Witten parameter and HPC on the lattice

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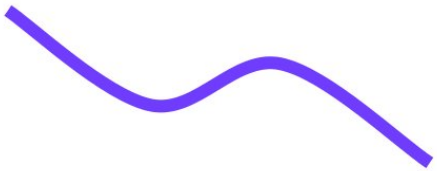
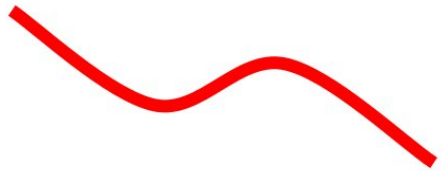
Operators in 4D Gauge Theory

- Dimension 0: **Local operators**
  (much studied in AdS/CFT)
- Dimension 1: **Line operators**



Wilson line

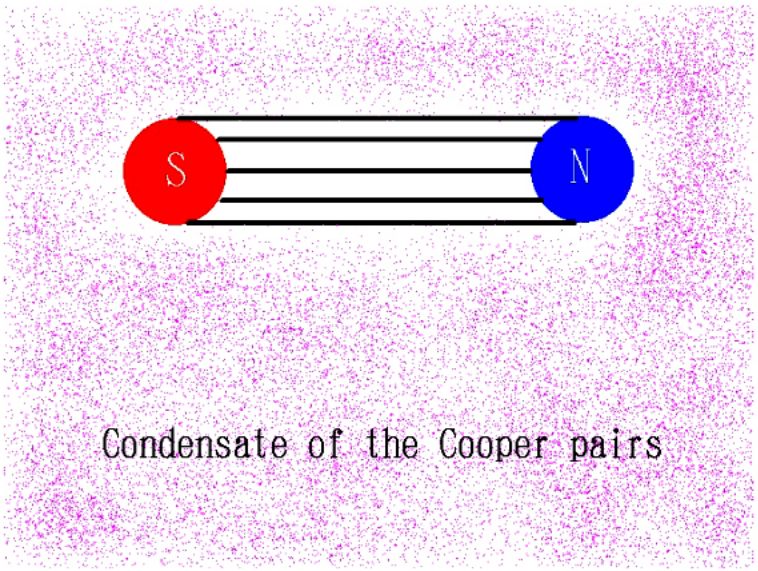


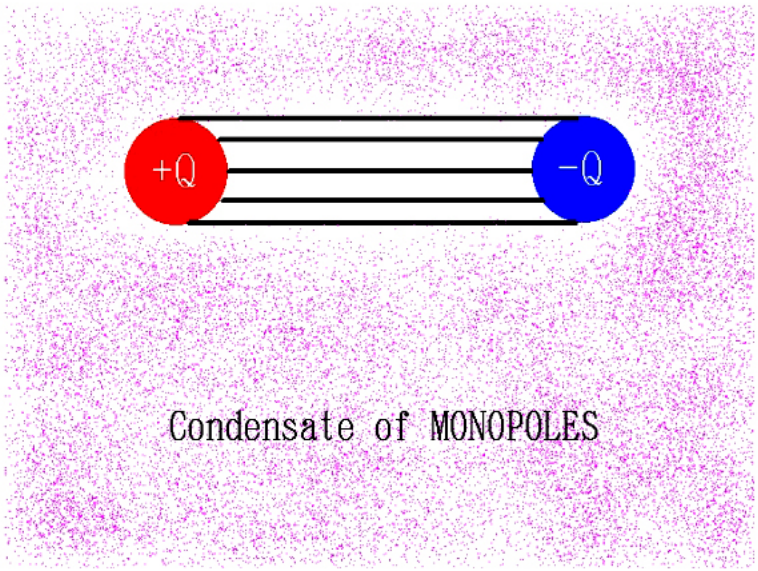
't Hooft line

- Dimension 2: **Surface operators**
  (Witten parameter)
- Dimension 3: **Boundaries**

Confinement

– a phenomenon which we can not see free quarks.





The monopole – the source of the magnetic field.

- The Dirac monopole
- The 't Hooft-Polyakov monopole

The value sensitive to monopoles – Witten parameter.

Witten parameter

In the $U(1)$ gauge field theory (the electrodynamics) $\Longrightarrow$ $e^{\imath\kappa \oint \mathbf{H}\cdot d\mathbf{S}} \equiv 1$

In the $SU(2)$-gluedynamics this identity does not necessarily hold $\Longrightarrow$ $e^{\imath\kappa \sum\limits_k \mathbf{H}_k \cdot d\mathbf{S}_k} \neq 1$

The parameter determine flow of the chromomagnetic field through a closed surface:

Witten parameter:

$$W(S) = Re \prod_S e^{\imath\theta_p}$$

We break the symmetry: $SU(2) \to U(1)$.

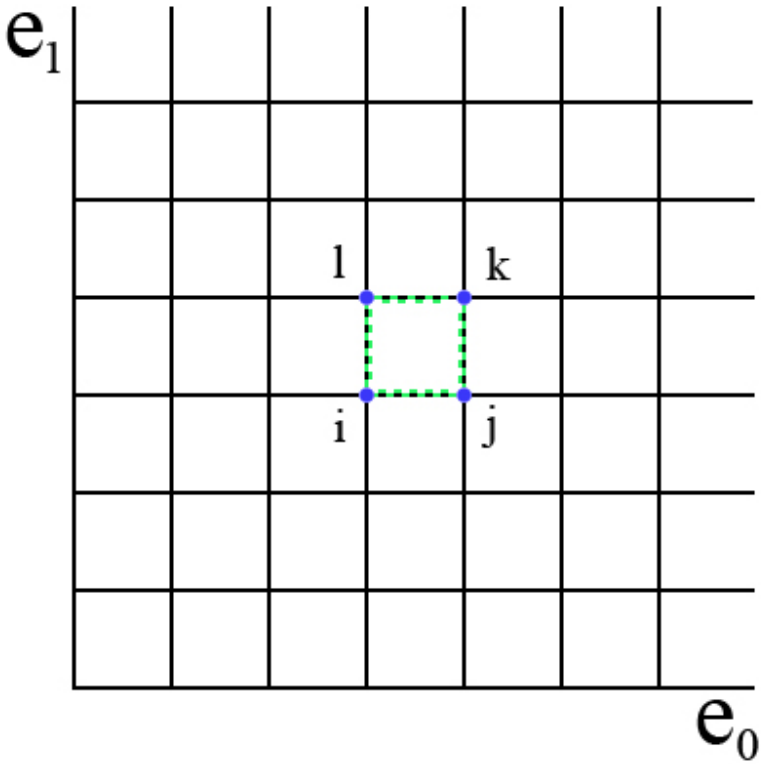
The θ_p is got from the plaquette tensor of the gauge field

$$F_p = \widehat{1}\, cos\,\theta_p + \imath\, n_i\sigma_i\, sin\,\theta_p$$

The lattice calculation

Problems of QFT:

- divergence (UV and IR)
- nonperturbative effects are playing a key role in the QCD



QCD on the lattice:

- vertex - field
  $\psi(x) \to \psi(x_i)$
- edge (link) - gauge field
  $A_\mu \to U(L) = e^{ig_0 \int_L A_\mu \, dx^\mu}$

gauge transformation:

$$U(L) \to g^{-1}(L_{end})\, U(L)\, g(L_{begin})$$

Wilson: $S_W = \sum_{plaquettes} S_P$, where $S_P = \beta\left(1 - \frac{1}{N} Tr\, U_P\right)$

The calculation of the average values

1. The configurations are generated with Boltzmann weight:

$$p(U)\, d(U) \sim e^{-S_W(U)}\, d(U)$$

2. The partition function has the form:

$$Z = \int (dU)\; e^{-S_W(U)}$$

3. The calculation of the average value:

$$\langle A \rangle = Z^{-1} \int (dU)\; A(U)\; e^{-S_W(U)} \qquad \text{or} \qquad \langle A \rangle = \sum_i A(U_i) \,/\, N_{conf}$$

Witten parameter on the lattice

$$W(S) = Re \prod_S e^{i\theta_p}, \text{ where } \theta_p = \arccos\left(\frac{1}{2} Tr\, F_p\right)$$

It used for best statistics:

- Multilevel
- Multi-hit

$+$ *HPC with using MPI*

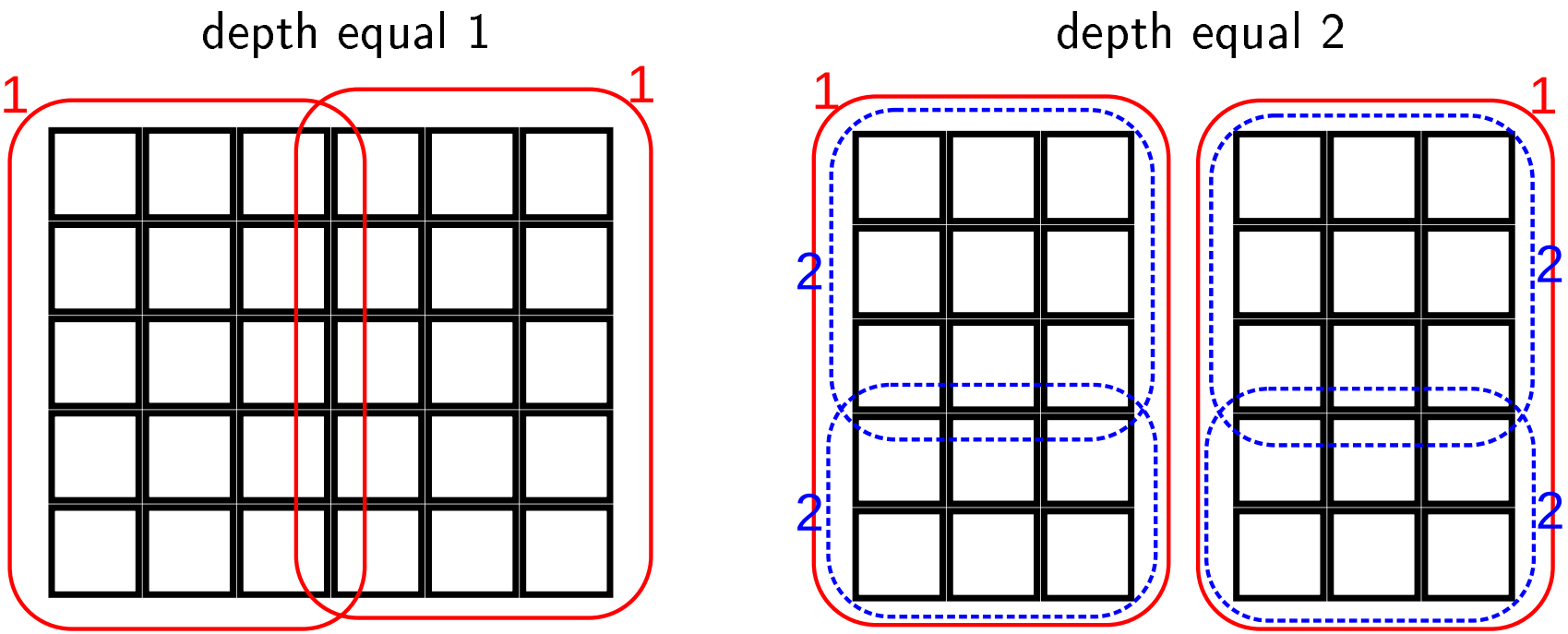
The configurations are prepared in two phases:

- confinement (41^4, Polyakov loop $= 0.0002 \pm 0.0006$)
- deconfinement ($4 * 30^3$, Polyakov loop $= 0.349 \pm 0.002$)

It has considered the closed surface:

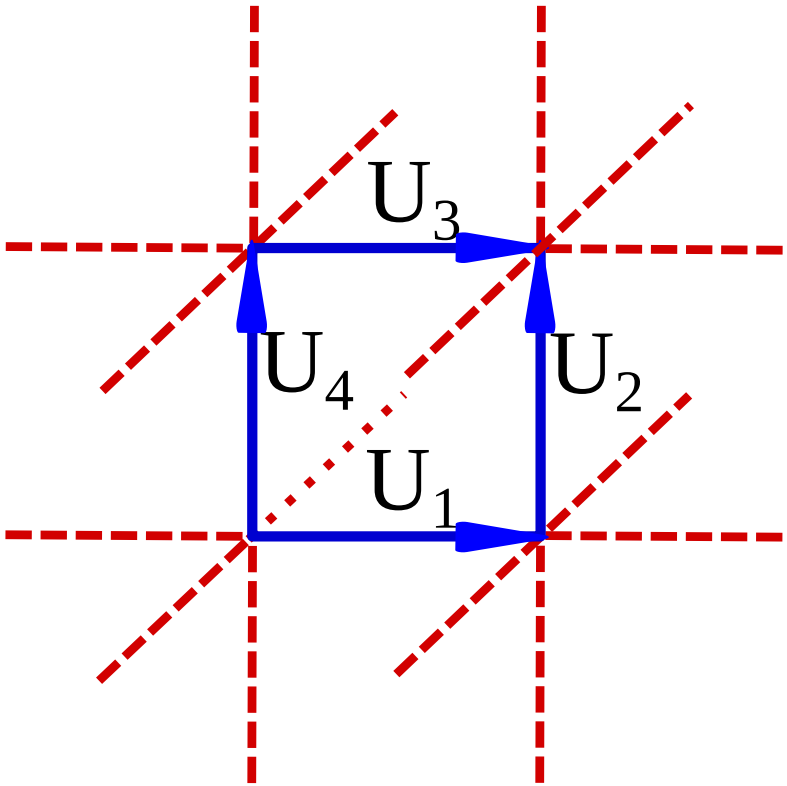
6 plaquettes (cube 1^3) $\cdots$ *1014 plaquettes* (cube 13^3)

Multilevel, hep-lat/0108014



Multilevel - *recursive algorithm, it averages the values from a lower level of recursion.*

$$A^{(d)} = \frac{\sum_i a_i^{(d)}}{N}, \qquad a_i^{(d)} = \{a^{(d)}\}, \qquad a^{(d)} = \sum_j A_j^{(d+1)}, \qquad A^{(d=d_{max})} = A$$



Multi-hit - *the studied value is determined by a bound conditions.*

- We can do it analytically
  $\Rightarrow$ *It uses at once*
- We can't do it analytically
  $\Rightarrow$ *It average with using the Monte-Carlo algorithm*

HPC with using MPI (Message Passing Interface)

The task of HPC: *The minimization of the calculation time!!!*

$$t = t_{writing} + t_{calc}$$

Parallelism is achieved by calculating Witten parameter at the same time in the different points on the lattice (I use 144 cores for each phase).

The two types of work

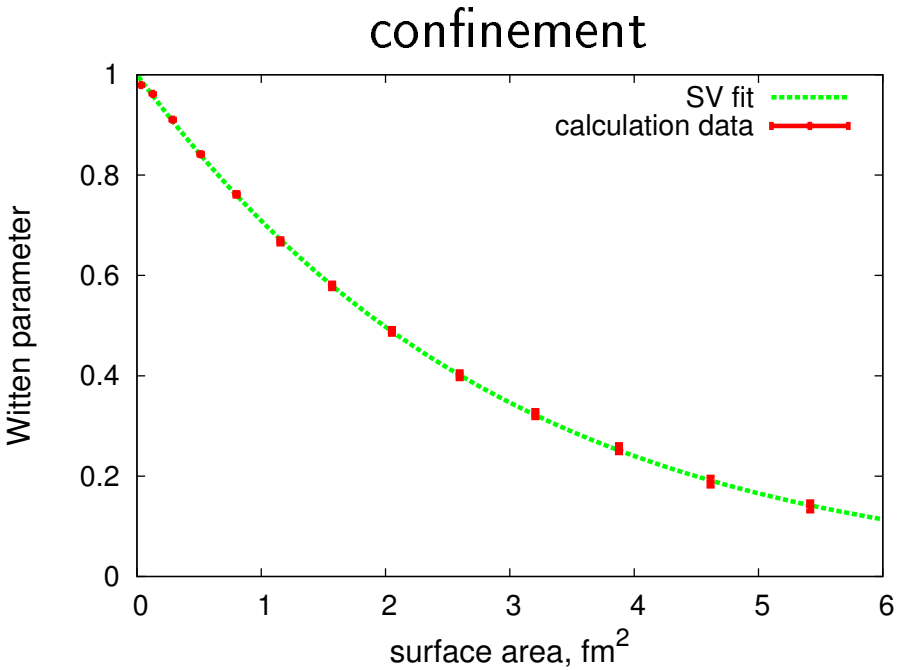
1. **The generation of configurations**
   - — Complex synchronization *(exchange of boundary conditions)*;
   - — The next configuration depend from current;
   - \+ We can save prepared configurations to use later;
   - \+ It is good for us to understand HPC.

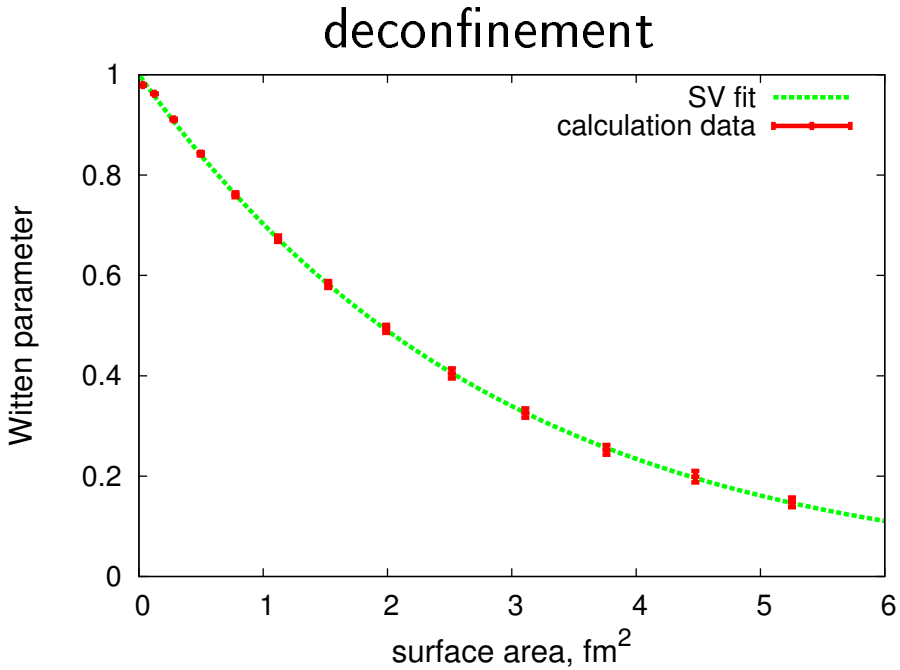
	with HPC	*without HPC*
write	*6 month*	*1 day*
generate	*1 day*	*1 weak*

2. **The averaging**
   - — It is a large load on the NFS;
   - \+ The minimum synchronization.

	with HPC	*without HPC*
write	*1 day*	*1 hour*
calculate	*1 weak*	*1 year*

The physical results



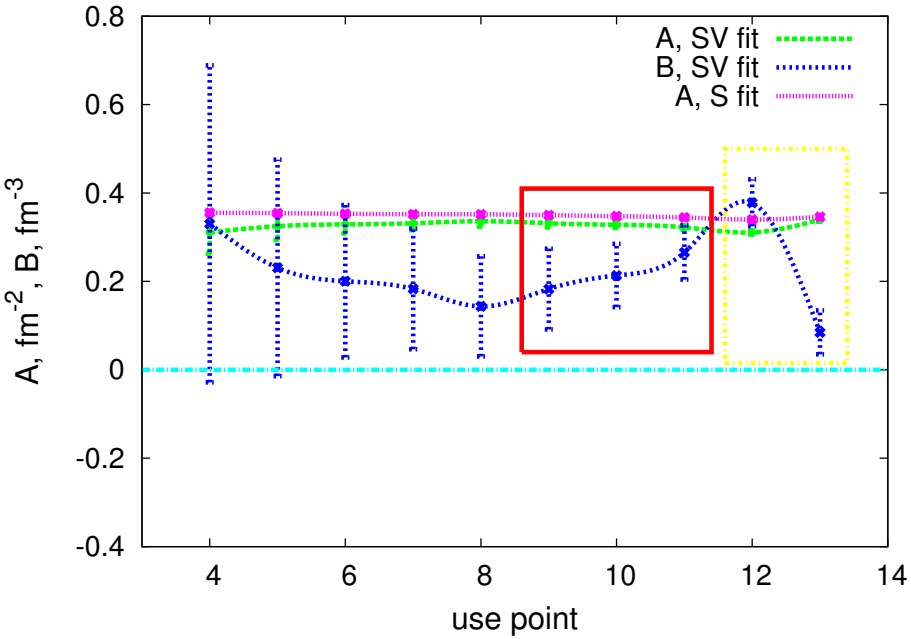


We see:

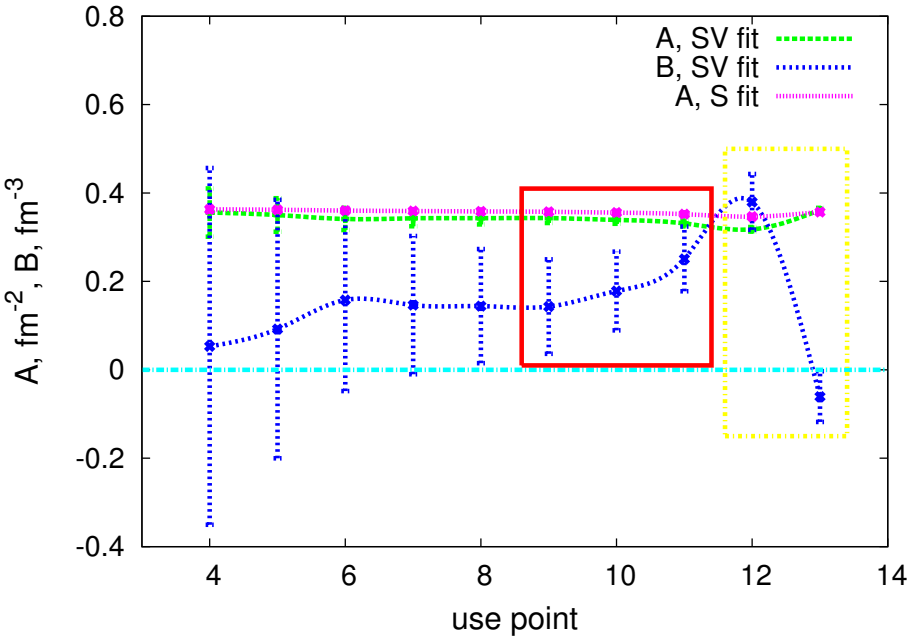
The exponential behavior $\Rightarrow$ Fit: $W(S, V) = e^{-AS-BV}$

Investigation of the dependency

confinement



deconfinement



The physical results:

- Witten parameter *depend from area surface and volume in two phases*.
- It *isn't the order parameter* for phase transition confinement-deconfinement.

The conclusions

1. **The physical results:**
   - Witten parameter *depends from area surface and volume in two phases*.
   - It *isn't the order parameter* for phase transition confinement-deconfinement.
2. **Multilevel algorithm is adapted for calculating a surface operators.**

Thank you for your attention!

The convergence of MLMH