On nature of bound and resonance states in ^{12}C.

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Outline

Motivations

1. Structure of ^{12}C:
   - Bound states

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   - Resonance states

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1. Structure of ^{12}C:
   - Bound states
   - Resonance states
   - Hoyle state

Motivations

1. Structure of ^{12}C:
   - Bound states
   - Resonance states
   - Hoyle state
   - **Nature of resonance states in three-cluster continuum**
2. Selfconsistency of different methods

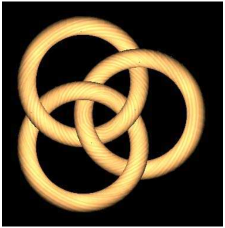
Resonance in three-cluster continuum

- Resonance state is generated in one channel, which is weakly coupled to other channels.

Resonance in three-cluster continuum

- Resonance state is generated in one channel, which is weakly coupled to other channels.
- Resonance is spread over all open channels. This type of resonance was predicted by A. Baz' and called as a diffusion-like resonance. The resonance is attributed to the effect that "the system spends most of its time wandering from one channel to another".
  A. I. Baz'. "Diffusion-like processes in the quantum theory of scattering," *Soviet J. Exp. Theor. Phys.*, vol. **43**, pp. 205–211, 1976.

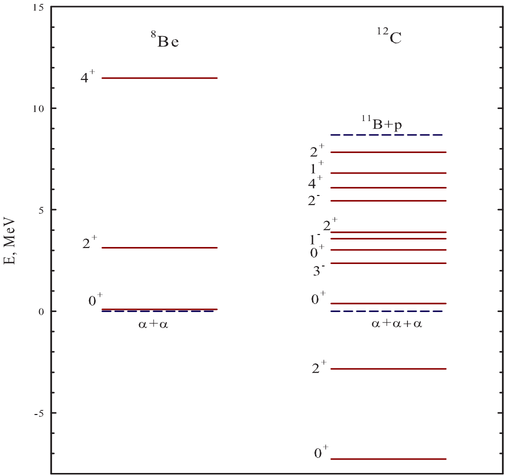
Borromean nucleus



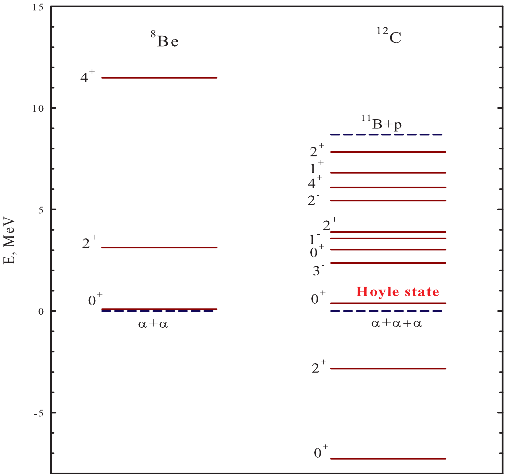
Wikipedia: Borromean rings.

The name "Borromean rings" comes from their use in the coat of arms of the aristocratic Borromeo family in Italy (XII-XIII century). The link itself is much older and has appeared in Gandhara (Afghan) Buddhist art from around the 2nd century, and in the form of the valknut on Norse image stones dating back to the 7th century.

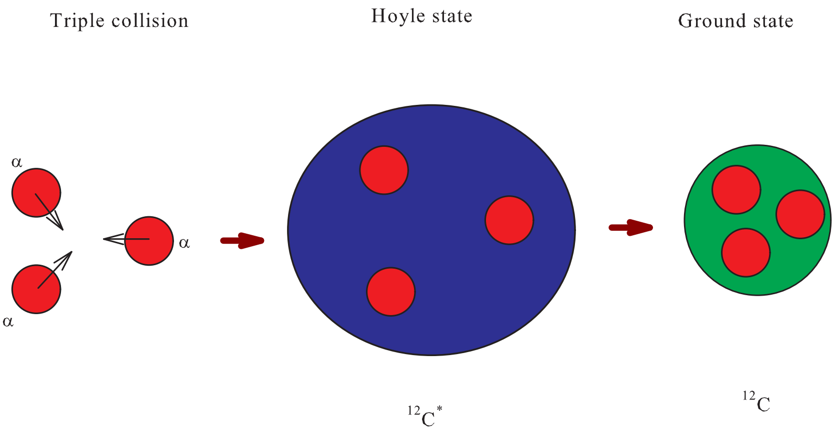
Borromean nucleus



Borromean nucleus. Hoyle state



Synthesis of ^{12}C. Hoyle state



$$3\alpha \Rightarrow {}^{12}C^* \Rightarrow {}^{12}C + \gamma$$

Hoyle state

- Hoyle state is the 0^+ resonance state in ^{12}C
- Energy of the Hoyle state is E = 0.4 MeV above the $\alpha + \alpha + \alpha$ threshold or E = 7.65 MeV above the ^{12}C ground state
- Very small width of the Hoyle state Γ = 8.5 eV, compare with width of the 0^+ resonance state in ^{8}Be Γ = 5.57 eV

F. Hoyle. “On Nuclear Reactions Occurring in Very Hot Stars. I. the Synthesis of Elements from Carbon to Nickel.” Astrophysical Journal Supplement, vol. 1, p.121 (1954).

Methods to study resonance states.

- Complex Scaling Method (CSM)
- Analytical Continuation on Couple Constant Method (ACCCM)
- Algebraic Model with the Hyperspherical Harmonics Basis (AMHHB)
- Models reducing three-cluster system to two-cluster system: $\alpha+\alpha+\alpha \Rightarrow \alpha+^{8}Be$
  - Microscopic *R* Matrix Method (MRM)
  - Algebraic Model with the Gaussian and Oscillator Basis (AMGOB)

Our model AMHHB

- V. Vasilevsky, A. V. Nesterov, F. Arickx, J. Broeckhove. *Phys. Rev. C*, vol. **63**, 034606 (16 pp), 2001.
- V. Vasilevsky, A. V. Nesterov, F. Arickx, J. Broeckhove. *Phys. Rev. C*, vol. **63**, 034607 (7 pp), 2001.
- J. Broeckhove, F. Arickx, P. Hellinckx, V. S. Vasilevsky, A. V. Nesterov. *J. Phys. G Nucl. Phys.*, vol. **34**, pp. 1955–1970, 2007.
- V. S. Vasilevsky, F. Arickx, J. Broeckhove, V. N. Romanov. *Ukr. J. Phys.*, vol. **49**, no. 11, pp. 1053–1059, 2004.
- V. Vasilevsky, A. V. Nesterov, F. Arickx, and J. Broeckhove. *Phys. Rev. C*, vol. **63**, 064604 (8 pp), 2001.

Application of AMHHB

- $^{6}He = \alpha + n + n$
- $^{6}Be = \alpha + p + p$
- $^{5}H = t + n + n$
- $^{5}B =^{3} He + p + p$
- $^{4}H = d + n + n$
- $^{4}Li = d + p + p$

Our model AM GOB (GOBLIN)

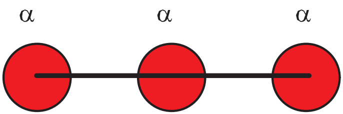
- V. S. Vasilevsky, F. Arickx, J. Broeckhove, T. P. Kovalenko. "A microscopic three-cluster model with nuclear polarization applied to the resonances of ^{7}Be and the reaction $^{6}Li(p,^{3}He)^{4}He$," *Nucl. Phys. A*, vol. **824**, pp. 37–57, 2009.
- A. V. Nesterov, V. S. Vasilevsky, T. P. Kovalenko. "Effect of cluster polarization on the spectrum of the ^{7}Li nucleus and on the reaction $^{6}Li(n,^{3}H)^{4}He$," *Phys. Atom. Nucl.*, vol. **72**, pp. 1450–1464, 2009.

Our model allows us

- to calculate S matrix
- to determine energy and total width of a resonance state
- to calculate partial widths of a resonance state
- to find wave function(s) of a resonance state
- to determine the most probable three-cluster configuration in coordinate and momentum space
- to determine optimal way for decay of resonance states

Linear chain of alpha-particles

There are around 100 theoretical papers suggesting linear configuration of three alpha-particles at the Hoyle state and other excited states of ^{12}C.



1. G. S. Anagnostatos, "Alpha-chain states in ^{12}C," *Phys. Rev. C*, vol. 51, pp. 152–159, 1995.
2. A. C. Merchant and W. D. M. Rae, "Systematics of alpha-chain states in 4N-nuclei," *Nucl. Phys. A*, vol. 549, pp. 431–438, 1992.
3. Y. Kanada-En'yo, "The Structure of Ground and Excited States of ^{12}C," *Progr. Theor. Phys.*, vol. 117, pp. 655–680, 2007.

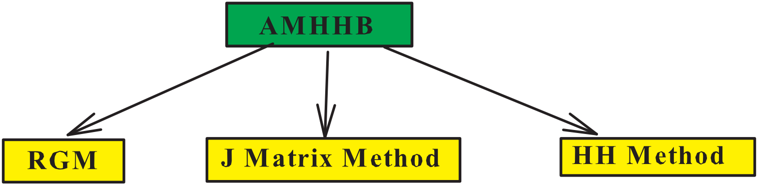
Microscopic models of ^{12}C.

1. K. Arai, “Resonance states of ^{12}C in a microscopic cluster model,” *Phys. Rev. C*, vol. 74, p. 064311, Dec. 2006. (CSM).
2. R. Pichler, H. Oberhummer, A. Csótó, and S. A. Moszkowski, “Three-alpha structures in ^{12}C,” *Nucl. Phys. A*, vol. 618, pp. 55–64, Feb. 1997. (CSM).
3. P. Descouvemont and D. Baye, “Microscopic theory of the $^{8}Be(\alpha,\gamma)^{12}C$ reaction in a three-cluster model,” *Phys. Rev. C*, vol. 36, pp. 54–59, July 1987. (MRM).

Peculiarities of a microscopic three-cluster model

- Due to antisymmetrization, the intercluster interaction is nonlocal and energy-dependent.
- It contains three-body forces, which originate from NN interaction, kinetic energy and overlap kernel.
- Square-integrable, orthogonal basis of functions is the best remedy to treat such systems

AMHHB and other methods



Three-cluster wave function

$$\Psi^J = \widehat{\mathcal{A}}\left\{\Phi\left(\alpha_1\right)\Phi\left(\alpha_2\right)\Phi\left(\alpha_3\right)\ f\left(\mathbf{x},\mathbf{y}\right)\right\}_J$$

where

- $\Phi\left(\alpha_\nu\right)$ is a shell-model wave function for the internal motion of α-particle ($\nu = 1, 2, 3$) (fixed)

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- $\widehat{\mathcal{A}}$ is the antisymmetrization operator

Three-cluster wave function

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where

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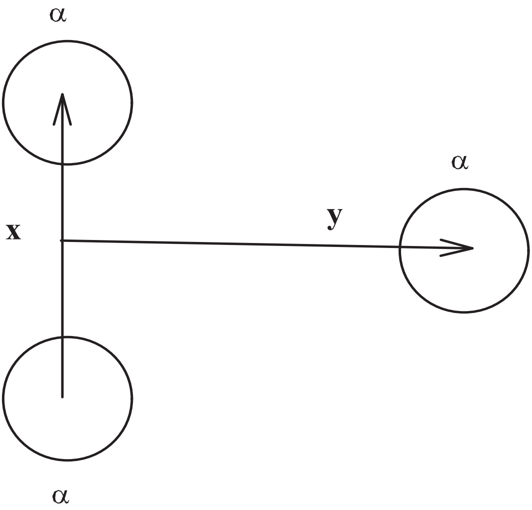
- $f\left(\mathbf{x},\mathbf{y}\right)$ is a wave function of inter-cluster motion (to be determined)
- $\widehat{\mathcal{A}}$ is the antisymmetrization operator
- ^{12}C is a system of 12 fermions and 3 bosons

Jacobi coordinates



Hyperspherical Harmonics

Hyperspherical coordinates

$$\rho = \sqrt{\mathbf{x}^2 + \mathbf{y}^2} \quad \text{hyper-radius,}$$

$$\theta = \arctan\left(\frac{|\mathbf{y}|}{|\mathbf{x}|}\right) \quad \text{hyper-angle,}$$

$$\widehat{\mathbf{x}} = \mathbf{x}/\left|\mathbf{x}\right|, \; \widehat{\mathbf{y}} = \mathbf{y}/\left|\mathbf{y}\right| \quad \text{unit vectors}$$

$$|\mathbf{x}| = \rho\cos\theta$$

$$|\mathbf{y}| = \rho\sin\theta$$

Hyperspherical Harmonics

Basis of the Hyperspherical Harmonics

$$\begin{aligned}\Psi &= \widehat{\mathcal{A}}\left\{\Phi\left(\alpha_1\right)\Phi\left(\alpha_2\right)\Phi\left(\alpha_3\right)f\left(\mathbf{x},\mathbf{y}\right)\right\} \\ &= \widehat{\mathcal{A}}\left\{\Phi\left(\alpha_1\right)\Phi\left(\alpha_2\right)\Phi\left(\alpha_3\right)f\left(\rho,\theta;\widehat{\mathbf{x}},\widehat{\mathbf{y}}\right)\right\} \\ &= \sum_{n_\rho,K,l_1,l_2} C_{n_\rho,K,l_1,l_2}\left|n_\rho,K,l_1,l_2;LM;\left(\rho,\theta;\widehat{\mathbf{x}},\widehat{\mathbf{y}}\right)\right\rangle\end{aligned}$$

where $\left|n_\rho,K,l_1,l_2;LM\right\rangle$ is the three-cluster oscillator functions (basis functions of the Hyperspherical Harmonic Method)

$$\left|n_\rho,K,l_1,l_2;LM\right\rangle =$$

$$= \widehat{\mathcal{A}}\left\{\Phi\left(\alpha_1\right)\Phi\left(\alpha_2\right)\Phi\left(\alpha_3\right)R_{n_\rho,K}\left(\rho\right)\chi_{K,l_1,l_2}\left(\theta\right)\left\{Y_{l_1}\left(\widehat{\mathbf{x}}\right)Y_{l_2}\left(\widehat{\mathbf{y}}\right)\right\}_{LM}\right\}$$

Hyperspherical Harmonics

Table: Relations between variables and quantum numbers, which determine dynamics of three-cluster triangle

Triangle	Variables	Quantum numbers
Size	ρ	n_ρ

Hyperspherical Harmonics

Table: Relations between variables and quantum numbers, which determine dynamics of three-cluster triangle

Triangle	Variables	Quantum numbers
Size	ρ	n_ρ
Shape	θ	K

Hyperspherical Harmonics

Table: Relations between variables and quantum numbers, which determine dynamics of three-cluster triangle

Triangle	Variables	Quantum numbers
Size	ρ	n_ρ
Shape	θ	K
Rotation	$\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$-unit vectors	l_1, l_2, LM

Numeration of the three-cluster channels:

$$c = \{K, l_1, l_2\}$$

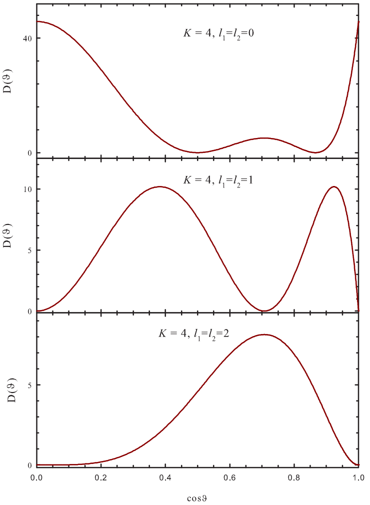
Hyperspherical Harmonics. Density distribution

Density distribution:

$$D(\theta) = \left|\chi_{K,l_1,l_2}(\theta)\right|^2$$

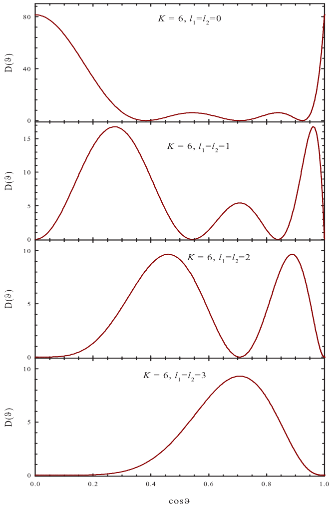
can be displayed as a function of θ or $\cos\theta = |\mathbf{x}|/\rho$.

Hyperspherical Harmonics, $K = 4$



From acute to obtuse triangle

Hyperspherical Harmonics, $K = 6$



Internal and asymptotic part of wave function

$$\begin{aligned}
\Psi &= \sum_{n_\rho,K,l_1,l_2} C_{n_\rho,K,l_1,l_2} \left|n_\rho, K, l_1, l_2; LM\right\rangle = \\
&= \sum_{n_\rho \le N_i} \sum_{K \le K_{\max}^{(i)}} \sum_{l_1,l_2} C^{(i)}_{n_\rho,K,l_1,l_2} \left|n_\rho, K, l_1, l_2; LM\right\rangle \\
&+ \sum_{n_\rho > N_i} \sum_{K \le K_{\max}^{(a)}} \sum_{l_1,l_2} C^{(a)}_{n_\rho,K,l_1,l_2} \left|n_\rho, K, l_1, l_2; LM\right\rangle
\end{aligned}$$

Here N_i marks border between internal and asymptotic regions,

$$K_{\max}^{(i)} \ge K_{\max}^{(a)}$$

and

$$C^{(a)}_{n_\rho,K,l_1,l_2} = C^{(a)}_{n_\rho,c} = \delta_{c_0 c}\psi_K^{(-)}(k\rho_n) - S_{c_0 c}\psi_K^{(-)}(k\rho_n)$$

Asymptotic part of Hamiltonian

The effective three-cluster potential which originates from the nucleon-nucleon interaction

$$V_{c,\widetilde{c}}^{(NN)} = \frac{V_{c,\widetilde{c}}}{\rho^3}$$

and from the Coulomb interaction

$$V_{c,\widetilde{c}}^{(C)} = \frac{Z_{c,\widetilde{c}}}{\rho}$$

Theoretical set-up

Input parameters

NN potential

Minnesota potential

Theoretical set-up

Input parameters

NN potential

Minnesota potential

Basis

Hypermomentum: $K_{\mathrm{max}} = 14$ for even parity states
Hypermomentum: $K_{\mathrm{max}} = 13$ for odd parity states
Hyperradial excitations: $n_\rho \leq 100$

Theoretical set-up

Input parameters

Adjustable parameters

- Oscillator length b: is adjusted to minimize energy of the α particle.
- Majorana parameter u: is adjusted to reproduce phase shifts of $\alpha + \alpha$ scattering and energy and width of 0^+, 2^+ and 4^+ resonance states in ^8Be.

Theoretical set-up. Minnesota potential

$$V_{ij} = \left[V_R + \frac{1}{2}\left(1 + P_{ij}^{\sigma}\right) V_T + \frac{1}{2}\left(1 - P_{ij}^{\sigma}\right) V_S\right] \left[\frac{1}{2}u + \frac{1}{2}(2 - u) P_{ij}^{r}\right]$$

where

$$V_R = V_{0R} \exp\left\{-k_R \left(\mathbf{r}_i - \mathbf{r}_j\right)^2\right\}$$

$$V_T = V_{0T} \exp\left\{-k_T \left(\mathbf{r}_i - \mathbf{r}_j\right)^2\right\}$$

$$V_S = V_{0S} \exp\left\{-k_S \left(\mathbf{r}_i - \mathbf{r}_j\right)^2\right\}$$

Theoretical set-up

Table: Number of channels for unsymmetrized and symmetrized Hyperspherical Harmonics.

J^π	0^+	2^+	4^+	1^-	3^-	
K_{max}	14	14	14	13	13	Clusters
N_{ch} $(\{K, l_1, l_2\})$	36	84	105	56	84	$A_1 \neq A_2 \neq A_3$
N_{ch} $(\{K, l_1, l_2\})$	20	44	54	28	42	$A_1 = A_2 \neq A_3$
N_{ch} $(\{K, \nu\})$	8	16	19	9	14	$A_1 = A_2 = A_3$

S-matrix representation

By solving system of the dynamic equations, we obtain scattering S-matrix $\|S_{cc'}\|$ for N_{ch} channel system and N_{ch} wave functions. Two different representations for S-matrix:

- Inelastic parameter $\eta_{c,c'}$ and corresponding phase shift $\delta_{c,c'}$

$$S_{c,c'} = \eta_{c,c'} \exp\left\{2i\delta_{c,c'}\right\}$$

- Eigenphase shift δ_ν or S-matrix

$$S_\nu = \exp\left\{2i\delta_\nu\right\},$$

where ν (=1,2,..., N_{ch}). The relation between the original $\left\|S_{c,c'}\right\|$ and diagonal $\|S_\nu\|$ forms of the S-matrix is

$$S_{c,c'} = \sum_\nu U_\nu^c S_\nu U_\nu^{c'}$$

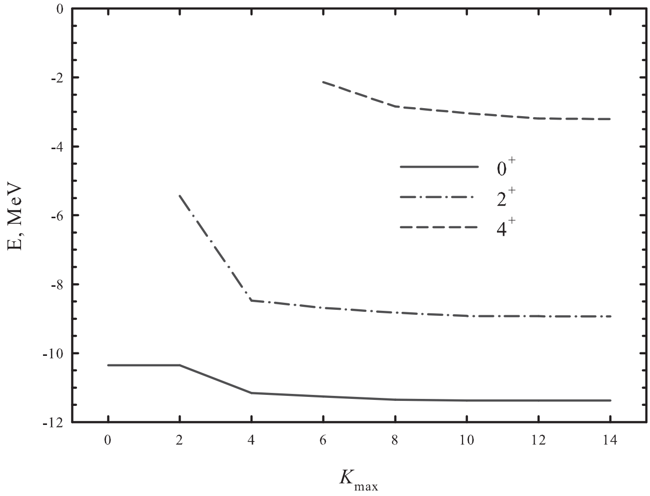
where $\|U_\nu^c\|$ is an orthogonal matrix.

Energy and width of resonance state

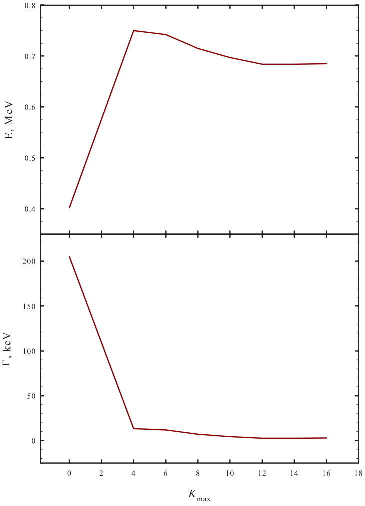
Energy E_r and total width Γ of the resonance state are determined in a traditional way through the first and second derivatives of the eigenphase shift δ_ν:

$$\frac{d^2\delta_\nu}{dE^2} = 0 \Longrightarrow E_r, \quad \Gamma = 2\left(\left.\frac{d\delta_\nu}{dE}\right|_{E_r}\right)^{-1}.$$

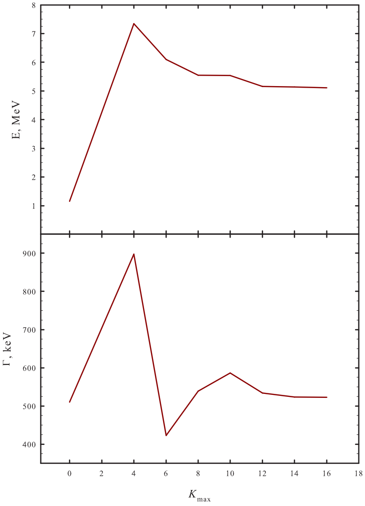
Bound states



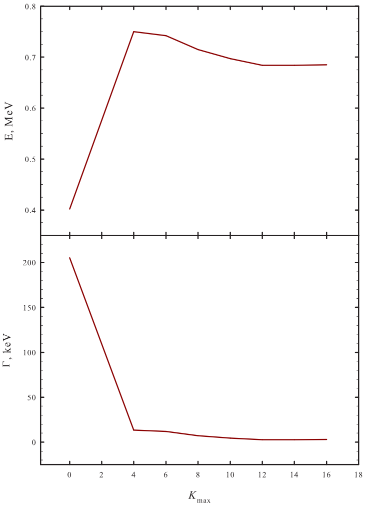
Resonance states. First 0^+ resonance. $K_{\max} = K_{\max}^{(i)} = K_{\max}^{(a)}$

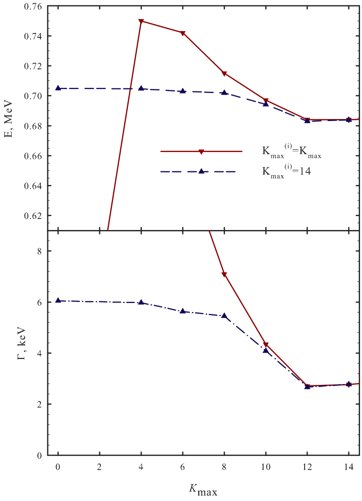


Resonance states. Second 0^+ resonance. $K_{max} = K^{(i)}_{max} = K^{(a)}_{max}$

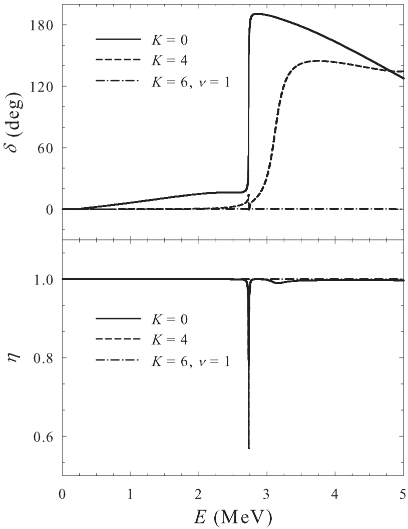


Internal and asymptotic regions

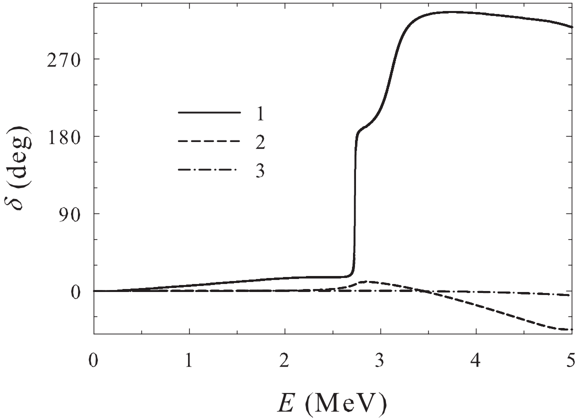




Resonance state $J^\pi = 2^+$. Phase shifts and inelastic parameters.



Resonance state $J^{\pi} = 2^{+}$. Eigenphase shifts.



Resonance states

Table: Energy (MeV) and width (keV) of the 0^+ and 2^+ states calculated with $K_{\max}^{(+)}$

L^π	$K_{\max}$	0	4	6	8	10	12	14
0^+	E	0.402	0.750	0.742	0.715	0.697	0.684	0.684
	Γ	205.1	13.40	11.79	7.10	4.35	2.71	2.77
0^+	E	1.151	7.340	6.094	5.546	5.538	5.158	5.141
	Γ	510	898	422	539	586	534	523
2^+	E	-	3.276	2.886	2.830	2.78	2.737	2.731
	Γ	-	30.19	13.07	11.85	9.95	8.84	8.75
2^+	E	-	3.504	3.269	3.215	3.170	3.143	3.113
	Γ	-	275	352	308	280	264	247

Partial widths of resonance states.

Table: Partial widths of resonance states of ^{12}C. Energy is in MeV, total and partial widths are given in keV.

L^π	0^+		2^+		2^+	
E	0.684		2.775		3.170	
Γ	2.786		9.95		280.24	
Γ_1	2.786	$K=0$	6.11	$K=2$	13.46	$K=2$
Γ_2	0	$K=4$	3.84	$K=4$	278.89	$K=4$
Γ_3	0	$K=6$	$<10^{-5}$	$K=6$	$<10^{-5}$	$K=6$

Total width $\Gamma = \sum_i \Gamma_i$.

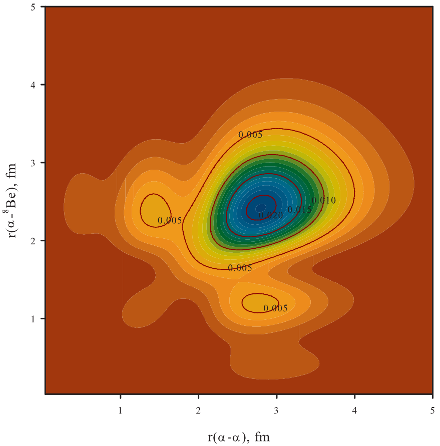
Partial widths of resonance states.

Table: Partial widths of resonance states of ^{12}C. Energy is in MeV, total and partial widths are given in keV.

L^{π}	4^+		1^-		3^-	
E	5.603		3.516		0.672	
Γ	0.55		0.210		8.34	
Γ_1	0.23	$K=4$	0.206	$K=3$	8.34	$K=3$
Γ_2	0.15	$K=6$	0.002	$K=5$	0	$K=5$
Γ_3,	0.16	$K=8$	$<10^{-5}$	$K=7$	0	$K=7$

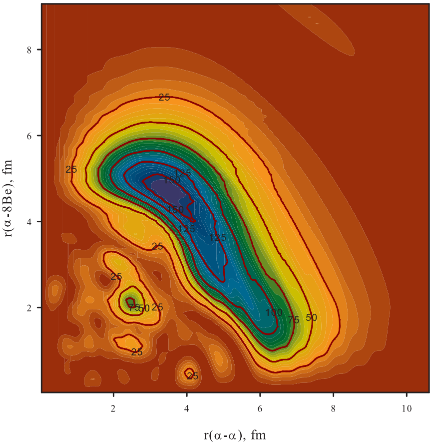
Dominant channels

Correlation function for the ground 0^+ state.



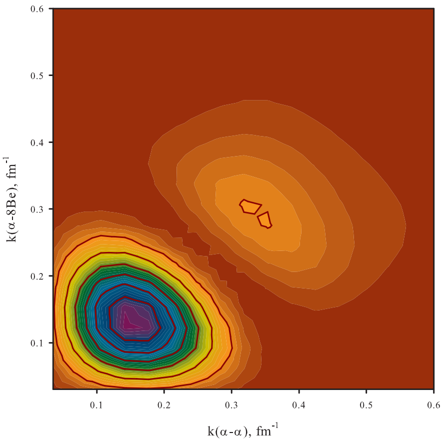
Dominant way for decay of resonance states.

Correlation function for the first 0^+ resonance.



LINEAR CONFIGURATION

Correlation function for the first 0^+ resonance. Momentum space



Comparison to the literature. I

Table: Bound and resonance states of ^{12}C obtained with the AMHHB model and CSM. E in MeV, Γ in keV.

Method	AMHHB		CSM-Arai [1]		CSM-Pichler [2]	
J^π	E	Γ	E	Γ	E	Γ
0^+	-11.372		-11.37		-10.43	
	0.684	2.78	0.4	< 1	0.64	14
	5.156	534.00	4.7	1000	5.43	920
2^+	-8.931		-8.93		-7.63	
	2.775	9.95	2.1	800	6.39	1100
	3.170	280.24	4.9	900		

1. K. Arai, "Resonance states of ^{12}C in a microscopic cluster model," *Phys. Rev. C*, 74, 064311, 2006.
2. R. Pichler, H. Oberhummer, A. Csótó, and S. A. Moszkowski, "Three-alpha structures in ^{12}C," *Nucl. Phys. A*, 618, 55, 1997.

Comparison to the literature. II

Table: Bound and resonance states of ^{12}C obtained with the AMHHB model and CSM. E in MeV, Γ in keV.

Method	AMHHB		CSM-Arai [1]		CSM-Pichler [2]	
J^π	E	Γ	E	Γ	E	Γ
4^+	-3.208		-3.21			
	5.603	0.55	5.1	2000		
1^-	3.516	0.21	3.4	200	3.71	360
3^-	0.672	8.34	0.6	< 50	1.16	25
	4.348	2.89	7.1	5400	11.91	1690
	5.433	334.90	9.6	400		

1. K. Arai, "Resonance states of ^{12}C in a microscopic cluster model," *Phys. Rev. C*, 74, 064311, 2006.
2. R. Pichler, H. Oberhummer, A. Csótó, and S. A. Moszkowski, "Three-alpha structures in ^{12}C," *Nucl. Phys. A*, 618, 55, 1997.

Comparison to experiment. I

Table: Bound and resonance states of ^{12}C. E in MeV, Γ in keV.

	AMHHB		Experiment [1]	
J^π	E	Γ	E	Γ
0^+	-11.372		-7.2746	
	0.684	2.78	0.3796 ± 0.0002	$(8.5 \pm 1.0) \times 10^{-3}$
	5.156	534.00	3.0 ± 0.3	3000 ± 700
2^+	-8.931		-2.8357 ± 0.0003	
	2.775	9.95	3.89 ± 0.05	430 ± 80
	3.170	280.24	8.17 ± 0.04	1500 ± 200
4^+	-3.208			
	5.603	0.55	6.808 ± 0.015	258 ± 15

1 F. Ajzenberg-Selove, “Energy levels of light nuclei A = 11-12,” *Nucl. Phys. A*, 506, 1, 1990.

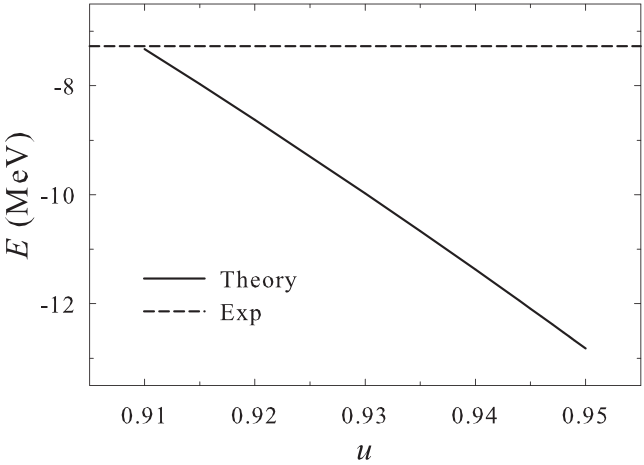
Comparison to experiment. II

Table: Resonance states of ^{12}C. E in MeV, Γ in keV.

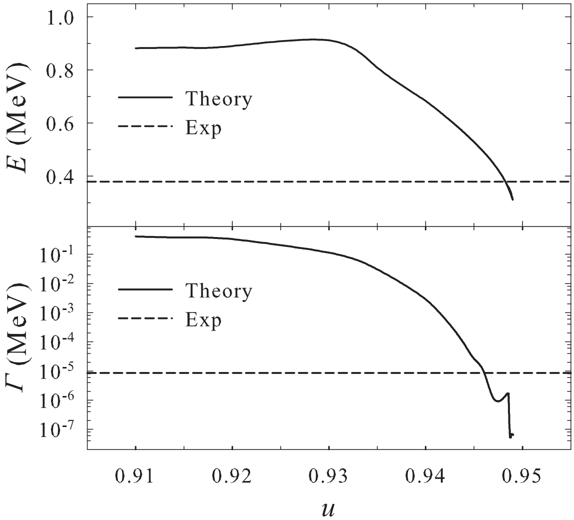
	AMHHB		Experiment [1]	
J^π	E	Γ	E	Γ
1^-	3.516	0.21	3.569 ± 0.016	315 ± 25
3^-	0.672	8.34	2.366 ± 0.005	34 ± 5
	4.348	2.89		
	5.433	334.90		

1. F. Ajzenberg-Selove, “Energy levels of light nuclei A = 11-12,” *Nucl. Phys. A*, 506, 1, 1990.

Energy of the ground 0^+ state as a function of u.



Energy and width of the Hoyle state as a function of u.



Summary

- Resonance state in three-cluster system is generated mainly by one (dominant) channel, which is weakly coupled to other open (nondominant) channels.
- However these (nondominant) channels effect very much parameters (energy and width) of the resonance state.
- There is a very small probability for linear chain configuration in bound and resonance states of ^{12}C.
- More details in V. Vasilevsky et al. *Phys. Rev. C*, vol. **85**, 034318, 2012.

Gratitude

THANK YOU VERY MUCH!