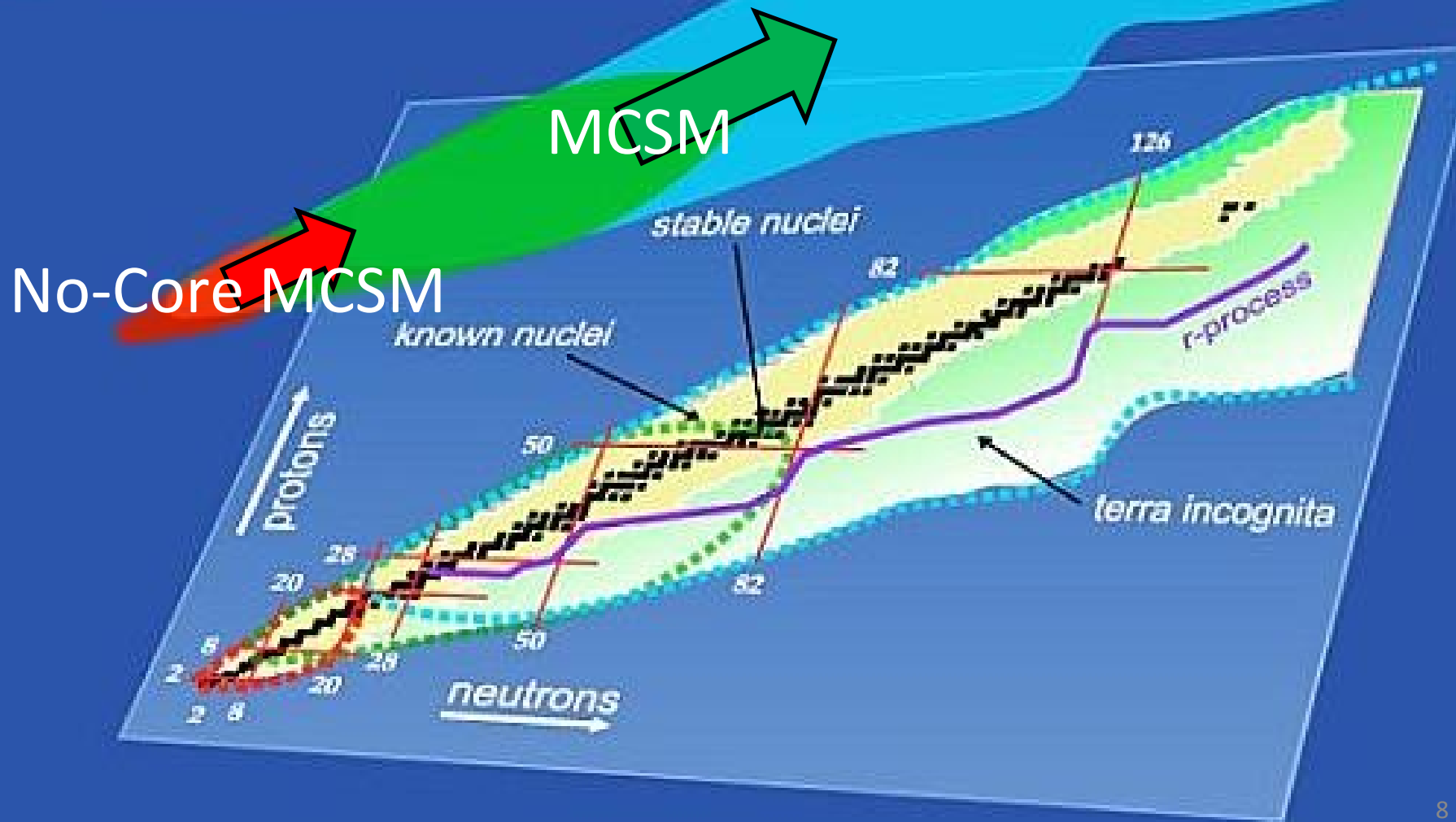
# Advantage of the MCSM

- MCSM w/ an assumed inert core is one of the powerful shell model algorithms.



# Nuclear Landscape

UNEDF SciDAC Collaboration: <http://unedf.org/>



# Monte Carlo shell model (MCSM)

- Importance truncation

## Standard shell model

$$H = \begin{pmatrix} * & * & * & * & * & \cdots \\ * & * & * & * & & \\ * & * & * & & & \\ * & * & & \ddots & & \\ * & & & & \ddots & \\ \vdots & & & & & \ddots \end{pmatrix}$$

All Slater determinants

Diagonalization

$$\begin{pmatrix} E_0 & & & & 0 \\ & E_1 & & & \\ & & E_2 & & \\ & & & \ddots & \\ 0 & & & & \ddots \end{pmatrix}$$

$d \sim O(10^{10})$

## Monte Carlo shell model

$$H \sim \begin{pmatrix} * & * & \cdots \\ * & \ddots & \\ \vdots & & \ddots \end{pmatrix}$$

Important bases stochastically selected

Diagonalization

$$\begin{pmatrix} E'_0 & & 0 \\ & E'_1 & \\ 0 & & \ddots \end{pmatrix}$$

$d_{\text{MCSM}} \sim O(100)$

# SM Hamiltonian & MCSM many-body w.f.

- 2nd-quantized non-rel. Hamiltonian (up to 2-body term, so far)

$$H = \sum_{\alpha\beta}^{N_{sps}} t_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta}^{N_{sps}} \bar{v}_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma} \quad \bar{v}_{ijkl} = v_{ijkl} - v_{ijlk}$$

- Eigenvalue problem

$$H|\Psi(J, M, \pi)\rangle = E|\Psi(J, M, \pi)\rangle$$

- MCSM many-body wave function & basis function

$$|\Psi(J, M, \pi)\rangle = \sum_{i=1}^{N_{basis}} f_i |\Phi_i(J, M, \pi)\rangle \quad |\Phi(J, M, \pi)\rangle = \sum_{K=-J}^J g_K P_{MK}^J P^{\pi} |\phi\rangle$$

These coeff. are obtained by the diagonalization.

- Deformed SDs

$$|\phi\rangle = \prod_i^A a_i^{\dagger} |-\rangle \quad a_i^{\dagger} = \sum_{\alpha}^{N_{sps}} c_{\alpha}^{\dagger} D_{\alpha i} \quad (c_{\alpha}^{\dagger} \dots \text{spherical HO basis})$$

This coeff. is obtained by a stochastic sampling.

# Sampling of basis functions

- Deformed Slater determinant basis

$$|\phi\rangle = \prod_i^A a_i^\dagger |-\rangle \quad a_i^\dagger = \sum_{\alpha}^{N_{sps}} c_{\alpha}^\dagger D_{\alpha i} \quad (c_{\alpha}^\dagger \dots \text{HO basis})$$

- Stochastic sampling of deformed SDs

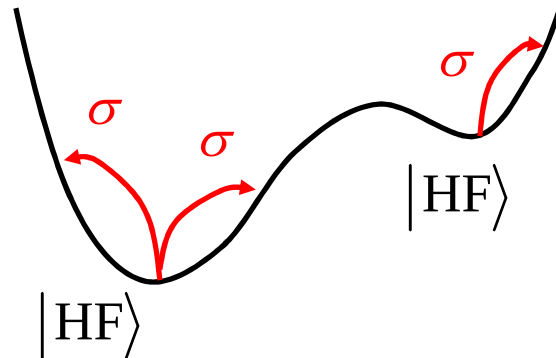
$$|\phi(\sigma)\rangle = e^{-h(\sigma)} |\phi\rangle$$

$$h(\sigma) = h_{HF} + \sum_i^{N_{AF}} s_i V_i \sigma_i O_i$$

c.f.) Imaginary-time evolution & Hubbard-Stratonovich transf.

$$|\phi(\sigma)\rangle = \prod_{N_{\tau}} e^{-\Delta\beta h(\sigma)} |\phi\rangle \quad e^{-\beta H} = \int_{-\infty}^{+\infty} \prod_i d\sigma_i \sqrt{\frac{\beta|V_i|}{2\pi}} e^{-\frac{\beta}{2}|V_i|\sigma_i^2} e^{-\beta h(\vec{\sigma})}$$

$$h(\sigma) = \sum_i^{N_{AF}} (\epsilon_i + s_i V_i \sigma_i) O_i \quad H = \sum_i \epsilon_i O_i + \frac{1}{2} \sum_i V_i O_i^2$$



# Rough image of the search steps

- Basis search
  - HF solution is taken as the 1<sup>st</sup> basis
  - Fix the n-1 basis states already taken
  - Requirement for the new basis: adopt the basis which makes the energy (of a many-body state) as low as possible by a stochastic sampling

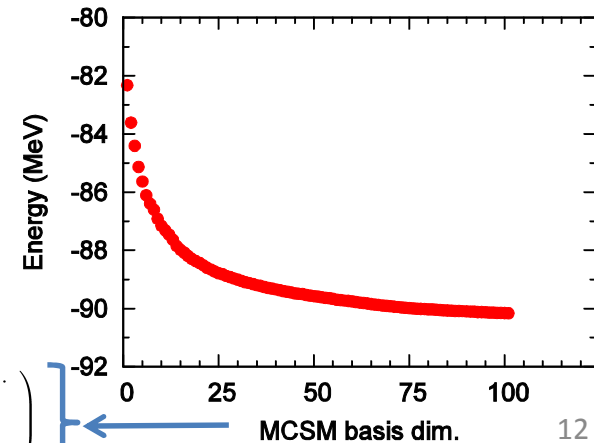
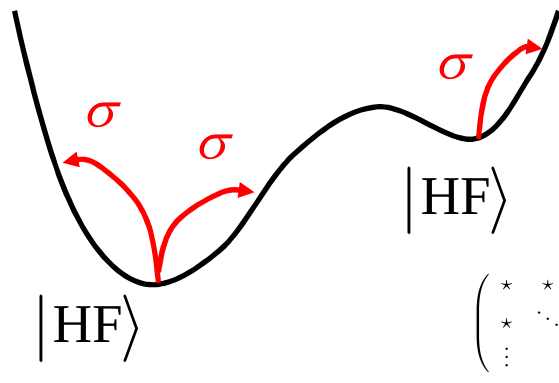
Hamiltonian  
kernel  
 $H(\Phi, \Phi') =$

(n-1)\*(n-1) matrix  
fixed

n-th  
(to be optimized)

$$|\phi(\vec{\sigma})\rangle = \prod_n e^{-\Delta\beta h(\vec{\sigma}_n)} |\phi\rangle$$

$$h(\vec{\sigma}_n) = h_{HF} + \sum_{\alpha} \sigma_{\alpha n} O_{\alpha}$$



# Feasibility study of MCSM for no-core calculations

PHYSICAL REVIEW C 86, 014302 (2012)

## No-core Monte Carlo shell-model calculation for $^{10}\text{Be}$ and $^{12}\text{Be}$ low-lying spectra

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(Received 24 April 2011; revised manuscript received 1 June 2012; published 3 July 2012)

# Recent developments in the MCSM

- Energy minimization by the CG method
  - N. Shimizu, Y. Utsuno, T. Mizusaki, M. Honma, Y. Tsunoda & T. Otsuka, Phys. Rev. C85, 054301 (2012)
- Efficient computation of TBMEs
  - Y. Utsuno, N. Shimizu, T. Otsuka & T. Abe, Compt. Phys. Comm. 184, 102 (2013)
- Energy variance extrapolation
  - N. Shimizu, Y. Utsuno, T. Mizusaki, T. Otsuka, T. Abe & M. Honma, Phys. Rev. C82, 061305 (2010)
- Summary of recent MCSM techniques
  - N. Shimizu, T. Abe, T. Tsunoda, Y. Utsuno, T. Yoshida, T. Mizusaki, M. Honma, T. Otsuka, Prog. Theor. Exp. Phys. 01A205 (2012)

# Energy minimization by Conjugate Gradient method

$$|\Psi(D)\rangle = \sum_{n=1}^{N_B} c_n \sum_{K=-J}^J g_K P_{MK}^{J,\Pi} |\phi(D^{(n)})\rangle$$

$$|\phi(D^{(n)})\rangle = \prod_{\alpha=1}^{N_p} \left( \sum_{i=1}^{N_{sp}} c_i^\dagger D_{i\alpha}^{(n)} \right) |-\rangle$$

$$E(D) = \langle \Psi(D) | H | \Psi(D) \rangle$$

Minimize  $E(D)$  as a function of  $D$  utilizing Conjugate Gradient method

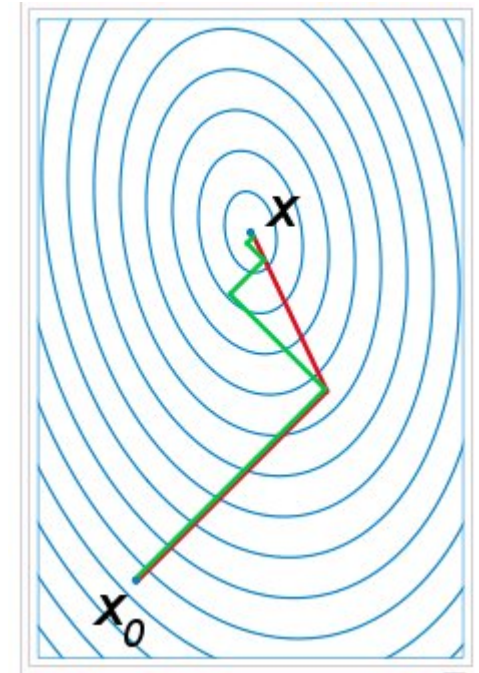
Step1 : Generate basis candidate by auxiliary field technique stochastically

$$|\phi(\sigma)\rangle = \prod e^{\Delta\beta \cdot h(\sigma)} \cdot |\phi^{(0)}\rangle$$

and select basis which lowers the energy

Step 2 : Energy expectation value is taken as a function of  $D$ , and optimize it using Conjugate Gradient method (VAP)

Iterate these steps every basis until the energy converges



Conjugate gradient  
taken from wikipedia

Few Determinant Approximation

M. Honma, B.A.Brown, T. Mizusaki, and T. Otsuka  
Nucl. Phys. A 704, 134c (2002)

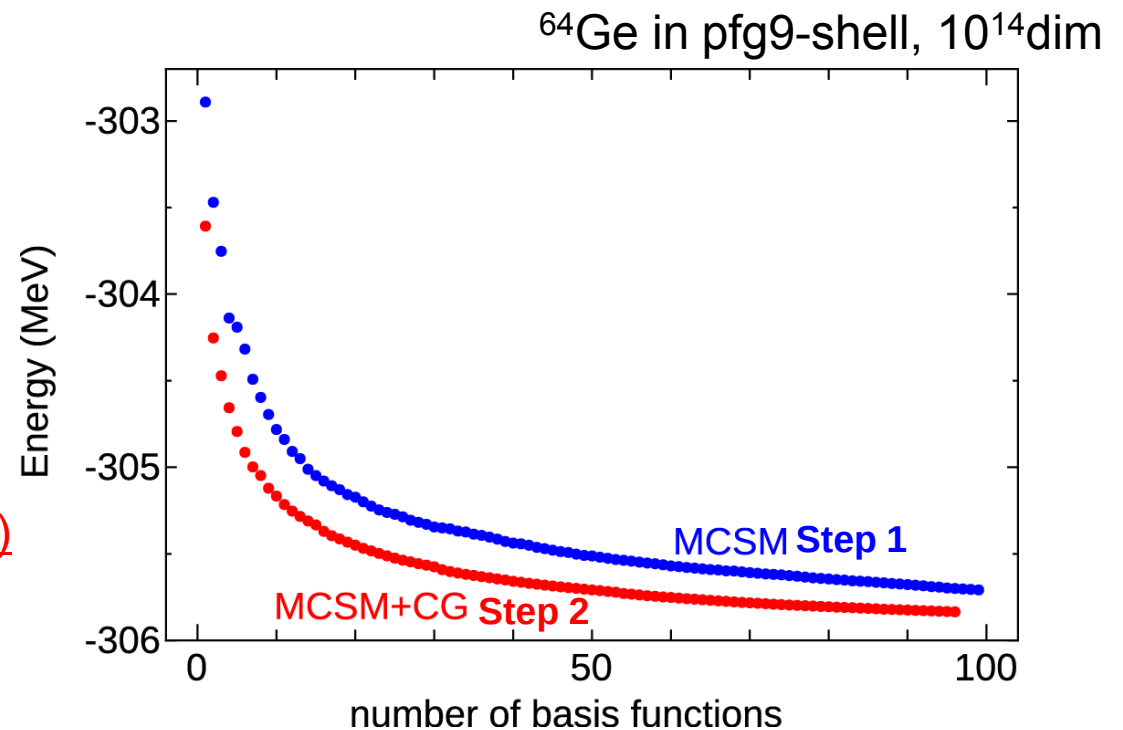
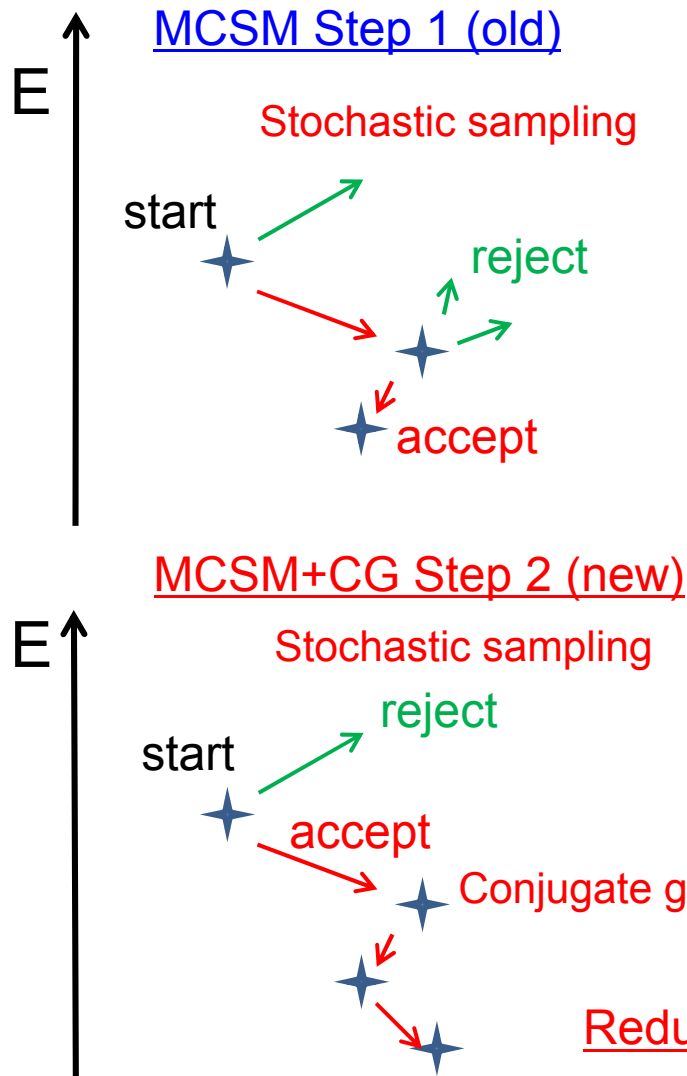
Hybrid Multi-Determinant

G. Puddu, Acta Phys. Polon. B42, 1287 (2011)

VAMPIR

K.W. Schmid, F. Glummer, M. Kyotoku, and A. Faessler  
Nucl. Phys. A 452, 493 (1986)

# Energy minimization by Conjugate Gradient method



Stochastic sampling before conjugate gradient to minimize the expectation value energy

Reduction of the number of basis function roughly 30%

# Efficient computation of the TBMEs

- hot spot: Computation of the TBMEs  
(w/o projections, for simplicity)

$$\frac{\langle \Phi' | V | \Phi \rangle}{\langle \Phi' | \Phi \rangle} = \frac{1}{2} \sum_{ijkl} \bar{v}_{ijkl} \rho_{ki} \rho_{lj} \quad \text{c.f.) Indirect-index method (list-vector method)}$$

- Utilization of the symmetry

$$j_z(i) + j_z(j) = j_z(k) + j_z(l) \rightarrow j_z(i) - j_z(k) = -(j_z(j) - j_z(l)) \equiv \Delta m$$

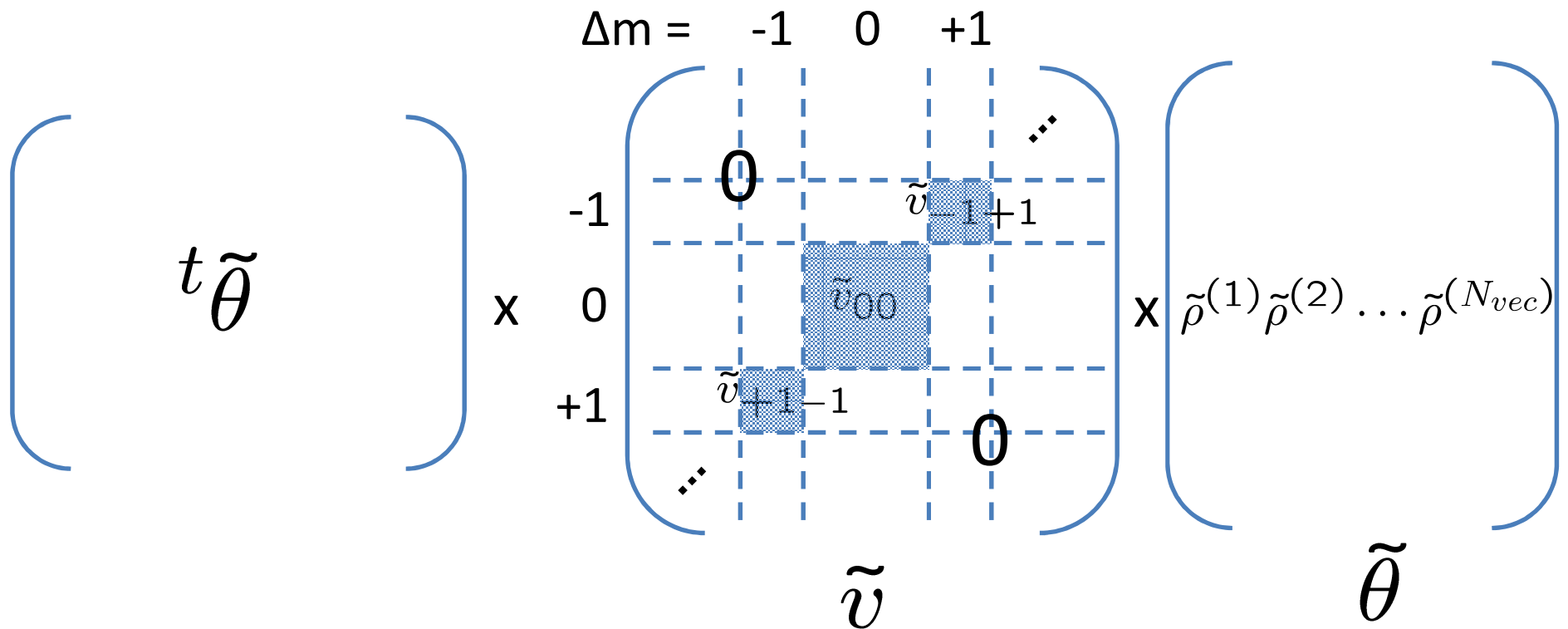
$$\sum_{ijkl} \bar{v}_{ijkl} \rho_{ki} \rho_{lj} = \sum_{\Delta m} \left[ \sum_{a \in J_z(a) = -\Delta m} \tilde{\rho}_a \left( \sum_{b \in J_z(b) = \Delta m} \tilde{v}_{ab} \tilde{\rho}_b \right) \right]$$

$$\begin{array}{ccc} \bar{v}_{ijkl} \rightarrow \tilde{v}_{ab} & \rho_{ki} \rightarrow \tilde{\rho}_a & \rho_{lj} \rightarrow \tilde{\rho}_b \\ \text{sparse} & \text{dense} & \end{array}$$

## Schematic illustration of the computation of TBMEs

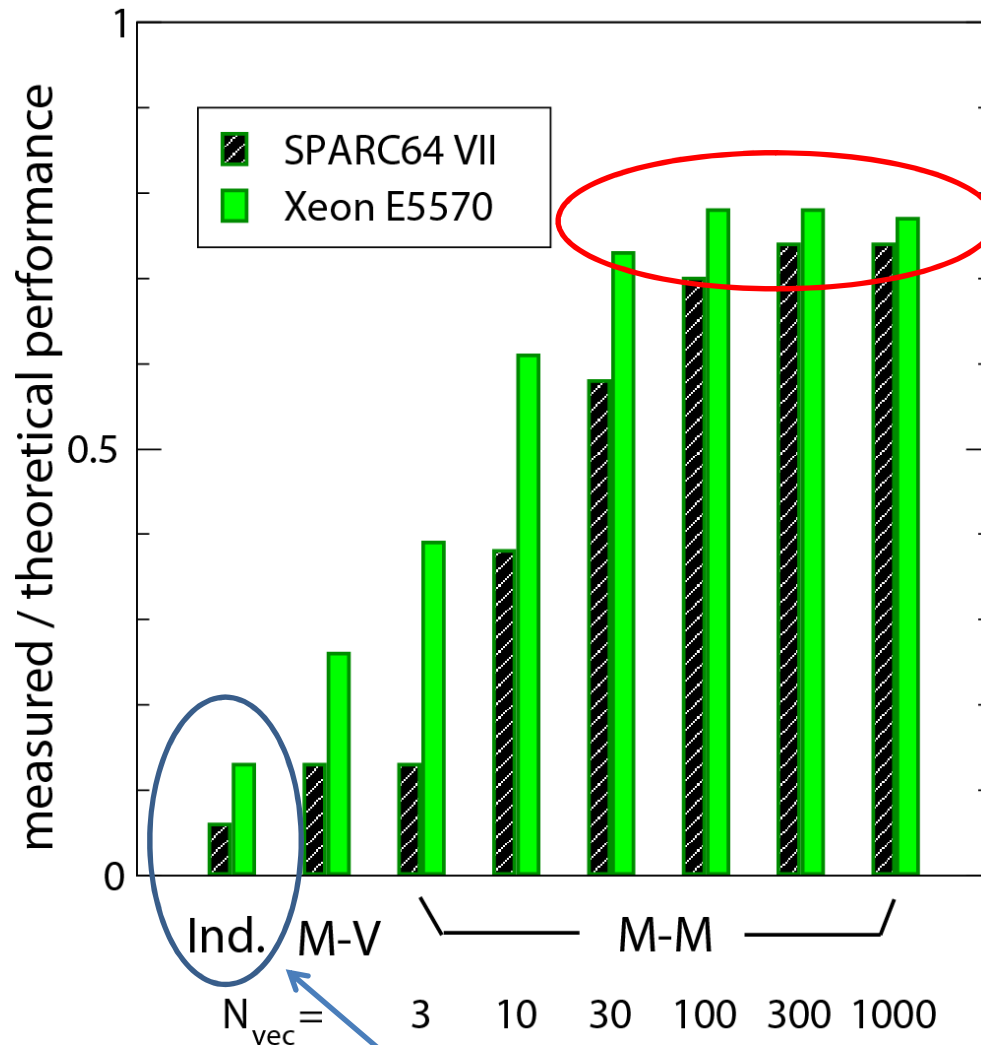
- Matrix-matrix method

$$\sum_{ijkl} \bar{v}_{ijkl} \rho_{ki} \rho_{lj} = \sum_{\Delta m} \left[ \sum_{a \in J_z(a) = -\Delta m} \tilde{\rho}_a \left( \sum_{b \in J_z(b) = \Delta m} \tilde{v}_{ab} \tilde{\rho}_b \right) \right]$$



→ BLAS Level 3

# Tuning of the density matrix product



Nshell = 5

The performance reaches 80% of the theoretical peak at hot spot.

SPARC64 requires large  $N_{\text{bunch}}$  in comparison to Xeon

Matrix product e.g.  
(390 x 390) x (390 x  $2N_{\text{bunch}}$ )

$N_{\text{bunch}}$  controllable tuning parameter  
chunk size

Standard technique

# Extrapolations in the MCSM

- Two steps of the extrapolation
  1. Extrapolation of our MCSM (approx.) results to the FCI (exact) results in fixed model space

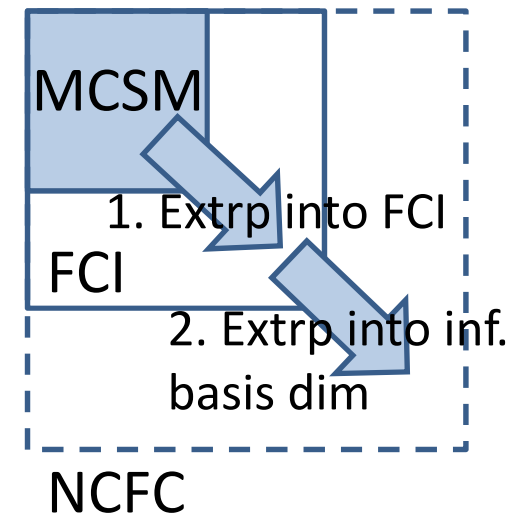
## Energy-variance extrapolation

N. Shimizu, Y. Utsuno, T. Mizusaki, T. Otsuka, T. Abe, & M. Honma, Phys. Rev. C82, 061305(R) (2010)

2. Extrapolation into the infinite model space

Exponential fit w.r.t.  $N_{\max}$  in the NCFC

Not applied in the MCSM, so far...



# Energy-variance extrapolation

- Originally proposed in condensed matter physics

Path Integral Renormalization Group method

M. Imada and T. Kashima, J. Phys. Soc. Jpn 69, 2723 (2000)

- Imported to nuclear physics

Lanczos diagonalization with particle-hole truncation

T. Mizusaki and M. Imada Phys. Rev. C65 064319 (2002)

T. Mizusaki and M. Imada Phys. Rev. C68 041301 (2003)

single deformed Slater determinant

T. Mizusaki, Phys. Rev. C70 044316 (2004)



Apply to the MCSM (multi deformed SDs)

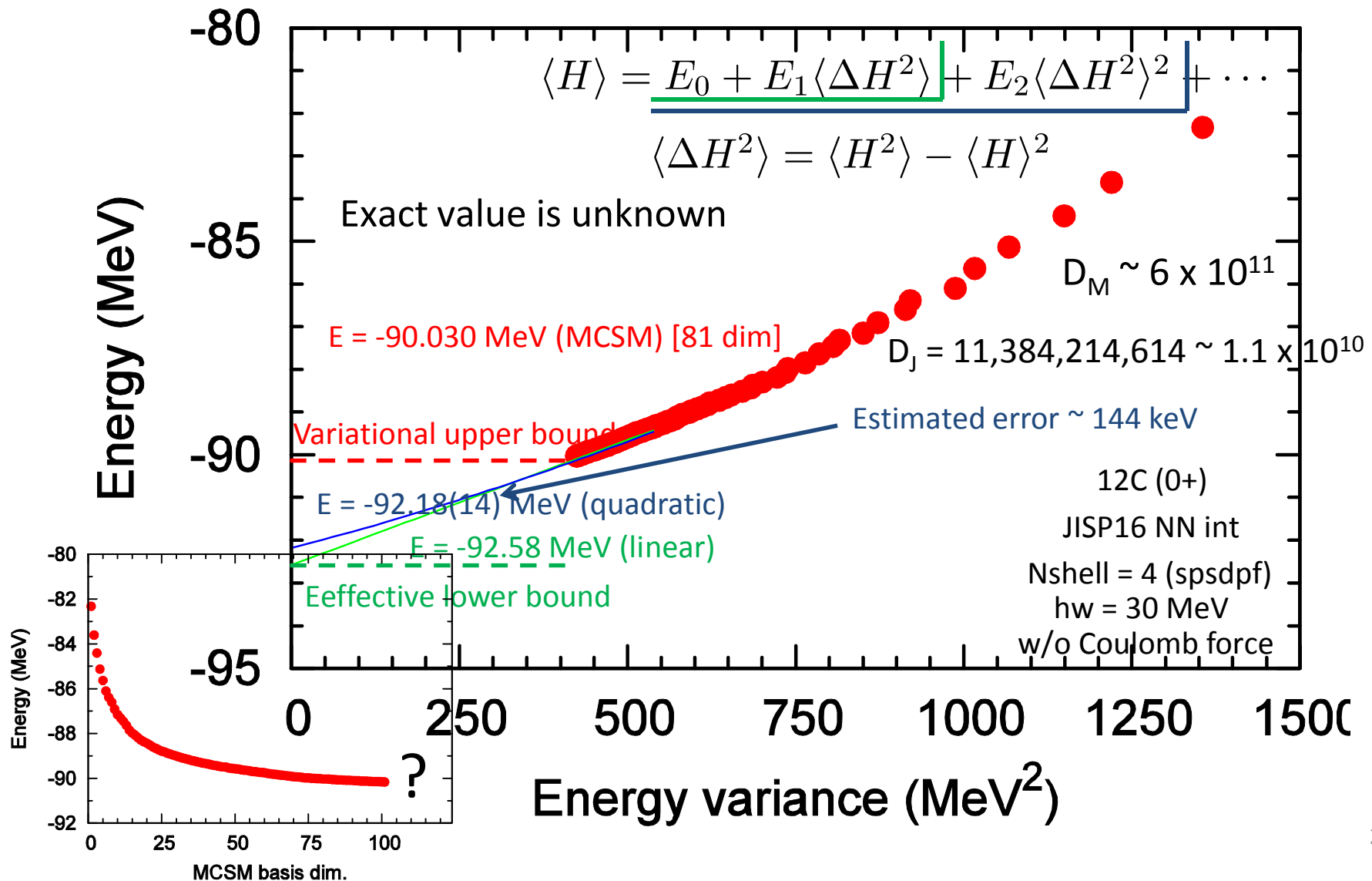
N. Shimizu, Y. Utsuno, T. Mizusaki, T. Otsuka, T. Abe & M. Honma, Phys. Rev. C82, 061305<sup>21</sup> (2010)

# Numerical effort

$$\begin{aligned}
 \frac{\langle \Phi' | \hat{V}^2 | \Phi \rangle}{\langle \Phi' | \Phi \rangle} &= \overset{\substack{\text{8-folded loop} \\ \sim O(N_{\text{sps}}^8)}}{\sum_{ijkl\alpha\beta\gamma\delta}} \bar{v}_{ijkl} \bar{v}_{\alpha\beta\gamma\delta} \left[ \frac{1}{4} (1 - \rho)_{k\alpha} (1 - \rho)_{l\beta} \rho_{\gamma i} \rho_{\delta j} \right. \\
 &\quad \left. + \rho_{\gamma\alpha} (1 - \rho)_{l\beta} \rho_{ki} \rho_{\delta j} + \frac{1}{4} \rho_{ki} \rho_{lj} \rho_{\gamma\alpha} \rho_{\delta\beta} \right] \\
 &= \frac{1}{4} \sum_{ij\alpha\beta} \left( \sum_{kl} \bar{v}_{ijkl} (1 - \rho)_{k\alpha} (1 - \rho)_{l\beta} \right) \left( \sum_{\gamma\delta} \bar{v}_{\alpha\beta\gamma\delta} \rho_{\gamma i} \rho_{\delta j} \right) \\
 &\quad \overset{\substack{\text{6-folded loop} \\ \sim O(N_{\text{sps}}^6)}}{+ \text{Tr}(\Gamma(1 - \rho)\Gamma\rho) + \frac{1}{4} [\text{Tr}(\rho\Gamma)]^2}
 \end{aligned}$$

$$\rho_{\beta\alpha} = \frac{\langle \Phi' | c_{\alpha}^{\dagger} c_{\beta} | \Phi \rangle}{\langle \Phi' | \Phi \rangle} \quad \Gamma_{ik} = \sum_{jl} \bar{v}_{ijkl} \rho_{lj} \quad \frac{\langle \Phi' | V | \Phi \rangle}{\langle \Phi' | \Phi \rangle} = \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \bar{v}_{\alpha\beta\gamma\delta} \rho_{\gamma\alpha} \rho_{\delta\beta}$$

# Energy-variance Extrapolation of $^{12}\text{C}$ 0+ g.s. Energy





PRESENTED BY  
UNIVERSITY OF  
MANNHEIM

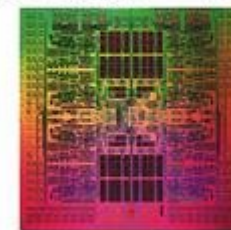
ICL  
INNOVATIVE  
COMPUTING LABORATORY  
at UNIVERSITY of TENNESSEE



Lawrence Berkeley  
National Laboratory

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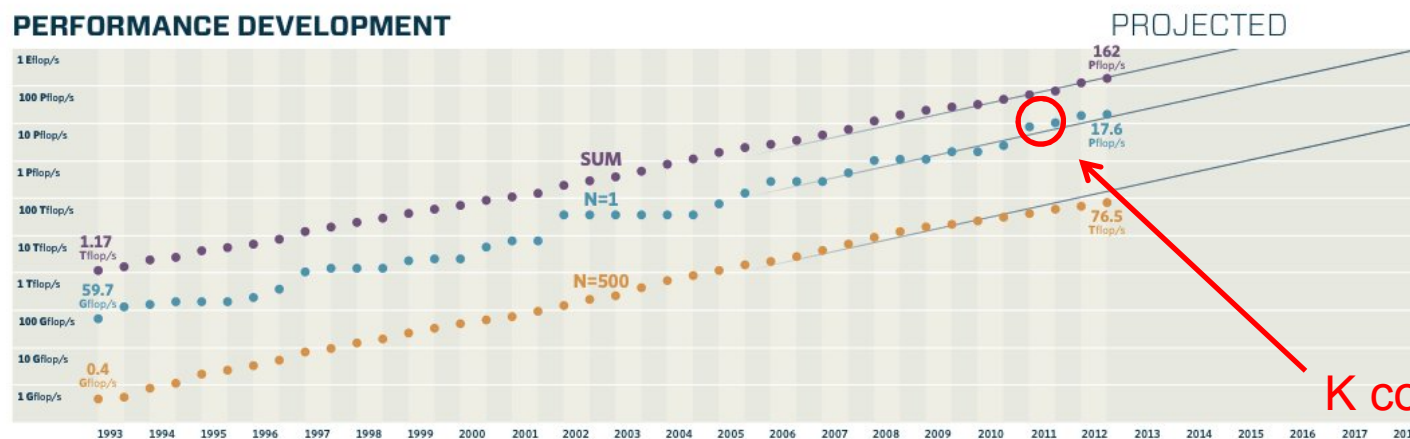
SPARC64™ VIIIfx



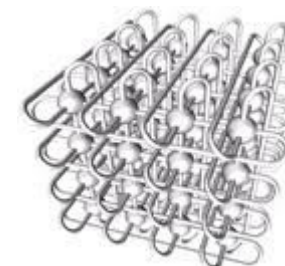
128 GFLOPS/CPU  
(8 cores/CPU)

|   | NAME       | SPECS                                                                      | SITE                     | COUNTRY | CORES     | RMAX<br>PFLOP/s | POWER<br>MW |
|---|------------|----------------------------------------------------------------------------|--------------------------|---------|-----------|-----------------|-------------|
| 1 | TITAN      | Cray XK7, Operon 6274 16C 2.2 GHz + Nvidia Kepler GPU, Custom interconnect | DOE/OS/ORNL              | USA     | 560,640   | 17.6            | 8.3         |
| 2 | SEQUOIA    | IBM BlueGene/Q, Power BQC 16C 1.60 GHz, Custom interconnect                | DOE/NNSA/LLNL            | USA     | 1,572,864 | 16.3            | 7.9         |
| 3 | K COMPUTER | Fujitsu SPARC64 VIIIfx 2.0GHz, Custom interconnect                         | RIKEN AICS               | Japan   | 705,024   | 10.5            | 12.7        |
| 4 | MIRA       | IBM BlueGene/Q, Power BQC 16C 1.60 GHz, Custom interconnect                | DOE/OS/ANL               | USA     | 786,432   | 8.16            | 3.95        |
| 5 | JUQUEEN    | IBM BlueGene/Q, Power BQC 16C 1.60 GHz, Custom interconnect                | Forschungszentrum Jülich | Germany | 393,216   | 4.14            | 1.97        |

## PERFORMANCE DEVELOPMENT



Tofu inter-connection  
6D Mesh/Torus

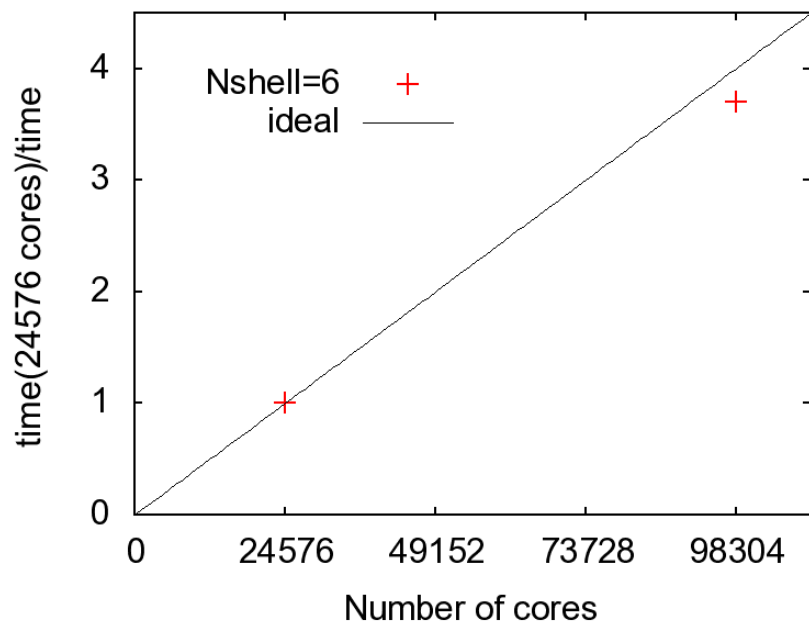
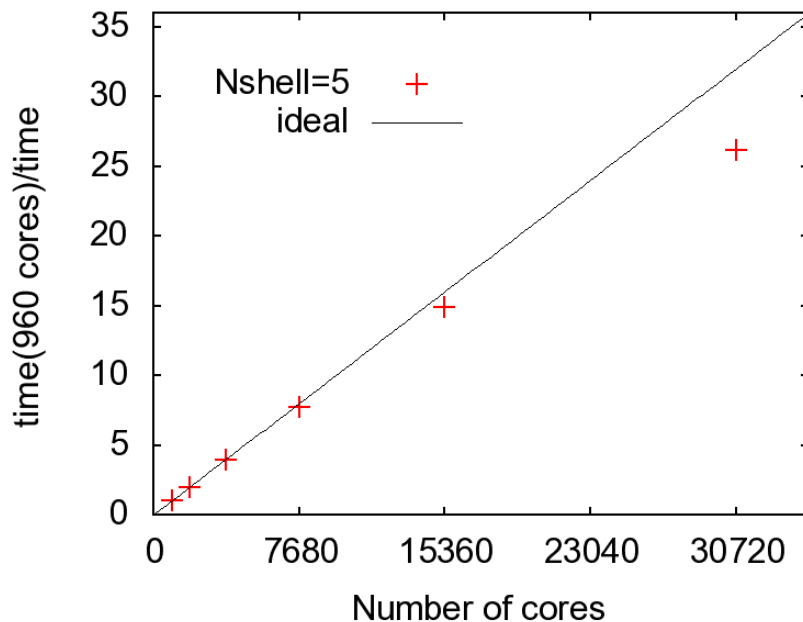


## HPCI Strategic Program Field 5 "The origin of matter and the universe"



# Parallel efficiency @ K-computer

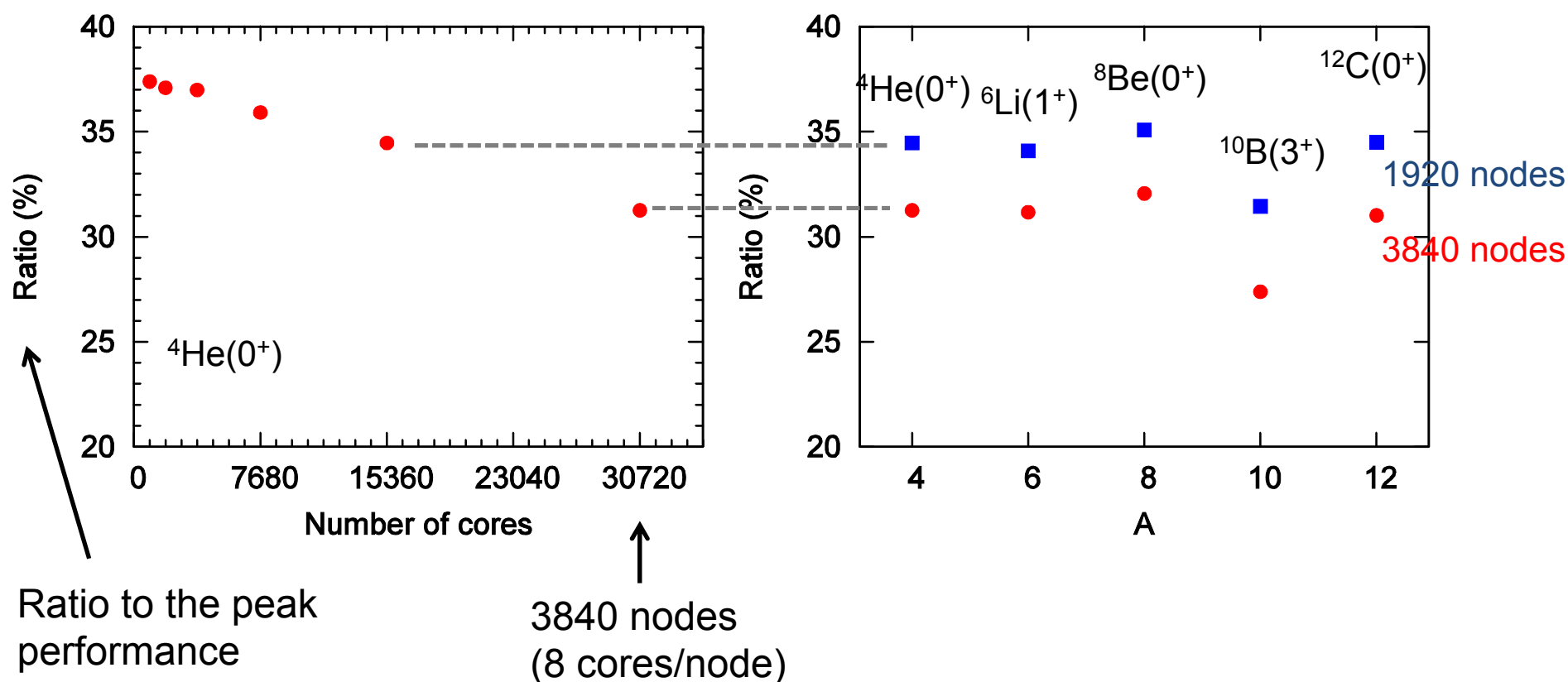
- Optimization of 15<sup>th</sup> basis dim. of the 4He (0+) w.f. in Nshell=5 w/ 100 CG iterations
- Optimization of 48<sup>th</sup> basis dim. of the 4He (0+) w.f. in Nshell=6 w/ 100 CG iterations



Note: it is a tentative result by early access to the K-computer at AICS, RIKEN.

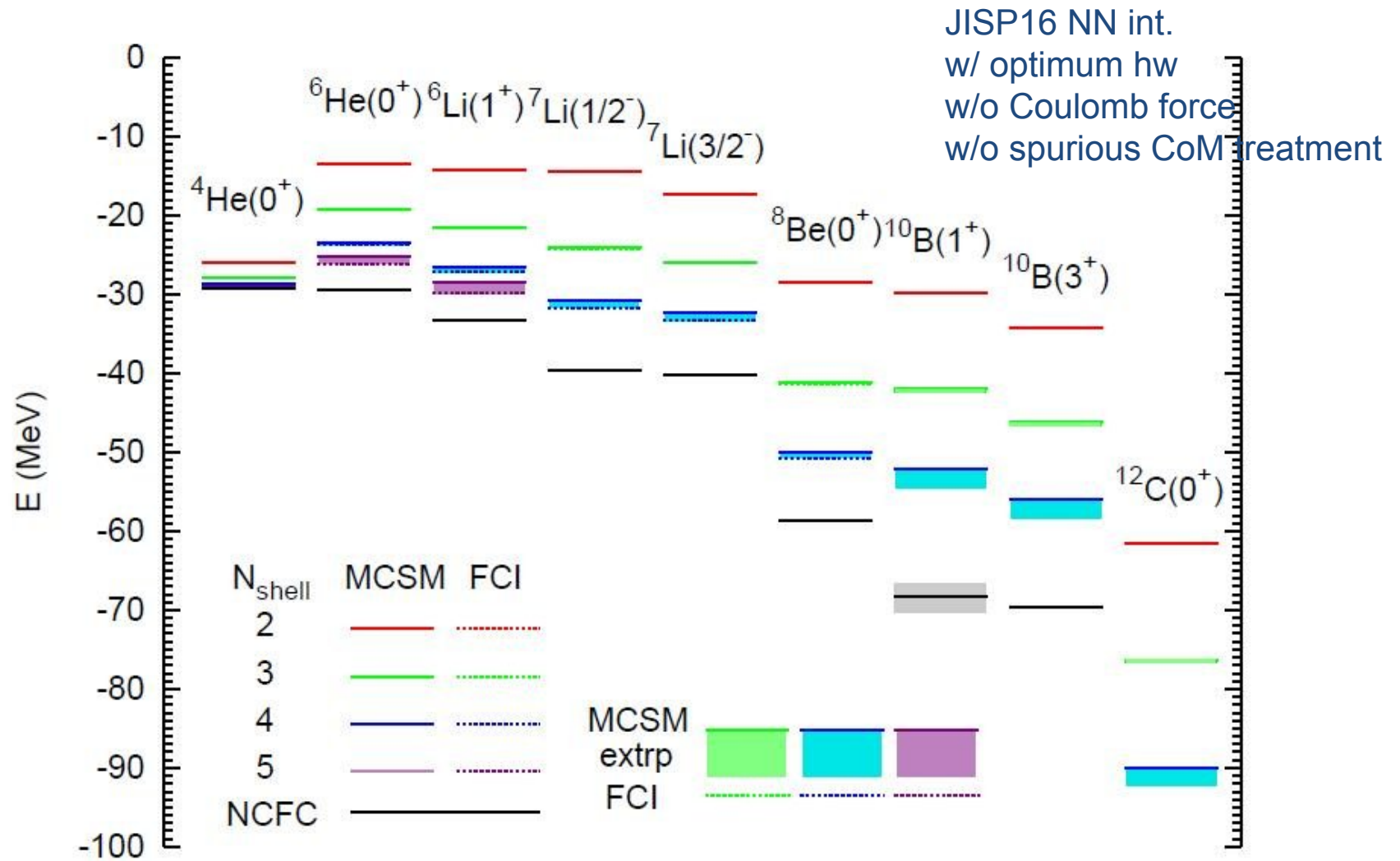
## Ratio to the peak performance @ K computer (phase IV-1)

- Test case: Optimization of 15<sup>th</sup> basis dim. of the w.f. in Nshell=5 w/ 100 CG iterations w/o preprocessing (MPI/OpenMP, 8 threads)



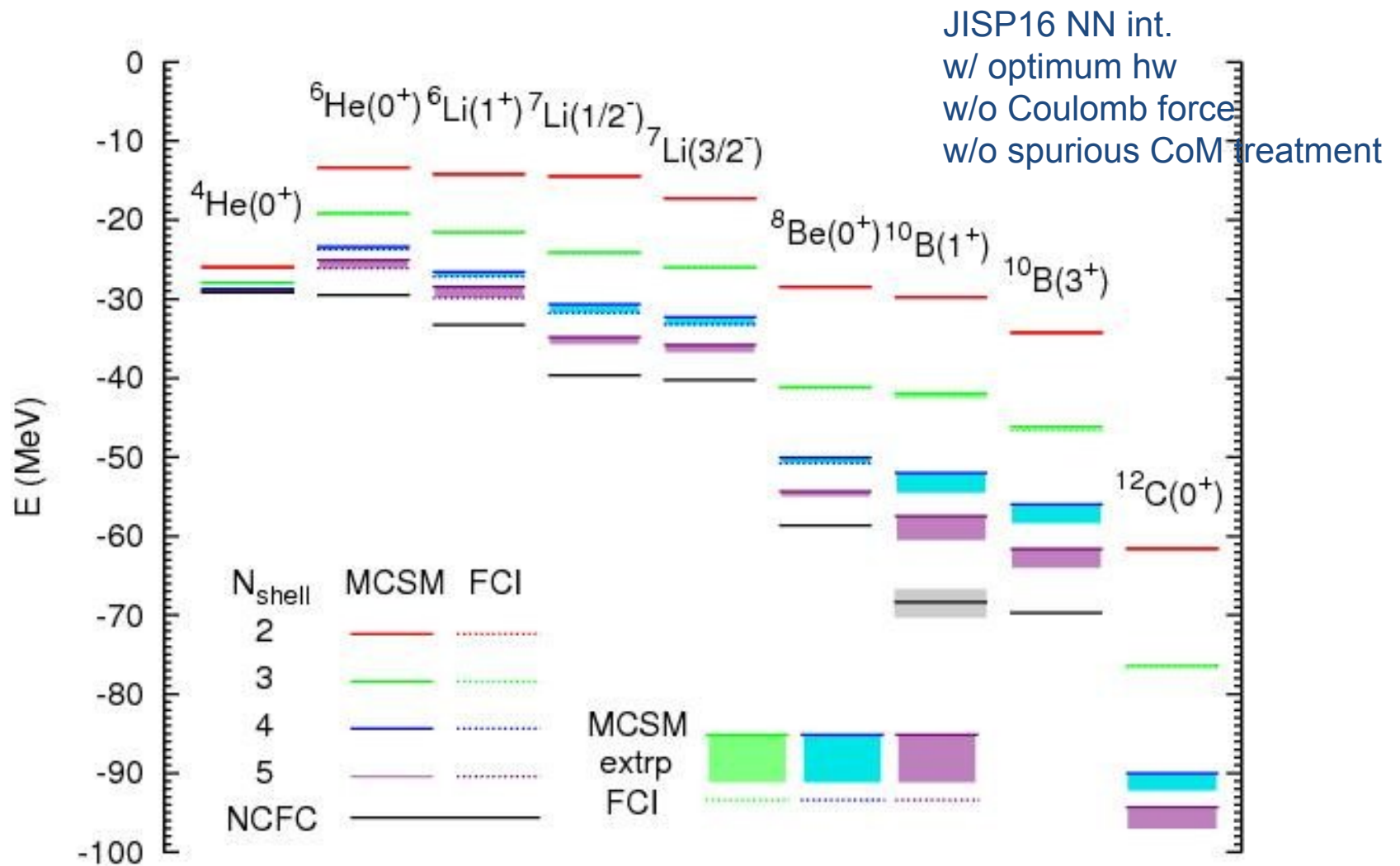
Note: it is a tentative result by early access to the K-computer at AICS, RIKEN.

# Energies of the Light Nuclei



Preliminary

# Energies of the Light Nuclei



# Density Profile from ab initio calc.

VMC

- Green's function Monte Carlo (GFMC)

- “Intrinsic” density is constructed by aligning the moment of inertia among samples

R. B. Wiringa, S. C. Pieper, J. Carlson & V. R. Pandharipande, Phys. Rev. C62, 014001 (2000)

- No-core full configuration (NCFC)

- Translationally-invariant density is obtained by deconvoluting the intrinsic & CM w.f.

C. Cockrell J. P. Vary & P. Maris, Phys. Rev. C86, 034325 (2012)

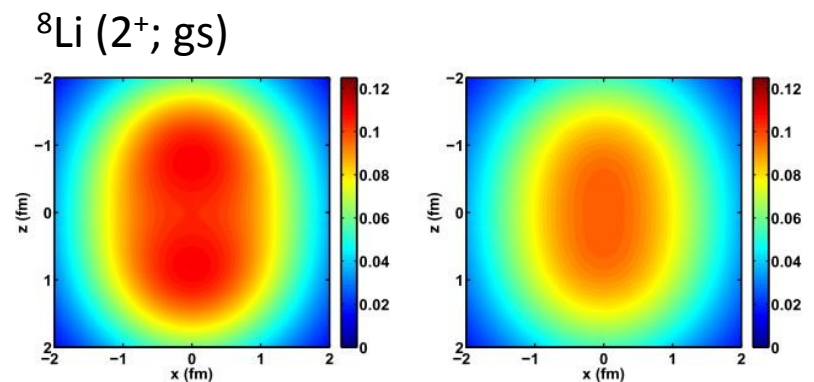
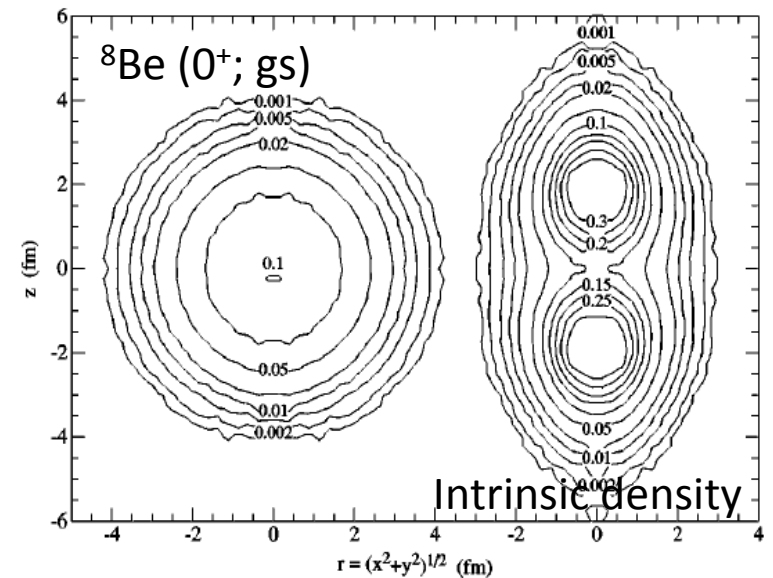


FIG. 12: (Color online) The  $y = 0$  slice of the translationally-invariant neutron density (left) of the  $2^+$  gs of  $^8\text{Li}$ . The space-fixed density for the same state is on the right. These densities were calculated at  $N_{\text{max}} = 12$  and  $\hbar\Omega = 12.5$  MeV.

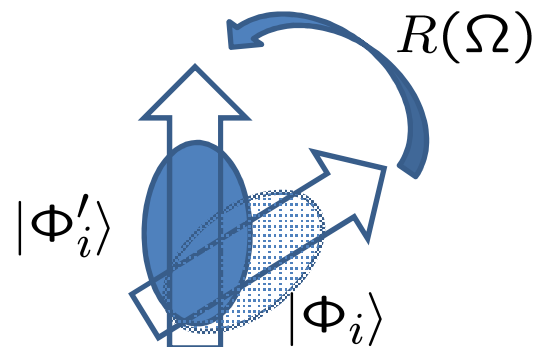
Translationally-  
invariant  
neutron density

Space-fixed  
neutron density

## How to construct an “intrinsic” density from MCSM w.f.

- MCSM wave function

$$|\Psi\rangle = \sum_{i=1}^{N_{basis}} c_i P^J P^\pi |\Phi_i\rangle$$



Rotation by diagonalizing Q-moment  
( $Q_{zz} > Q_{yy} > Q_{xx}$ )

- Wave function w/o the projections

$$\sum_{i=1}^{N_{basis}} c_i |\Phi_i\rangle = c_1 \text{ [diagram]} + c_2 \text{ [diagram]} + \dots + c_{N_{basis}} \text{ [diagram]}$$

- Wave function w/o the projection w/ the alignment of Q-moment

$$\sum_{i=1}^{N_{basis}} c_i |\Phi'_i\rangle = c_1 \text{ [diagram]} + c_2 \text{ [diagram]} + \dots + c_{N_{basis}} \text{ [diagram]}$$

# Density profiles in MCSM

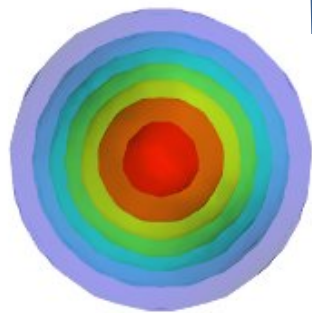
$$|\Phi\rangle = \sum_{i=1}^{N_{basis}} c_i |\Phi_i\rangle = c_1 \text{ (img)} + c_2 \text{ (img)} + c_3 \text{ (img)} + c_4 \text{ (img)} + \dots$$

Angular-momentum projection

Rotation of each basis

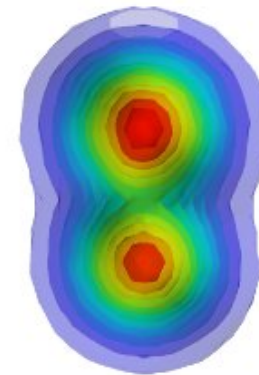
$$|\Psi\rangle = \sum_{i=1}^{N_{basis}} c_i P^J P^\pi |\Phi_i\rangle$$

$$|\Phi'\rangle = \sum_{i=1}^{N_{basis}} c_i R(\Omega_i) |\Phi_i\rangle$$



Laboratory frame

$^8\text{Be } 0^+ \text{ g.s. state}$

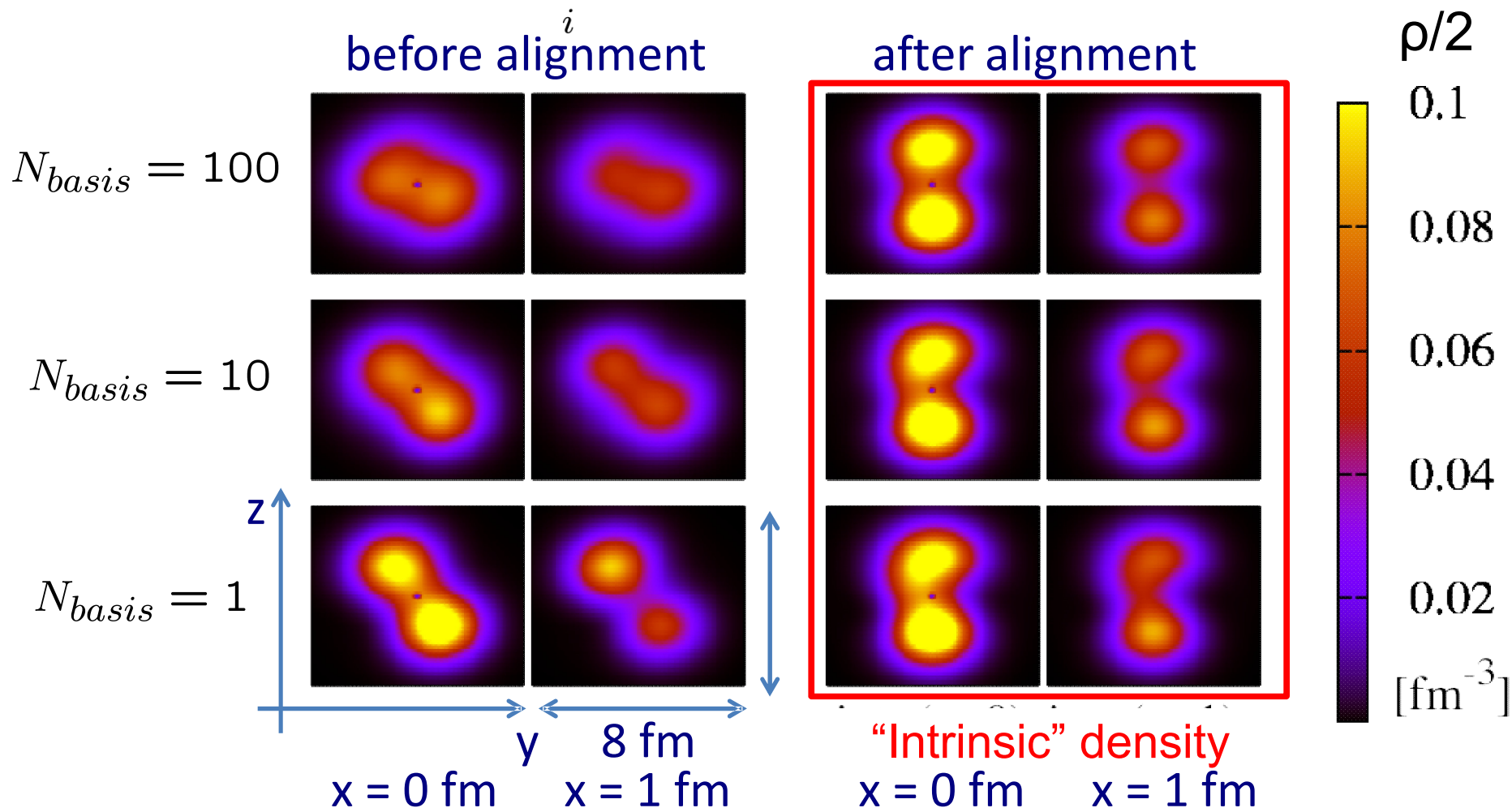


"Intrinsic" (body-fixed) frame

## Density profile of $^8\text{Be}$ $0^+$ g.s. state from MCSM w.f.

- Test calculation of the density by using the MCSM w.f. in Nshell = 4

$$\rho(\vec{r}) = \langle \Phi(\{\vec{r}_i'\}) | \sum_i \delta(\vec{r} - \vec{r}_i') | \Phi(\{\vec{r}_i'\}) \rangle$$



# Summary

- MCSM can be applied to no-core calculations of the p-shell nuclei.
  - Benchmarks for the p-shell nuclei have been performed and gave good agreements w/ FCI results.
  - Density profiles from MCSM many-body w.f. are preliminarily investigated and the cluster-like distributions are reproduced.

# Perspective

- MCSM algorithm/computation
  - Extension to larger model spaces ( $N_{\text{shell}} = 6, 7, \dots$ ), extrapolation to infinite model space, & comparison with another truncations
  - Inclusion of the 3-body force (thru. effective 2-body force)
- Physics
  - Cluster(-like) states (He & Be isotopes,  $^{12}\text{C}$  Hoyle state, ...)
  - sd-shell nuclei

Happy birthday, James!!